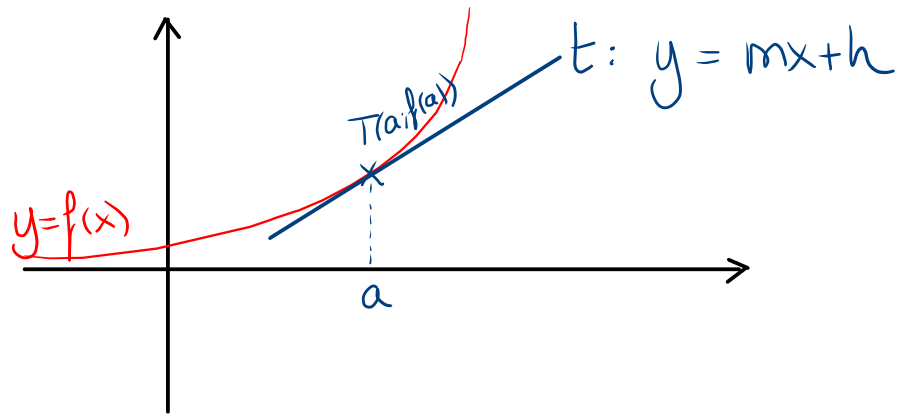


Tangente à une courbe (rappel)



$$m = f'(a) \quad \Rightarrow \quad t: y = f'(a)x + h$$

$$T(a; f(a)) \in t \Rightarrow f(a) = f'(a) \cdot a + h \Leftrightarrow h = f(a) - f'(a) \cdot a \quad \left. \vphantom{f(a)} \right\} \Rightarrow$$

$$t: y = f'(a)x + f(a) - f'(a) \cdot a$$

$$y = f'(a)(x-a) + f(a) \quad (\text{CRM})$$

$$y - f(a) = f'(a)(x-a) \quad (\text{Binet})$$

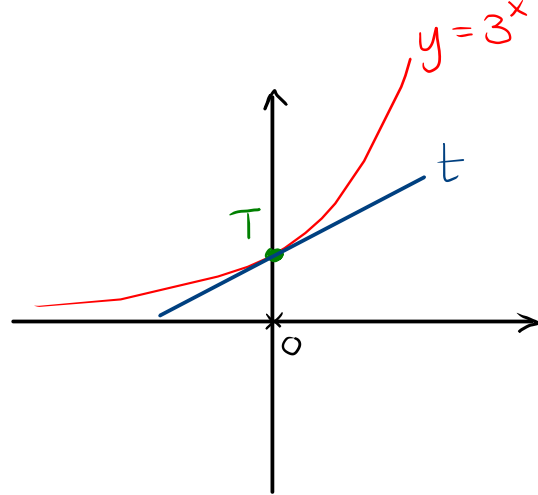
Ex 2.3.22

$$f(x) = 3^x$$

T intersection avec Oy

$$\Rightarrow T(0; 1)$$

$$f(0) = 3^0 = 1$$



$$t : y = mx + 1 \quad \text{car } T \in t$$

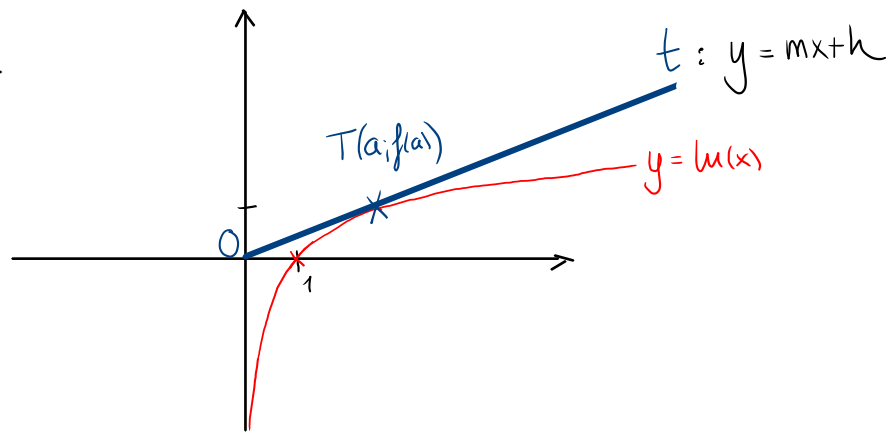
$$m = f'(0) = 3^0 \cdot \ln(3) = \ln(3)$$

$$\text{avec } f'(x) = 3^x \cdot \ln(3)$$

$$(a^x)' = a^x \cdot \ln(a)$$

$$\Rightarrow t : y = \ln(3)x + 1$$

Ex 2.3.23



$$0 \in t \Rightarrow y = mx$$

$$t: y - f(a) = f'(a)(x - a) \quad \text{avec} \quad f(a) = \ln(a)$$
$$f'(a) = \frac{1}{a} \quad \text{car} \quad f'(x) = \frac{1}{x}$$

$$y - \ln(a) = \frac{1}{a}(x - a)$$

$$t: y - \ln(a) = \frac{1}{a}x - 1$$

$$0 \in t \Rightarrow 0 - \ln(a) = \frac{1}{a} \cdot 0 - 1 \quad \Leftrightarrow \quad -\ln(a) = -1$$

$$\Leftrightarrow \ln(a) = 1$$

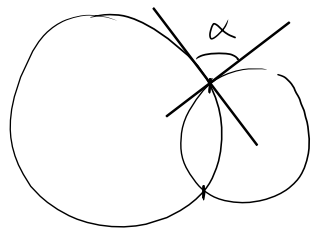
$$\Leftrightarrow a = e$$

$$\Rightarrow y - \underbrace{\ln(e)}_{=1} = \frac{1}{e}x - 1$$

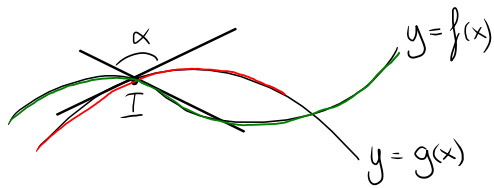
$$\Rightarrow t: y = \frac{1}{e}x$$

Angle entre deux courbes

Angle entre les tangentes à ces courbes
au point d'intersection.



Ex 2.3.30 / 33



Ex 2.3.30

$$f(x) = (x-1)e^x$$

$$g(x) = -e^{x-a} + b$$

$$I(1, ?) \begin{cases} \in y=f(x) \Rightarrow f(1) = 0 \cdot e = 0 \Rightarrow I(1, 0) \\ \in y=g(x) \Rightarrow I(1, 0) \in y=g(x) \quad \otimes \end{cases}$$

angle droit \Leftrightarrow tangentes sont $\perp \Leftrightarrow m_f \cdot m_g = -1$

$$m_f = f'(1) : f'(x) = e^x + (x-1)e^x \\ = e^x(1+x-1) \\ = xe^x \Rightarrow m_f = 1 \cdot e^1 = e$$

$$\otimes g(1) = 0 \Leftrightarrow -e^{1-a} + b = 0$$

$$m_g = g'(1) : g'(x) = -e^{x-a} \Rightarrow m_g = -e^{1-a}$$

$$\Rightarrow e \cdot (-e^{1-a}) = -1$$

$$-e^{1-a} = -\frac{1}{e}$$

$$e^{1-a} = \frac{1}{e} = e^{-1} \Leftrightarrow 1-a = -1 \Leftrightarrow a=2$$

$$\otimes \Rightarrow -e^{1-2} + b = 0 \Leftrightarrow -e^{-1} + b = 0 \Leftrightarrow b = e^{-1} = \frac{1}{e}$$