Dérivée des fonctions exponentielles et logarithme de base a

Soit $a \in \mathbb{R}_+^* - \{1\}$

$$\boxed{\text{Thm :} \quad (a^x)' = \ln(a) \cdot a^x}$$

preuve : on pose $a^x = e^{\ln(a^x)}$

$$\Rightarrow (a^x)' = \left(e^{\ln(a^x)}\right)' = (\ln(a^x))' \cdot e^{\ln(a^x)}$$

$$= (x \cdot \underbrace{\ln(a)}_{\text{nbre}})' \cdot e^{\ln(a^x)}$$

$$= \ln(a) \cdot a^x \qquad \#$$

Exple : $(5^x)' = \ln(5) \cdot 5^x$

$$\Rightarrow \boxed{\left(a^{u(x)}\right)' = \ln(a) \cdot a^{u(x)} \cdot u'(x)}$$

Exple : $\left(5^{x^2+x+1}\right)' = \ln(5) \cdot 5^{x^2+x+1} \cdot (2x+1)$

$$\left(e^{x^2+x+1}\right)' = e^{x^2+x+1} \cdot (2x+1)$$

$$\boxed{\text{Thm : } \left(\log_a(x)\right)' = \frac{1}{\ln(a)\cdot x}}$$

preuve : $\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

$$\Rightarrow \left(\log_a(x)\right)' = \left(\frac{\ln(x)}{\ln(a)}\right)' = \left(\underbrace{\frac{1}{\ln(a)}}_{\text{nbre}} \cdot \ln(x)\right)'$$

$$= \frac{1}{\ln(a)} \cdot \frac{1}{x} = \frac{1}{\ln(a)\, x} \qquad \#$$

Exple : $\left(\log_5(x)\right)' = \dfrac{1}{\ln(5)\, x}$

$$\Rightarrow \boxed{\left(\log_a(u(x))\right)'} = \frac{1}{\ln(a)\cdot u(x)} \cdot u'(x) = \boxed{\frac{u'(x)}{\ln(a)\, u(x)}}$$

Exple : $\left(\log_3(\sqrt{x})\right)' = \dfrac{\frac{1}{2\sqrt{x}}}{\ln(3)\cdot\sqrt{x}} = \dfrac{1}{2\sqrt{x}} \cdot \dfrac{1}{\ln(3)\cdot\sqrt{x}} = \dfrac{1}{2\ln(3)\, x}$

ex 2.3.14

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a) $f(x) = \log_2(2x+3)$ cond: $2x+3 > 0 \Leftrightarrow x > -\frac{3}{2}$

$ED(f) = \left]-\frac{3}{2}\,;\,+\infty\right[$

$$f'(x) = \frac{2}{(2x+3)\ln(2)}$$

b) $f(x) = \log_3(x^2-2x+1)$ cond: $x^2-2x+1 > 0$

$\underbrace{(x-1)^2}_{\text{tjrs positif}} > 0$

tjrs positif au nul si $x=1$

$ED(f) = \mathbb{R} - \{1\}$

$$f'(x) = \frac{2x-2}{(x^2-2x+1)\cdot\ln(3)} = \frac{2(x-1)}{(x-1)^2\ln(3)} = \frac{2}{(x-1)\ln(3)}$$

c) $f(x) = 3^{2x-4}$

$ED(f) = \mathbb{R}$

$$f'(x) = \ln(3)\cdot 3^{2x-4}\cdot 2 = \underline{\underline{2\ln(3)\,3^{2x-4}}}$$

$e^{(2x-4)\ln(3)}$

$= e^{\ln(3^{2x-4})} = 3^{2x-4}$

d) $f(x) = 2^{\sqrt{x^2+1}}$ cond: $x^2+1 \geq 0$ tjrs vrai

$ED(f) = \mathbb{R}$

$$f'(x) = \ln(2)\,2^{\sqrt{x^2+1}}\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \underline{\underline{\frac{\ln(2)\cdot x\cdot 2^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}}}$$