Ex 1.3.7 Rem : 3 coords, on travaille dans l'espace (V_3)

a) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 8 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 8-5 \\ 0-2 \\ 5+3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$

b) $\vec{BD} = \vec{OD} - \vec{OB} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 4-8 \\ -6 \\ 3-5 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix}$

c) $\vec{CA} = \vec{OA} - \vec{OC} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 5+2 \\ 2+4 \\ -3-1 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ -4 \end{pmatrix}$

d) $\vec{AD} + \vec{CB} = \vec{OD} - \vec{OA} + \vec{OB} - \vec{OC} = \begin{pmatrix} 4-5 \\ -6-2 \\ 3+3 \end{pmatrix} + \begin{pmatrix} 8+2 \\ 0+4 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -1 \\ -8 \\ 6 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -4 \\ 10 \end{pmatrix}$

$\vec{DA} = -\vec{AD}$

e) $\vec{BC} - \vec{AC} + \vec{DB} = \vec{DB} + \vec{BC} + \vec{CA} \underset{\text{Chasles}}{=} \vec{DA} = \begin{pmatrix} 1 \\ 8 \\ -6 \end{pmatrix}$

variante :

$= \begin{pmatrix} -2-8 \\ -4-0 \\ 1-5 \end{pmatrix} - \begin{pmatrix} -2-5 \\ -4-2 \\ 1+3 \end{pmatrix} + \begin{pmatrix} 8-4 \\ 0+6 \\ 5-3 \end{pmatrix} = \begin{pmatrix} -10 \\ -4 \\ -4 \end{pmatrix} - \begin{pmatrix} -7 \\ -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ -6 \end{pmatrix}$

f) $4\vec{CD} - 3(\underbrace{\vec{CA} + \vec{BC}}_{\vec{BC}+\vec{CA}}) = 4\vec{CD} - 3\vec{BA} = 4\vec{CD} + 3\vec{AB}$

$= 4\begin{pmatrix} 4+2 \\ -6+4 \\ 3-1 \end{pmatrix} + 3\underbrace{\begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}}_{a)} = 4\begin{pmatrix} 6 \\ -2 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 24+9 \\ -8-6 \\ 8+24 \end{pmatrix}$

$= \begin{pmatrix} 33 \\ -14 \\ 32 \end{pmatrix}$

Ex 1.3.8 $\quad A(1;1) \quad B(10;5) \quad C(4;12)$

a) ABCD parallélogramme $\Leftrightarrow \vec{AB} = \vec{DC}$ et $\vec{AB} = \begin{pmatrix} 10-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

On cherche D :

$$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} - \vec{DC} = \vec{OC} - \vec{AB} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} - \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}$$

$\Rightarrow$ $D(-5;8)$

Variante : on pose $D(d_1, d_2)$

$$\vec{AB} = \vec{DC} \Leftrightarrow \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 4-d_1 \\ 12-d_2 \end{pmatrix} \Leftrightarrow \begin{cases} 9 = 4 - d_1 \\ 4 = 12 - d_2 \end{cases} \Leftrightarrow \begin{cases} d_1 = -5 \\ d_2 = 8 \end{cases}$$

b) ABDC //-gramme $\Leftrightarrow \vec{AB} = \vec{CD}$

On cherche D :

$$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} + \vec{AB} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} + \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \end{pmatrix}$$

$\Rightarrow$ $D(13;16)$

Variante : on pose $D(d_1, d_2)$

$$\vec{AB} = \vec{CD} \Leftrightarrow \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} d_1-4 \\ d_2-12 \end{pmatrix} \Leftrightarrow \begin{cases} 9 = d_1 - 4 \\ 4 = d_2 - 12 \end{cases} \Leftrightarrow \begin{cases} d_1 = 13 \\ d_2 = 16 \end{cases}$$

Ex 1.3.12

$$A, B \text{ et } C \text{ alignés} \iff \vec{AB} \sim \vec{BC}$$

a) $A(1;2)$ $B(-3;3)$ et $C(k;1)$

$\vec{AB} = \begin{pmatrix} -3-1 \\ 3-2 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$ et $\vec{BC} = \begin{pmatrix} k+3 \\ 1-3 \end{pmatrix} = \begin{pmatrix} k+3 \\ -2 \end{pmatrix}$

$\vec{AB} \sim \vec{BC} \iff \det(\vec{AB}; \vec{BC}) = 0$

$\iff \begin{vmatrix} -4 & k+3 \\ 1 & -2 \end{vmatrix} = 0$

$\iff -4 \cdot (-2) - 1 \cdot (k+3) = 0$

$\iff 8 - k - 3 = 0 \iff k = 5 \Rightarrow C(5;1)$

b) $A(2;k)$, $B(7k-29;5)$ et $C(-4;2)$

$\vec{AB} = \begin{pmatrix} 7k-29-2 \\ 5-k \end{pmatrix} = \begin{pmatrix} 7k-31 \\ 5-k \end{pmatrix}$ et $\vec{BC} = \begin{pmatrix} -4-(7k-29) \\ 2-5 \end{pmatrix} = \begin{pmatrix} -7k+25 \\ -3 \end{pmatrix}$

$\vec{AB} \sim \vec{BC} \iff \det(\vec{AB}; \vec{BC}) = 0$

$\iff \begin{vmatrix} 7k-31 & -7k+25 \\ 5-k & -3 \end{vmatrix} = 0$

$\iff (7k-31) \cdot (-3) - (5-k)(-7k+25) = 0$

$\iff -21k + 93 - (-35k + 125 + 7k^2 - 25k) = 0$

$\iff -21k + 93 + 35k - 125 - 7k^2 + 25k = 0$

$\iff -7k^2 + 39k - 32 = 0$

$\Delta = 39^2 - 4 \cdot (-7) \cdot (-32) = 625 \Rightarrow k_{1,2} = \dfrac{-39 \pm 25}{-14} = \begin{cases} \dfrac{-14}{-14} = 1 \\ \dfrac{-64}{-14} = \dfrac{32}{7} \end{cases}$

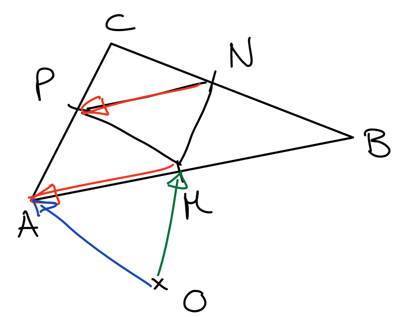
<u>Ex 1.3.14</u> $A(7;-3)$ $B(23;-6)$ et $C \in Ox \Leftrightarrow C(x;0)$

"appartient à"

$$\vec{AB} = \begin{pmatrix} 23-7 \\ -6+3 \end{pmatrix} = \begin{pmatrix} 16 \\ -3 \end{pmatrix} \quad \text{et} \quad \vec{AC} = \begin{pmatrix} x-7 \\ 0+3 \end{pmatrix} = \begin{pmatrix} x-7 \\ 3 \end{pmatrix}$$

$$\begin{aligned}
A, B \text{ et } C \text{ alignés} &\Leftrightarrow \vec{AB} \sim \vec{AC} \\
&\Leftrightarrow \det(AB; AC) = 0 \\
&\Leftrightarrow \begin{vmatrix} 16 & x-7 \\ -3 & 3 \end{vmatrix} = 0 \\
&\Leftrightarrow 16 \cdot 3 - (-3)(x-7) = 0 \\
&\Leftrightarrow 48 + 3(x-7) = 0 \\
&\Leftrightarrow 48 + 3x - 21 = 0 \\
&\Leftrightarrow 3x = -27 \Leftrightarrow x = -9 \Rightarrow \underline{C(-9;0)}
\end{aligned}$$

<u>Ex 1.3.18</u>



$*$ $\vec{OA} = \vec{OM} + \vec{MA} = \vec{OM} + \vec{NP}$

Calculons $\vec{NP} = \begin{pmatrix} -2-(-1) \\ 2-4 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$\Rightarrow \vec{OA} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow \underline{A(1;-3)}$

$*$ $\vec{OB} = \vec{OM} + \vec{MB} = \vec{OM} + \vec{PN} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \underline{B(3;1)}$

$*$ $\vec{OC} = \vec{OP} + \vec{PC} = \vec{OP} + \vec{MN}$

Calculons $\vec{MN} = \begin{pmatrix} -1-2 \\ 4-(-1) \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

$\Rightarrow \vec{OC} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \Rightarrow \underline{C(-5;7)}$

Ex 1.3.19 $A(-4;2)$ $B(1;3)$ $C(2;5)$

$$M_{AB}\left(\frac{-4+1}{2};\frac{2+3}{2}\right) = \underline{M_{AB}\left(\frac{-3}{2};\frac{5}{2}\right)}$$

$$M_{AC}\left(\frac{-4+2}{2};\frac{2+5}{2}\right) = \underline{M_{AC}\left(-1;\frac{7}{2}\right)}$$

$$M_{BC}\left(\frac{1+2}{2};\frac{3+5}{2}\right) = \underline{M_{BC}\left(\frac{3}{2};4\right)}$$

$$G_{ABC}\left(\frac{-4+1+2}{3};\frac{2+3+5}{3}\right) = \underline{G_{ABC}\left(-\frac{1}{3};\frac{10}{3}\right)}$$

Ex 1.3.21 $A(2;-1)$ $B(0;3)$

a) On cherche C tq. $O(0;0)$ est le centre de gravité du ΔABC

On peut poser $C(c_1;c_2)$

$$\left(\frac{2+0+c_1}{3};\frac{-1+3+c_2}{3}\right)=(0;0)$$

$$\Leftrightarrow \begin{cases}\frac{2+c_1}{3}=0\\ \frac{2+c_2}{3}=0\end{cases} \Leftrightarrow \begin{cases}2+c_1=0\\ 2+c_2=0\end{cases} \Leftrightarrow \begin{cases}c_1=-2\\ c_2=-2\end{cases} \Rightarrow C(-2;-2)$$

b) On cherche D tq ABCD //-gramme $\Leftrightarrow \vec{AB}=\vec{DC}$

$$\vec{AB}=\begin{pmatrix}0-2\\3+1\end{pmatrix}=\begin{pmatrix}-2\\4\end{pmatrix}$$

$$\vec{OD}=\vec{OC}+\vec{CD}=\vec{OC}-\vec{DC}=\vec{OC}-\vec{AB}=\begin{pmatrix}-2\\-2\end{pmatrix}-\begin{pmatrix}-2\\4\end{pmatrix}=\begin{pmatrix}0\\-6\end{pmatrix}\Rightarrow D(0;-6)$$

Variante : on pose $D(d_1,d_2)$

$$\vec{AB}=\vec{DC} \Leftrightarrow \begin{pmatrix}-2\\4\end{pmatrix}=\begin{pmatrix}-2-d_1\\-2-d_2\end{pmatrix}$$

$$\Leftrightarrow \begin{cases}-2-d_1=-2\\ -2-d_2=4\end{cases} \Leftrightarrow \begin{cases}d_1=0\\ d_2=-6\end{cases}$$