Ex 1.3.1

cf réponses au cours

Ex 1.3.2

a) $\begin{pmatrix}1\\5\end{pmatrix} \sim \begin{pmatrix}-2\\m+4\end{pmatrix} \Leftrightarrow 1(m+4) - 5\cdot(-2) = 0$ (2^e critère)

$m+4+10 = 0$

$m+14 = 0 \Leftrightarrow \underline{m=-14}$

vérif : $\begin{pmatrix}1\\5\end{pmatrix} \sim \begin{pmatrix}-2\\-10\end{pmatrix}$ car $-2\cdot\begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}-2\\-10\end{pmatrix}$ ✓

Variante avec 1^e critère : $\begin{pmatrix}-2\\m+4\end{pmatrix} = k\begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}k\\5k\end{pmatrix} \Leftrightarrow \begin{cases}-2 = k\\ m+4 = 5k\end{cases}$ substitution $\Rightarrow m+4 = 5\cdot(-2)$

$\Leftrightarrow m+4 = -10$

$\Leftrightarrow m = -14$

b) $\begin{pmatrix}m\\m+4\end{pmatrix} \sim \begin{pmatrix}3\\m-1\end{pmatrix} \Leftrightarrow m(m-1) - (m+4)\cdot 3 = 0$

$m^2 - m - 3m - 12$

$m^2 - 4m - 12 = 0$

$(m+2)(m-6) = 0$

$\underline{m=-2}$ ou $\underline{m=6}$

vérif : si $m=6$: $\begin{pmatrix}6\\10\end{pmatrix} = 2\cdot\begin{pmatrix}3\\5\end{pmatrix}$ ✓ si $m=-2$: $\begin{pmatrix}-2\\2\end{pmatrix} = -\frac{2}{3}\cdot\begin{pmatrix}3\\-3\end{pmatrix}$ ✓

Variante avec 1^e critère : $\begin{pmatrix}m\\m+4\end{pmatrix} = k\begin{pmatrix}3\\m-1\end{pmatrix} \Leftrightarrow \begin{cases}m = 3k\\ m+4 = k(m-1)\end{cases}$ substitution $\Rightarrow 3k+4 = k(3k-1)$

$\Rightarrow 3k+4 = 3k^2 - k$

$0 = 3k^2 - 4k - 4$ $\Delta = (-4)^2 - 4\cdot 3\cdot(-4) = 64$

$\Rightarrow k_{1,2} = \frac{4\pm 8}{6} = \begin{cases}\frac{12}{6} = 2 & \Rightarrow m = 3\cdot 2 = 6\\ -\frac{4}{6} = -\frac{2}{3} & \Rightarrow m = 3\cdot\left(-\frac{2}{3}\right) = -2\end{cases}$

Ex 1.3.3

$\vec{x} \sim \vec{a} \iff$ il existe un nbre réel k tel que $\vec{x} = k\cdot\vec{a}$ (1e critère)

$\underbrace{\sim}_{\text{est colinéaire à}}$

$\vec{x} = k\begin{pmatrix}7\\-2\end{pmatrix}$

$\vec{x} = \begin{pmatrix}7k\\-2k\end{pmatrix}$

$\vec{x} + \lambda\vec{b} = \vec{c} \iff \begin{pmatrix}7k\\-2k\end{pmatrix} + \lambda\begin{pmatrix}-3\\5\end{pmatrix} = \begin{pmatrix}0\\5\end{pmatrix}$

$\iff \begin{pmatrix}7k\\-2k\end{pmatrix} + \begin{pmatrix}-3\lambda\\5\lambda\end{pmatrix} = \begin{pmatrix}0\\5\end{pmatrix}$

$\iff \begin{pmatrix}7k-3\lambda\\-2k+5\lambda\end{pmatrix} = \begin{pmatrix}0\\5\end{pmatrix}$

$\iff \begin{cases} 7k - 3\lambda = 0 \\ -2k + 5\lambda = 5 \end{cases} \left|\begin{matrix}5\\3\end{matrix}\right|\begin{matrix}2\\7\end{matrix}$

$\Rightarrow$

$$+\begin{array}{l} 35k - 15\lambda = 0 \\ -6k + 15\lambda = 15 \\ \hline 29k = 15 \end{array}$$

$$k = \frac{15}{29}$$

$$+\begin{array}{l} 14k - 6\lambda = 0 \\ -14k + 35\lambda = 35 \\ \hline 29\lambda = 35 \end{array}$$

$$\lambda = \frac{35}{29}$$

$$\Rightarrow \vec{x} = \frac{15}{29}\begin{pmatrix}7\\-2\end{pmatrix} = \begin{pmatrix}\frac{105}{29}\\-\frac{30}{29}\end{pmatrix}$$