Dérivée

Rappels : 1)

$$(3)' = 0$$

$$(3x)' = 3$$

$$(3x^2)' = 6x$$

$$(3x^2+3x+3)' = 6x+3$$

$$\left[(x^2+2x+5)(3x^4-2x-3)\right]' = (2x+2)(3x^4-2x-3) + (x^2+2x+5)(12x^3-2)$$

$$(u\cdot v)' = u'v + uv'$$

$$\left(\frac{2x+1}{4x^2+x-1}\right)' = \frac{2(4x^2+x-1) - (2x+1)(8x+1)}{(4x^2+x-1)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

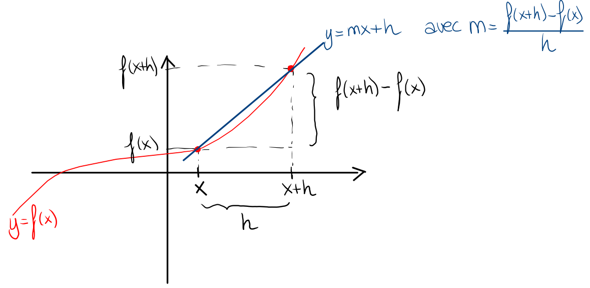
$$\left[(2x^3+4x)^5\right]' = 5(2x^3+4x)^4 \cdot (6x^2+4)$$

$$(u^n)' = n \cdot u^{n-1} \cdot u'$$

$$\left[\sqrt{x^2+1}\right]' = \left[(x^2+1)^{1/2}\right]' = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}$$

$$\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$$ (cas particulier de $(u^n)'$)

2) $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



pente de la tangente au graphe de f au point d'abscisse x

3) $e \cong 2{,}71\ldots$ nbre d'euler

$$e = \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = \lim_{x\to 0}(1+x)^{1/x} \quad ⊛$$

chgmt de variable : $x = \frac{1}{n} \Leftrightarrow n = \frac{1}{x}$

<u>Thm1</u>: $$\boxed{\lim_{x\to 0}\frac{e^x-1}{x} = 1}$$

preuve : $$\lim_{x\to 0}\frac{e^x-1}{x} = \lim_{y\to 0}\frac{y}{\ln(y+1)} = \lim_{y\to 0}\frac{1}{\frac{1}{y}\ln(1+y)}$$

$$= \lim_{y\to 0}\frac{1}{\ln\big[\underbrace{(1+y)^{1/y}}_{e\ ⊛}\big]}$$

$$= \frac{1}{\ln(e)} = \frac{1}{1} = 1$$

chgmt de variable

$y = e^x - 1 \Leftrightarrow y+1 = e^x$

$\Leftrightarrow x = \ln(y+1)$

$x\to 0,\ y \to e^0 - 1 = 0$

<u>Thm2</u> : $$\boxed{(e^x)' = e^x}$$

preuve: $$(e^x)' = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0}\frac{e^x e^h - e^x}{h}$$

$$= \lim_{h\to 0}\frac{e^x(e^h-1)}{h} = e^x\cdot\lim_{h\to 0}\frac{e^h-1}{h} \overset{\text{thm1}}{=} e^x\cdot 1 = e^x$$

$$\Rightarrow \boxed{\left(e^{u(x)}\right)' = e^{u(x)}\cdot u'(x)}$$

<u>Exples</u> 1) $f(x) = e^{x^2+1} \Rightarrow f'(x) = e^{x^2+1}\cdot 2x = 2xe^{x^2+1}$

2) $f(x) = e^{\frac{1}{x+1}} \Rightarrow f'(x) = e^{\frac{1}{x+1}}\cdot\left(-\frac{1}{(x+1)^2}\right) = -\frac{1}{(x+1)^2}e^{\frac{1}{x+1}}$

$$\left(\frac{1}{x+1}\right)' = \frac{-1}{(x+1)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \quad \text{avec} \quad u=1 \Rightarrow u'=0, \quad v = x+1 \Rightarrow v'=1$$

ex 2.3.1
2.3.2

Thm 3 : $\boxed{\left(\ln(x)\right)' = \frac{1}{x}}$

preuve : on utilise $e^{\ln(x)} = x$

et on dérive les deux côtés : $e^{\ln(x)} \cdot \left(\ln(x)\right)' = 1$

$\iff \quad x \cdot \left(\ln(x)\right)' = 1$

$\iff \quad \left(\ln(x)\right)' = \frac{1}{x}$

$$\Rightarrow \quad \boxed{\left(\ln\left(u(x)\right)\right)'} = \frac{1}{u(x)} \cdot u'(x) = \boxed{\frac{u'(x)}{u(x)}}$$

Exple : $\left(\ln\left(x^2-x-6\right)\right)' = \dfrac{2x-1}{x^2-x-6}$

ex 2.3.6