

Exercice 6

Factoriser par la méthode des groupements :

$$a) x^3 - 2x^2 - 4x + 8 = x^2(x-2) - 4(x-2) = (x-2)(x^2-4) \stackrel{P.R.}{=} \underline{(x-2)^2(x+2)}$$

$$b) x^3 + 7x^2 + x + 7 = x^2(x+7) + 1(x+7) = \underline{(x+7)(x^2+1)}$$

$$c) x^4 - 4x^3 + 9x^2 - 36x = x^3(x-4) + 9x(x-4) = (x-4)(x^3+9x) \stackrel{Mee}{=} \underline{x(x-4)(x^2+9)}$$

$$d) x^5 + 3x^4 - 16x - 48 = x^4(x+3) - 16(x+3) = (x+3)(x^4-16) \stackrel{P.R.}{=} (x+3)(x^2+4)(x^2-4) \\ \stackrel{P.R.}{=} \underline{(x+3)(x^2+4)(x+2)(x-2)}$$

$$e) x^4 + 3x^3 - 8x - 24 = x^3(x+3) - 8(x+3) = (x+3)(x^3-8) \\ = \underline{(x+3)(x-2)(x^2+2x+4)}$$

$$f) 2x^3 + 4x^2 - 3x - 6 = 2x^2(x+2) - 3(x+2) = (x+2)(2x^2-3) \stackrel{P.R.}{=} \underline{(x+2)(\sqrt{2}x+\sqrt{3})(\sqrt{2}x-\sqrt{3})}$$

$$g) 2x^4 + 3x^3 + 2x^2 + 3x = x^3(2x+3) + x(2x+3) = (2x+3)(x^3+x) \stackrel{Mee}{=} \underline{x(2x+3)(x^2+1)}$$

$$h) 27x^4 - 54x^3 + 8x - 16 = 27x^3(x-2) + 8(x-2) = (x-2)(27x^3+8) \\ \stackrel{P.R.}{=} \underline{(x-2)(3x+2)(9x^2-6x+4)}$$

Exercice 7

Factoriser au maximum :

$$a) 2(x-3)(2x-5) - (2x-5)^2 \stackrel{Mee}{=} (2x-5)[2(x-3) - (2x-5)] = (2x-5)(2x-6-2x+5) \\ = \underline{-(2x-5)}$$

$$b) 36x^3 - 48x^2 + 16x \stackrel{Mee}{=} 4x(9x^2 - 12x + 4) \stackrel{P.R.}{=} \underline{4x(3x-2)^2}$$

$$c) 2x^2 + 22x - 24 \stackrel{Mee}{=} 2(x^2 + 11x - 12) \stackrel{SP}{=} \underline{2(x+12)(x-1)}$$

$$d) 3x^2 + x - 10 \stackrel{\Delta}{=} 3(x - \frac{5}{3})(x+2) = \underline{(3x-5)(x+2)} \quad \Delta = 1 - 4 \cdot 3 \cdot (-10) = 121 \\ x_{1,2} = \frac{-1 \pm 11}{6} = \begin{matrix} < -2 \\ \frac{5}{3} \end{matrix}$$

$$e) 5x^3 + 30x^2 + 60x + 40 \stackrel{Mee}{=} 5(x^3 + 6x^2 + 12x + 8) \stackrel{P.R.}{=} \underline{5(x+2)^3}$$

$$f) x^3 - x^2 - 4x + 4 \stackrel{GR}{=} x^2(x-1) - 4(x-1) = (x-1)(x^2-4) \stackrel{P.R.}{=} \underline{(x-1)(x+2)(x-2)}$$

$$g) x^6 + x^4 - x^2 - 1 \stackrel{GR}{=} x^4(x^2+1) - 1(x^2+1) = (x^2+1)(x^4-1) \stackrel{PR}{=} (x^2+1)^2(x^2-1) \stackrel{PR}{=} \underline{(x^2+1)^2(x+1)(x-1)}$$

$$h) x^6 + 7x^3 - 8 \stackrel{SP}{=} (x^3+8)(x^3-1) \stackrel{PR}{=} \underline{(x+2)(x^2-2x+4)(x-1)(x^2+x+1)}$$

$$i) 7x^2 + 63xy + 98y^2 \stackrel{Mee}{=} 7(x^2 + 9xy + 14y^2) \stackrel{SP}{=} \underline{7(x+7y)(x+2y)} \quad \left| \begin{array}{l} S: 9y \\ P: 14y^2 \end{array} \right.$$

$$j) 2000x^2 - 100x - 1200 \stackrel{Mee}{=} 100(20x^2 - x - 12) \stackrel{\Delta}{=} 100 \cdot 20 \left(x - \frac{1}{5}\right) \left(x + \frac{3}{4}\right) \quad \left| \begin{array}{l} \Delta = 1 - 4 \cdot 20 \cdot (-12) = 961 \\ x_{1,2} = \frac{1 \pm 31}{40} = \frac{41}{40} \quad \frac{15}{-34} \end{array} \right.$$

$$= \underline{100(5x-4)(4x+3)}$$

$$k) 3x^4 + 15x^2 - 42 \stackrel{Mee}{=} 3(x^4 + 5x^2 - 14) \stackrel{SP}{=} 3(x^2+7)(x^2-2) \stackrel{PR}{=} \underline{3(x^2+7)(x+\sqrt{2})(x-\sqrt{2})}$$

$$l) 2x^3 - 5x^2 - 8x + 20 \stackrel{Gr.}{=} x^2(2x-5) - 4(2x-5) = (2x-5)(x^2-4) \stackrel{PR}{=} \underline{(2x-5)(x+2)(x-2)}$$

$$m) -81x^4 + 3x \stackrel{Mee}{=} -3x(27x^3-1) \stackrel{PR}{=} \underline{-3x(3x-1)(9x^2+3x+1)}$$

$$n) x^4 - 6x^2 + 9 \stackrel{PR}{=} (x^2-3)^2 = \underline{(x+\sqrt{3})^2(x-\sqrt{3})^2}$$

$$o) -x^4 + x^3 + 8x - 8 \stackrel{GR}{=} x^3(-x+1) - 8(-x+1) = (-x+1)(x^3-8) \stackrel{PR}{=} \underline{(-x+1)(x-2)(x^2+2x+4)}$$

$$p) x^6 - x^4 - 12x^2 \stackrel{Mee}{=} x^2(x^4 - x^2 - 12) \stackrel{SP}{=} x^2(x^2-4)(x^2+3) \stackrel{PR}{=} \underline{x^2(x^2+3)(x+2)(x-2)}$$

$$q) (3x+5)(x^2-1) - (1-x^2)(2x-7) = (3x+5)(x^2-1) + (x^2-1)(2x-7) \stackrel{Mee}{=} (x^2-1)(3x+5+2x-7)$$

$$= (x^2-1)(5x-2) \stackrel{PR}{=} \underline{(x+1)(x-1)(5x-2)}$$

$$r) 5(x-1)^2 - 4(x^2-1) \stackrel{Mee}{=} \underbrace{(x-1)(x-1)}_{(x+1)(x-1)} \left[5(x-1) - 4(x+1) \right] = (x-1)(5x-5-4x-4) = \underline{(x-1)(x-9)}$$

$$s) 2x^6 - 6x^4 + 6x^2 - 2 \stackrel{Mee}{=} 2(x^6 - 3x^4 + 3x^2 - 1) \stackrel{PR}{=} 2(x^2-1)^3 \stackrel{PR}{=} \underline{2(x+1)^3(x-1)^3}$$

$$t) (x-3)^2 - (2x-5)^2 \stackrel{PR}{=} [(x-3)+(2x-5)][(x-3)-(2x-5)] = (3x-8)(x-3-2x+5)$$

$$= \underline{(3x-8)(-x+2)}$$

$$u) x^4 + 1 \stackrel{CC}{=} x^4 + 2x^2 + 1 - 2x^2 = (x^2+1)^2 - 2x^2$$

$$= [(x^2+1) + \sqrt{2}x][(x^2+1) - \sqrt{2}x] = \underline{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}$$

$$v) x^4 + x^2 + 1 \stackrel{CC}{=} x^4 + x^2 + x^2 + 1 - x^2$$

$$= x^4 + 2x^2 + 1 - x^2 = (x^2+1)^2 - x^2 = [(x^2+1)+x][(x^2+1)-x] = \underline{(x^2+x+1)(x^2-x+1)}$$