

Etude de fonctions

1. ED(f)

2. Parité

3. Signe

4. Asymptotes

5. Croissance

6. Courbure

7. Graphe

Ex 2.3.19

c) $f(x) = \frac{\ln(x)}{x}$

1) $ED(f) = \mathbb{R}_+^*$

2) parité : $f(-x) = \frac{\ln(-x)}{-x}$ ni paire ni impaire / $ED(f)$ n'est pas sym. p.r. à zero.

3) zéros de f : $\ln(x) = 0 \Leftrightarrow x = 1$

signe :

x	0	1
$\text{sgn}(f)$		- 0 +

4) AV/hou : $\lim_{x \rightarrow 0_+} f(x) = \frac{\infty}{0} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = -\infty \Rightarrow AV : x = 0_+$

AH/AO : $m = \lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} = \lim_{x \rightarrow +\infty} \frac{\ln(x)}{x^2} \stackrel{\frac{\infty}{\infty}}{=} \lim_{x \rightarrow +\infty} \frac{1}{2x} = \lim_{x \rightarrow +\infty} \frac{1}{2x^2} = 0$

$h = \lim_{x \rightarrow +\infty} f(x) = \frac{\infty}{\infty} \stackrel{\text{B.H.}}{=} \lim_{x \rightarrow +\infty} \frac{1}{1} = \frac{1}{\infty} = 0$

\Rightarrow AHD : $y = 0$

5) Croissance :

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln(x) \cdot 1}{x^2} = \frac{1 - \ln(x)}{x^2}$

zéro de f' : $1 - \ln(x) = 0 \Leftrightarrow \ln(x) = 1 \Leftrightarrow x = e$

x	0	e
$\text{sgn}(f')$		+ 0 -
$\text{carac}(f)$		Max

$f'(1) : + \quad f'(3) : -$

$f(e) = \frac{\ln(e)}{e} = \frac{1}{e}$

$\Rightarrow \text{Max}(e; \frac{1}{e}) \sim (2,7; 0,4)$

6. Courbure

$$f'(x) = \frac{1 - \ln(x)}{x^2}$$

$$u = 1 - \ln(x) \quad v = x^2$$

$$u' = -\frac{1}{x} \quad v' = 2x$$

$$f''(x) = \frac{-\frac{1}{x} \cdot x^2 - (1 - \ln(x)) \cdot 2x}{x^4}$$

$$= \frac{-x - 2x(1 - \ln(x))}{x^4} = \frac{x(-1 - 2(1 - \ln(x)))}{x^{4-3}}$$

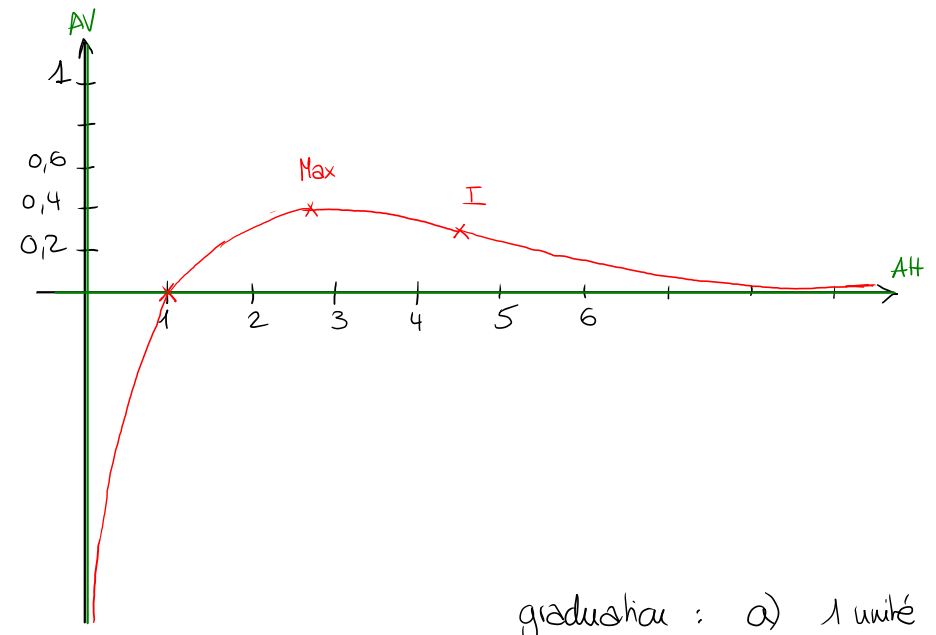
$$= \frac{-1 - 2 + 2\ln(x)}{x^3} = \frac{2\ln(x) - 3}{x^3}$$

zéro de f'' : $2\ln(x) - 3 = 0 \Leftrightarrow \ln(x) = \frac{3}{2} \Leftrightarrow x = e^{3/2} = \sqrt{e^3}$

x	0	$e^{3/2}$
sgn(f'')	/	- 0 +
carb.(f)	/	∩ I ∪

$$f(e^{3/2}) = \frac{\ln(e^{3/2})}{e^{3/2}} = \frac{3/2}{e^{3/2}} = \frac{3}{2\sqrt{e^3}}$$

$$\Rightarrow I(\sqrt{e^3}; \frac{3}{2\sqrt{e^3}}) \cong (4,5; 0,3)$$



gradushau : a) 1 unité 5 carrés
 b) 1u 2c.
 e) 1u 3c.
 f) 1u 2c.

ex 2.3.18 a) b) e) f)
 sans courbure

ex 2.3.19 a) e) f)
 sans courbure
 a) 1u 5c.
 e) 1u 2c.
 f) 1u 1c.