Ex 2.4.2

a) $\lim\limits_{x\to1}(4x^3-2x^2+x-1)=4-2+1-1=2$

b) $\lim\limits_{x\to-2}(x^2-5x)=4+10=14$

c) $\lim\limits_{x\to0}\dfrac{x+3x^2}{x+1}=\dfrac{0}{1}=0$

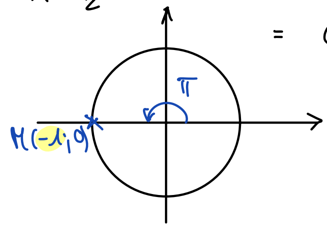
d) $\lim\limits_{x\to4}(-5)=-5$

e) $\lim\limits_{x\to3}\sqrt{x^2-5}=\sqrt{9-5}=2$

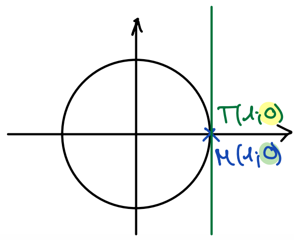
f) $\lim\limits_{x\to1}\dfrac{x^2+2x+1}{x^3+x^2+x}=\dfrac{1+2+1}{1+1+1}=\dfrac{4}{3}$

g) $\lim\limits_{x\to\frac{\pi}{2}}\cos\left(x+\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$

$=\cos(\pi)=-1$



h) $\lim\limits_{x\to0}\dfrac{\sin(x)+1}{2-\tan(x)}=\dfrac{\sin(0)+1}{2-\tan(0)}=\dfrac{0+1}{2-0}=\dfrac{1}{2}$



Ex 2.4.3

a) $\lim\limits_{x\to3}\dfrac{x-3}{2x-6}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to3}\dfrac{\overset{1}{\cancel{x-3}}}{2(\cancel{x-3})}=\lim\limits_{x\to3}\dfrac{1}{2}=\dfrac{1}{2}$

b) $\lim\limits_{x\to0}\dfrac{100x^2}{x}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to0}\dfrac{100x}{1}=0$

c) $\lim\limits_{x\to2}\dfrac{(x-2)(x+1)}{(x+4)(x-2)}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to2}\dfrac{x+1}{x+4}=\dfrac{3}{6}=\dfrac{1}{2}$

d) $\lim\limits_{x\to1}\dfrac{x^2-1}{x-1}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to1}\dfrac{(\cancel{x-1})(x+1)}{\cancel{x-1}}=\lim\limits_{x\to1}\dfrac{x+1}{1}=2$

e) $\lim\limits_{x\to5}\dfrac{x^2+2x-15}{x^2+8x+15}=\dfrac{20}{80}=\dfrac{1}{4}$

f) $\lim\limits_{x\to4}\dfrac{x^2-4x}{x^2-16}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to4}\dfrac{x(x-4)}{(x-4)(x+4)}=\lim\limits_{x\to4}\dfrac{x}{x+4}=\dfrac{4}{8}=\dfrac{1}{2}$

g) $\lim\limits_{x\to1}\dfrac{x^2-x}{x^3-4x^2-7x+10}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to1}\dfrac{x(\cancel{x-1})}{\underset{\cancel{(x-1)}}{\cancel{x}}(x^2-3x-10)}=\lim\limits_{x\to1}\dfrac{x}{x^2-3x-10}=-\dfrac{1}{12}$

$$\begin{array}{c|cccc}
 & 1 & -4 & -7 & 10 \\
1 & & 1 & -3 & -10 \\
\hline
 & 1 & -3 & -10 & 0
\end{array}$$

h) $\lim\limits_{x\to1}\dfrac{2x^3-3x^2+1}{x^2+3x-4}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to1}\dfrac{(\cancel{x-1})(2x^2-x-1)}{(x+4)(\cancel{x-1})}=\lim\limits_{x\to1}\dfrac{2x^2-x-1}{x+4}=\dfrac{0}{5}=0$

$$\begin{array}{c|cccc}
 & 2 & -3 & 0 & 1 \\
1 & & 2 & -1 & -1 \\
\hline
 & 2 & -1 & -1 & 0
\end{array}$$

Ex 2.4.14

a) $\lim\limits_{x\to -2} \dfrac{x^2+3x+6}{(x+2)^2} \overset{\text{"}\frac{4}{0_+}\text{"}}{=} +\infty$

b) $\lim\limits_{x\to -3} \dfrac{x^2+2x-15}{x^2+8x+15} \overset{\text{"}\frac{-12}{0}\text{"}}{=} \infty$

c) $\lim\limits_{x\to 0} \dfrac{x^2-3x}{x^3} \underset{f.i.}{\overset{\text{"}\frac{0}{0}\text{"}}{=}} \lim\limits_{x\to 0} \dfrac{x(x-3)}{x^3} = \lim\limits_{x\to 0} \dfrac{x-3}{x^2} \overset{\text{"}\frac{-3}{0_+}\text{"}}{=} -\infty$

d) $\lim\limits_{\substack{x\to 5 \\ >}} \dfrac{x-3}{5-x} \overset{\frac{2}{0_-}}{=} -\infty$

e) $\lim\limits_{x\to 1} (2x^2-5x+3)\dfrac{1}{x-1} \underset{f.i.}{\overset{\text{"}0\cdot\infty\text{"}}{=}} \lim\limits_{x\to 1} \dfrac{(2x-3)(x-1)}{x-1} = \lim\limits_{x\to 1} (2x-3) = -1$

$\Delta = 1 \Rightarrow x_{1,2} = \dfrac{5\pm 1}{4} = \begin{cases} 3/2 \\ 1 \end{cases}$

f) $\lim\limits_{x\to 1} \left(\dfrac{x^2}{x-1} - \dfrac{1}{x-1}\right) \underset{f.i.}{\overset{\text{"}\infty-\infty\text{"}}{=}} \lim\limits_{x\to 1} \dfrac{(x+1)(x-1)}{x-1} = \lim\limits_{x\to 1} (x+1) = 2$

g) $\lim\limits_{\substack{x\to -2 \\ <}} \dfrac{x-1}{x+2} \overset{\text{"}\frac{-3}{0_-}\text{"}}{=} +\infty$

h) $\lim\limits_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4}\right) \underset{f.i.}{\overset{\text{"}\infty-\infty\text{"}}{=}} \lim\limits_{x\to 2} \dfrac{x+2-4}{(x+2)(x-2)} = \lim\limits_{x\to 2} \dfrac{x-2}{(x+2)(x-2)}$

$= \lim\limits_{x\to 2} \dfrac{1}{x+2} = \dfrac{1}{4}$

Rem. $\lim\limits_{\substack{x\to 5 \\ >}} f(x)$ peut aussi être noté $\lim\limits_{x\to 5_+} f(x)$

$\lim\limits_{\substack{x\to -2 \\ <}} f(x)$ $\qquad \lim\limits_{x\to -2_-} f(x)$

Ex 2.4.16

a) $\lim\limits_{x\to\infty} \dfrac{2x-4}{-3x+1} = \lim\limits_{x\to\infty} \dfrac{2\cancel{x}}{-3\cancel{x}} = -\dfrac{2}{3}$

b) $\lim\limits_{x\to-\infty} \dfrac{-3x^2+1}{x+2} = \lim\limits_{x\to-\infty} \dfrac{-3x^2}{x} = \lim\limits_{x\to-\infty} -3x = -3\cdot(-\infty) = +\infty$

c) $\lim\limits_{x\to\infty} \dfrac{x^2+4x+49}{x^2-2x+4} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}^{\,1}}{\cancel{x^2}_{\,1}} = 1$

d) $\lim\limits_{x\to\infty} \dfrac{(3x+4)(x-1)}{(2x+7)(1-5x)} = \lim\limits_{x\to\infty} \dfrac{3x^2+\dots}{-10x^2+\dots} = \lim\limits_{x\to\infty} \dfrac{3\cancel{x^2}}{-10\cancel{x^2}} = -\dfrac{3}{10}$

e) $\lim\limits_{x\to\infty} \dfrac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8} = \lim\limits_{x\to\infty} \dfrac{(x^7+\dots)(16x^4+\dots)}{(8x^3+12x^2+\dots)(x^8+\dots)} = \lim\limits_{x\to\infty} \dfrac{\cancel{16}^{\,2}\cancel{x^{11}}}{\cancel{8}_{\,1}\cancel{x^{11}}} = 2$

f) $\lim\limits_{x\to\infty} \left(\dfrac{2x^2-1}{x-1} + 1 - 2x\right) = \lim\limits_{x\to\infty} \dfrac{2x^2-1}{x-1} + \dfrac{(1-2x)(x-1)}{x-1} = \lim\limits_{x\to\infty} \dfrac{2x^2-1-2x^2+3x-1}{x-1}$

$= \lim\limits_{x\to\infty} \dfrac{3\cancel{x}-2}{x-1} = \lim\limits_{x\to\infty} \dfrac{3\cancel{x}}{\cancel{x}} = 3$

g) $\lim\limits_{x\to+\infty} \left(\dfrac{2x-x^3}{3x+1} + x - 1\right) = \lim\limits_{x\to+\infty} \left(\dfrac{2x-x^3}{3x+1} + \dfrac{(x-1)(3x+1)}{3x+1}\right) = \lim\limits_{x\to+\infty} \dfrac{2x-x^3+3x^2-2x-1}{3x+1}$

$= \lim\limits_{x\to+\infty} \dfrac{-x^3}{3x} = \lim\limits_{x\to+\infty} \dfrac{-x^2}{3} = \dfrac{-(+\infty)^2}{3} = -\infty$

h) $\lim\limits_{x\to\infty} \left(\dfrac{1+5x-3x^2}{x-2} + 3x + 1\right) = \lim\limits_{x\to\infty} \left(\dfrac{1+5x-3x^2}{x-2} + \dfrac{(3x+1)(x-2)}{x-2}\right)$

$= \lim\limits_{x\to\infty} \dfrac{1+5x-3x^2+3x^2-5x-2}{x-2} = \lim\limits_{x\to\infty} \dfrac{-1}{x-2} = \text{“}\dfrac{-1}{\infty}\text{”} = 0$