

Ex 2.2.1

a) → i) et k) l) voir réponses brochure

$$\begin{aligned} \text{j) } \underbrace{(2a+3b)(2x+y)}_{2 \text{ termes}} + \underbrace{(3a+5b)(2x+y)}_{2 \text{ termes}} &= (2x+y) \left[\underbrace{(2a+3b)}_{2 \text{ termes}} + \underbrace{(3a+5b)}_{2 \text{ termes}} \right] \\ &= \underline{(2x+y)(5a+8b)} \end{aligned}$$

$$\begin{aligned} \text{m) } \underbrace{(x-3)(x+1)}_{3 \text{ termes}} + \underbrace{2(x-3)^2}_{3 \text{ termes}} - \underbrace{(x-3)}_{3 \text{ termes}} &= (x-3) \left[\underbrace{(x+1)}_{3 \text{ termes}} + \underbrace{2(x-3)}_{3 \text{ termes}} - \underbrace{1}_{3 \text{ termes}} \right] \\ &= (x-3)(x+1+2x-6-1) \\ &= (x-3) \underbrace{(3x-6)}_{3(x-2)} = \underline{3(x-3)(x-2)} \end{aligned}$$

$$\begin{aligned} \text{n) } \underbrace{(u+v)^3}_{2 \text{ termes}} - \underbrace{(u+v)^2}_{2 \text{ termes}} &= (u+v)^2 \left[\underbrace{(u+v)}_{2 \text{ termes}} - \underbrace{1}_{2 \text{ termes}} \right] = (u+v)^2(u+v-1) \end{aligned}$$

$$\begin{aligned} \text{o) } \underbrace{2a(a-b)}_{2 \text{ termes}} - \underbrace{(a-b)^2}_{2 \text{ termes}} &= (a-b) \left[\underbrace{2a}_{2 \text{ termes}} - \underbrace{(a-b)}_{2 \text{ termes}} \right] = (a-b)(2a-a+b) \\ &= \underline{(a-b)(a+b)} \end{aligned}$$

Ex 2.2.2

a) \rightarrow c), m) \rightarrow r) voir réponses brochure

$$d) \underline{(a+b)^2 - x^2} = [(a+b)+x][(a+b)-x] = \underline{(a+b+x)(a+b-x)}$$

$$\begin{array}{l} A^2 - B^2 \\ \text{différence de 2 carrés} \end{array} = (A+B)(A-B)$$

$$e) \begin{array}{l} \underline{(ax+2y)^2 - (2x-3y)^2} \\ A^2 - B^2 \end{array} = [(ax+2y)+(2x-3y)][(ax+2y)-(2x-3y)] \\ = (ax+2x-y)(ax+2y-2x+3y) \\ = \underline{(ax+2x-y)(ax-2y+5y)}$$

$$f) \underline{(a-b)^2 - 1} = \underline{(a-b+1)(a-b-1)}$$
$$A^2 - B^2$$

$$g) 3a^2 - 3 \stackrel{\text{Hee}}{=} 3(a^2 - 1) \stackrel{\text{PR}}{=} \underline{3(a+1)(a-1)}$$

$$h) 4x^5y^2 - 9x^3 \stackrel{\text{Hee}}{=} x^3(4x^2y^2 - 9) \stackrel{\text{PR}}{=} \underline{x^3(2xy+3)(2xy-3)}$$

$$i) a^4 - b^4 \stackrel{\text{PR}}{=} (a^2 + b^2)(a^2 - b^2) \stackrel{\text{PR}}{=} \underline{(a^2 + b^2)(a+b)(a-b)}$$

$$j) a^5 - a \stackrel{\text{Hee}}{=} a(a^4 - 1) \stackrel{\text{PR}}{=} a(a^2 + 1)(a^2 - 1) \stackrel{\text{PR}}{=} \underline{a(a^2 + 1)(a+1)(a-1)}$$

$$k) \frac{u^4}{625} - \frac{v^4}{81} \stackrel{\text{PR}}{=} \left(\frac{u^2}{25} + \frac{v^2}{9}\right) \left(\frac{u^2}{25} - \frac{v^2}{9}\right)$$

$$\stackrel{\text{PR}}{=} \underline{\left(\frac{u^2}{25} + \frac{v^2}{9}\right) \left(\frac{u}{5} + \frac{v}{3}\right) \left(\frac{u}{5} - \frac{v}{3}\right)}$$

$$l) x^5y^4 - x \stackrel{\text{Hee}}{=} x(x^4y^4 - 1) \stackrel{\text{PR}}{=} x(x^2y^2 + 1)(x^2y^2 - 1) \\ \stackrel{\text{PR}}{=} \underline{x(x^2y^2 + 1)(xy + 1)(xy - 1)}$$

$$s) (a+b)^2 - 2(a+b)c + c^2 \stackrel{\text{PR}}{=} \underline{(a+b-c)^2}$$

$$A^2 - 2AB + B^2 = (A-B)^2$$

$$t) 5x^2 - 10x + 5 \stackrel{\text{Hee}}{=} 5(x^2 - 2x + 1) \stackrel{\text{PR}}{=} 5(x-1)^2$$

$$u) \underline{x^2(a+b) + 2(a+b)x + (a+b)} \stackrel{\text{Hee}}{=} (a+b)(\underline{x^2 + 2x + 1}) \stackrel{\text{PR}}{=} \underline{(a+b)(x+1)^2}$$

Ex 2.2.3 d) e) f) g) voir réponses brochure

Ex 2.2.4 sauf h) et o)

$$a) \quad x^2 + 5x + 6 = \underline{(x+2)(x+3)}$$

$\downarrow \quad \downarrow$
 $2+3 \quad 2 \cdot 3$

$$b) \quad x^2 + 5x + 4 = \underline{(x+1)(x+4)}$$

$\downarrow \quad \downarrow$
 $1+4 \quad 1 \cdot 4$

$$c) \quad u^2 - 6u + 8 = \underline{(x-2)(x-4)}$$

$\downarrow \quad \downarrow$
 $-2+(-4) \quad -2 \cdot (-4)$

$$d) \quad x^2 - 2x - 35 = \underline{(x-7)(x+5)}$$

$\downarrow \quad \downarrow$
 $-7+5 \quad -7 \cdot 5$

$$e) \quad 9x^2 + 6x + 1 \stackrel{\text{PR.}}{=} \underline{(3x+1)^2} \quad (\Delta=0)$$

$$f) \quad 4z^2 + 5z + 1 = 4\left(z + \frac{1}{4}\right)(z+1) = \underline{(4z+1)(z+1)}$$

$$\Delta = 25 - 4 \cdot 4 \cdot 1 = 9$$

$$z_{1,2} = \frac{-5 \pm 3}{8} = \begin{cases} -\frac{2}{8} = -\frac{1}{4} \\ -1 \end{cases}$$

$$g) \quad x^2 - 2x - 80 = \underline{(x-10)(x+8)}$$

$\downarrow \quad \downarrow$
 $-10+8 \quad -10 \cdot 8$

$$i) \quad 6x^2 + 5x + 1 = \overset{3 \cdot 2}{6} \left(x + \frac{1}{3}\right) \left(x + \frac{1}{2}\right) = \underline{(3x+1)(2x+1)}$$

$$\Delta = 25 - 4 \cdot 6 \cdot 1 = 1$$

$$x_{1,2} = \frac{-5 \pm 1}{12} = \begin{cases} -\frac{4}{12} = -\frac{1}{3} \\ -\frac{6}{12} = -\frac{1}{2} \end{cases}$$

$$j) \quad x^2 - 22x + 85 = \underline{(x-5)(x-17)}$$

avec SP: \downarrow \downarrow
 $-5-17$ $-5 \cdot (-17)$ et si on ne trouve pas avec SP

$$\text{avec } \Delta : \Delta = 22^2 - 4 \cdot 1 \cdot 85 = 144$$

$$x_{1,2} = \frac{22 \pm 12}{2} = \begin{cases} 17 \\ 5 \end{cases}$$

$$k) \quad x^2 + x + 1$$

avec SP on ne trouve pas donc on calcule $\Delta = 1 - 4 \cdot 1 \cdot 1 = -3 < 0$
 et on peut conclure que le polynôme n'est pas factorisable.

$$l) \quad 16u^2 - 72u + 81 \stackrel{\text{PR}}{=} \underline{(4u-9)^2} \quad (\Delta=0)$$

$$m) \quad 40x^2 + 3x - 28 = 40 \left(x - \frac{4}{5}\right) \left(x + \frac{7}{8}\right) = \underline{(5x-4)(8x+7)}$$

$$\Delta = 9 - 4 \cdot 40 \cdot (-28) = 4489$$

$$x_{1,2} = \frac{-3 \pm 67}{80} = \begin{cases} 64/80 = 8/10 = 4/5 \\ -70/80 = -7/8 \end{cases}$$

$$n) \quad a^2 + 9a - 10 = \underline{(a+10)(a-1)}$$

\downarrow \downarrow
 $10-1$ $-1 \cdot 10$

$$p) \quad 4m^2 + 25m - 21 = 4 \left(m - \frac{3}{4}\right) (m+7) = \underline{(4m-3)(m+7)}$$

$$\Delta = 25^2 - 4 \cdot 4 \cdot (-21) = 961$$

$$x_{1,2} = \frac{-25 \pm 31}{8} = \begin{cases} 6/8 = 3/4 \\ -7 \end{cases}$$

Ex 2.2.5

a) $x^4 - 13x^2 + 36 = (x^2)^2 - 13x^2 + 36$ chgmt de variable

$$\begin{aligned} \underset{y=x^2}{\Rightarrow} \quad y^2 - 13y + 36 &\stackrel{SP}{=} (y-4)(y-9) \end{aligned}$$

$$\underset{y=x^2}{\Rightarrow} \quad (x^2-4)(x^2-9) \stackrel{PR}{=} \underline{(x+2)(x-2)(x+3)(x-3)}$$

b) $a^6 + 19a^3 - 216 = (a^3)^2 + 19a^3 - 216$ chgmt de variable

$$\underset{y=a^3}{\Rightarrow} \quad y^2 + 19y - 216 \stackrel{SP}{=} (y-8)(y+27)$$

$$\underline{a} \quad \Delta = 19^2 - 4 \cdot 1 \cdot (-216) = 1225 = 35^2$$

$$\Rightarrow y_{1,2} = \frac{-19 \pm 35}{2} = \left\langle \begin{array}{l} -27 \\ 8 \end{array} \right. \Rightarrow (y+27)(y-8)$$

$$\underset{y=a^3}{\Rightarrow} \quad (a^3-8)(a^3+27) \stackrel{PR}{=} \underline{(a-2)(a^2+2a+4)(a+3)(a^2-3a+9)}$$

c) $x^8 - 257x^4 + 256 = (x^4)^2 - 257x^4 + 256$ chgmt de variable

$$\underset{y=x^4}{\Rightarrow} \quad y^2 - 257y + 256 \stackrel{SP}{=} (y-256)(y-1)$$

$$\underset{y=x^4}{\Rightarrow} \quad (x^4-256)(x^4-1) \stackrel{PR}{=} (x^2+16)(x^2-16)(x^2+1)(x^2-1)$$

$$\stackrel{PR}{=} \underline{(x^2+16)(x+4)(x-4)(x^2+1)(x+1)(x-1)}$$

d) $7x^4 - 61x^2 - 18 = 7(x^2)^2 - 61x^2 - 18$ chgmt de variable

$$\underset{y=x^2}{\Rightarrow} \quad 7y^2 - 61y - 18$$

$$\Delta = 61^2 - 4 \cdot 7 \cdot (-18) = 4225 = 65^2$$

$$y_{1,2} = \frac{61 \pm 65}{14} = \left\langle \begin{array}{l} \frac{126}{14} = 9 \\ -\frac{4}{14} = -\frac{2}{7} \end{array} \right. \Rightarrow 7(y-9)(y+\frac{2}{7}) = (y-9)(7y+2)$$

$$\underset{y=x^2}{\Rightarrow} \quad (x^2-9)(7x^2+2) \stackrel{PR}{=} \underline{(x+3)(x-3)(7x^2+2)}$$

Ex 2.2.6

$$a) \underline{ax+bx} + \underline{ay+by} = x(a+b) + y(a+b) = \underline{(a+b)(x+y)}$$

$$b) \underline{a+b} + \underline{ax+bx} + \underline{ay+by} = 1(a+b) + x(a+b) + y(a+b) = \underline{(a+b)(1+x+y)}$$

$$\text{ou } \underline{a+b+ax+bx+ay+by} = a(1+x+y) + b(1+x+y) = (1+x+y)(a+b)$$

$$c) \underline{ax-bx} - \underline{ay+by} = x(a-b) - y(a-b) = \underline{(a-b)(x-y)}$$

$$d) \underline{ax-4x} + \underline{4y-ay} = x(a-4) + y(4-a) = x(a-4) - y(-4+a) \\ = x(a-4) - y(a-4) = \underline{(a-4)(x-y)}$$

$$e) \underline{ax+x} - \underline{a-1} = x(a+1) - 1(a+1) = \underline{(a+1)(x-1)}$$

$$f) \underline{x^3+x} - \underline{x^2-1} = x(x^2+1) - 1(x^2+1) = \underline{(x^2+1)(x-1)}$$

↑
pas factorisable (somme de deux carrés)

$$g) \underline{\frac{xy}{2} - \frac{x}{4}} + \underline{\frac{yz}{3} - \frac{z}{6}} = \frac{x}{2} \left(y - \frac{1}{2}\right) + \frac{z}{3} \left(y - \frac{1}{2}\right) = \underline{\left(y - \frac{1}{2}\right) \left(\frac{x}{2} + \frac{z}{3}\right)}$$

$$h) \underline{10xz - 10z} - \underline{x^2+x} = 10z(x-1) - x(x-1) = \underline{(x-1)(10z-x)}$$

$$i) \underline{a^2-2ab+b^2} - \underline{1} = (a-b)^2 - 1 \stackrel{\text{PR}}{=} [(a-b)+1][(a-b)-1] = \underline{(a-b+1)(a-b-1)}$$

$$j) \underline{4x^2+2x-9y^2-3y} = (4x^2-9y^2) + (2x-3y) \stackrel{\text{PR}}{=} (2x+3y)(2x-3y) + (2x-3y) \\ = \underline{(2x-3y)(2x+3y+1)}$$

$$k) \underline{1+x+x^2} + \underline{x^3+x^4+x^5} = 1(1+x+x^2) + x^3(1+x+x^2) = (1+x+x^2)(1+x^3) \\ \stackrel{\text{PR}}{=} \underline{(1+x+x^2)(1+x)(1-x+x^2)}$$

$$\left(\text{ou } \underline{1+x+x^2+x^3+x^4+x^5} = (1+x) + x^2(1+x) + x^4(1+x) = \underline{(1+x)(1+x^2+x^4)} \right)$$

↓
pas fini mais on n'a pas la méthode

$$l) \quad \underline{8y^4 - 8y^3} + \underline{y-1} = 8y^3(y-1) + 1(y-1) = (y-1)(8y^3+1) \stackrel{PR}{=} \underline{(y-1)(2y+1)(4y^2-2y+1)}$$

$$m) \quad \underline{x^3+x} - \underline{x^2-1} = x(x^2+1) - 1(x^2+1) = \underline{(x^2+1)(x-1)}$$

$$n) \quad \underline{2a^4-3-2a^3+3a} = a(2a^3+3) - 1(2a^3+3) = \underline{(2a^3+3)(a-1)}$$

$$o) \quad \underline{6x^2+xy} + \underline{18xz+3yz} = x(6x+y) + 3z(6x+y) = \underline{(6x+y)(x+3z)}$$

