Asymptotes

AV : $\lim_{x \to a} f(x) = \infty$ $\Rightarrow$ AV : $x = a$

a v.i. se trouvant au milieu ou bord
de l'ED(f)

$\lim_{x \to a} f(x) = b$ $\Rightarrow$ "trou" $(a; b)$

AH : $\lim_{x \to +\infty} f(x) = c$ $\Rightarrow$ AHD : $y = c$

$\lim_{x \to -\infty} f(x) = c$ $\Rightarrow$ AHG : $y = c$

AO : $y = mx + h$

avec $m = \lim_{x \to \infty} \frac{f(x)}{x}$

$h = \lim_{x \to \infty} \left(f(x) - mx \right)$ (avec la distinction gauche, droite si $x \to +\infty$ ou si $x \to -\infty$)

Exples :

1) $f(x) = x^2 e^{-2x}$ $\qquad ED(f) = \mathbb{R}$

AV, trou : rien vu $ED(f)$

AH - AO : * $\lim\limits_{x \to +\infty} f(x) = +\infty \cdot e^{-\infty} = \text{"} +\infty \cdot 0 \text{"} = \lim\limits_{x \to +\infty} \frac{x^2}{e^{2x}} = \text{"}\frac{\infty}{\infty}\text{"}$

f.i.

$\overset{B.H.}{=} \lim\limits_{x \to +\infty} \frac{\cancel{2}x}{\cancel{2}e^{2x}} = \text{"}\frac{\infty}{\infty}\text{"} \overset{B.H.}{=} \lim\limits_{x \to +\infty} \frac{1}{2e^{2x}} = \frac{1}{\infty} = 0$

$\Longrightarrow$ AHD : $y = 0$

* $\lim\limits_{x \to -\infty} f(x) = +\infty e^{+\infty} = \text{"}\infty \cdot \infty\text{"} = \infty$ $\Longrightarrow$ pas d'AHG.

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{x^{\cancel{2}} e^{-2x}}{\cancel{x}} = \text{"}-\infty \cdot e^{+\infty}\text{"} = \text{"}-\infty \cdot (+\infty)\text{"} = -\infty$

$\Longrightarrow$ pas d'AOG

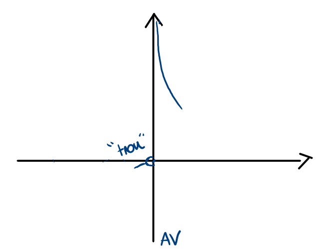
2) $f(x) = x \cdot e^{1/x}$ $\qquad ED(f) = \mathbb{R}^*$

<u>AV-trou</u> $*$ $\lim\limits_{x \to 0_+} f(x) = 0 \cdot e^{+\infty} = \underset{f.i.}{``0 \cdot \infty"} = \lim\limits_{x \to 0_+} \dfrac{e^{1/x}}{\frac{1}{x}}$

$$\overset{B.H.}{=} \lim_{x \to 0_+} \frac{e^{1/x} \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \lim_{x \to 0_+} e^{\frac{1}{x}} = e^{+\infty} = +\infty$$

$\Rightarrow$ AV : $x = 0_+$

$*$ $\lim\limits_{x \to 0_-} f(x) = 0 \cdot e^{-\infty} = 0 \cdot 0 = 0 \Rightarrow$ "trou" $(0_- ; 0)$



AH-AO : $*$ $\lim\limits_{x \to \pm\infty} x e^{\frac{1}{x}} = \pm\infty \cdot e^0 = \pm\infty$ $\qquad$ pas d'AH

$*$ $\lim\limits_{x \to \pm\infty} \dfrac{\cancel{x} e^{\frac{1}{x}}}{\cancel{x}} = e^0 = 1 = m$

$\lim\limits_{x \to \pm\infty} \left(x e^{\frac{1}{x}} - x\right) = \lim\limits_{x \to \pm\infty} x\left(e^{1/x} - 1\right) = \underset{f.i.}{``\pm\infty \cdot 0"}$

$$= \lim_{x \to \pm\infty} \frac{e^{1/x} - 1}{\frac{1}{x}} \underset{B.H}{``\frac{0}{0}"} \lim_{x \to \pm\infty} \frac{e^{1/x} \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = e^0 = 1 = h$$

$\Rightarrow$ AO : $y = x + 1$