

Ex 2.4.1

$$a) \frac{54a^3b^3}{15a^5b^2} = \frac{18b}{5a^2}$$

$$b) \frac{-16u^2v^2w^3}{-4u^3vw^2} = \frac{4vw}{u}$$

$$c) \frac{x-1}{2x-2} = \frac{\cancel{x-1}}{2(\cancel{x-1})} = \frac{1}{2}$$

$$d) \frac{2x-2y}{3y-3x} = \frac{2(x-y)}{3(y-x)} = \frac{2(\cancel{x-y})}{-3(\cancel{x-y})} = -\frac{2}{3}$$

$$e) \frac{a^2-b^2}{(a-b)^2} = \frac{(a+b)(a-b)}{(a-b)^2} = \frac{a+b}{a-b}$$

$$f) \frac{x^2-16}{x^2-5x+4} = \frac{(x+4)(x-4)}{(x-4)(x-1)} = \frac{x+4}{x-1}$$

$$g) \frac{x-x^3}{x^4+2x^3+x^2} = \frac{x(1-x^2)}{x^2(x^2+2x+1)} = \frac{\cancel{x}(1-x)(1+x)}{x^2(x+1)^2} = \frac{1-x}{x(x+1)}$$

$$h) \frac{3z^2-21z+36}{2z^2-12z+18} = \frac{3(z^2-7z+12)}{2(z^2-6z+9)} = \frac{3(z-4)(z-3)}{2(z-3)^2} = \frac{3(z-4)}{2(z-3)}$$

$$i) \frac{x^3-15x^2+75x-125}{x^2-25} = \frac{(x-5)^3}{(x+5)(x-5)} = \frac{(x-5)^2}{x+5}$$

$$j) \frac{x^4-y^4}{x^5-x^3y^2} = \frac{(x^2+y^2)(x^2-y^2)}{x^3(x^2-y^2)} = \frac{x^2+y^2}{x^3}$$

$$k) \frac{10x^2-10xy}{5x^2y^2-5x^4} = \frac{10x(x-y)}{5x^2(y^2-x^2)} = \frac{10x(x-y)}{5x^2(y+x)(y-x)} = \frac{2}{-x(y+x)} = -\frac{2}{x(x+y)}$$

$$l) \frac{6x^2+2x}{27x^3+1} = \frac{2x(3x+1)}{(3x+1)(9x^2-3x+1)} = \frac{2x}{9x^2-3x+1}$$

$$m) \frac{1-x^2+x^3-x^5}{x+x^2-x^3-x^4} = \frac{(1-x^2)+x^3(1-x^2)}{x(1+x-x^2-x^3)} = \frac{(1-x^2)(1+x^3)}{x[(1+x)-x^2(1+x)]} = \frac{\cancel{(1+x)}\cancel{(1-x)}(1+x)(1-x+x^2)}{x(1+x)(1-x^2)}$$

$$= \frac{1-x+x^2}{x}$$

$$n) \frac{x^3+x^2-x-1}{x^3+2x^2-x-2} = \frac{x^2(x+1)-(x+1)}{x^2(x+2)-(x+2)} = \frac{(x+1)(x^2-1)}{(x+2)(x^2-1)} = \frac{x+1}{x+2}$$

$$o) \frac{2x^3+9x^2+7x-6}{2x^3+x^2-13x+6}$$

$$= \frac{(x+2)(2x^2+5x-3)}{(x-2)(2x^2+5x-3)} = \frac{x+2}{x-2}$$

$$N(-2) = -16 + 36 - 14 - 6 = 0$$

$$\begin{array}{r|rrrr} -2 & 2 & 9 & 7 & -6 \\ & & -4 & -10 & 6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$D(2) = 16 + 4 - 26 + 6$$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

Ex 2.4.2

$$a) \frac{a+7}{a-1} \cdot \frac{a^2-1}{2a+14} = \frac{(a+7)(a+1)(a-1)}{(a-1) \cdot 2(a+7)} = \frac{a+1}{2}$$

$$b) \frac{x+5}{7} : \frac{2x+10}{x-8} = \frac{x+5}{7} \cdot \frac{x-8}{2(x+5)} = \frac{x-8}{14}$$

$$c) (x+y) : \frac{x+y}{x-y} = \frac{x+y}{1} \cdot \frac{x-y}{x+y} = \frac{(x+y)(x-y)}{x+y} = x-y$$

$$d) \frac{z^2+z}{z-1} \cdot \frac{z-z^2}{z^3} = \frac{z(z+1) \cdot z(1-z)}{(z-1) \cdot z^3} = \frac{-z^2(z+1)(z-1)}{z^3(z-1)} = -\frac{(z+1)}{z}$$

$$e) \frac{x+2}{2x-3} : \frac{x^2-4}{2x^2-3x} = \frac{x+2}{2x-3} \cdot \frac{x(2x-3)}{(x+2)(x-2)} = \frac{x}{x-2}$$

$$f) \frac{9x^2-4}{3x^2-5x+2} \cdot \frac{9x^4-6x^3+4x^2}{27x^4+8x} = \frac{(3x+2)(3x-2)}{(3x-2)(x-1)} \cdot \frac{x^2(9x^2-6x+4)}{x(3x+2)(9x^2-6x+4)} = \frac{x}{x-1}$$

$$g) \frac{x^2-6x+9}{x^2-1} \cdot \frac{2x-2}{x-3} = \frac{(x-3)^2 \cdot 2(x-1)}{(x+1)(x-1)(x-3)} = \frac{2(x-3)}{x+1}$$

$$h) \frac{6x^2-5x-6}{x^2-4} : \frac{2x^2-3x}{x+2} = \frac{(3x+2)(2x-3)}{(x+2)(x-2)} \cdot \frac{x+2}{x(2x-3)} = \frac{3x+2}{x(x-2)}$$

Ex 2.4.3

$$a) \frac{x}{x+3} + \frac{x+6}{x+3} = \frac{x+x+6}{x+3} = \frac{2x+6}{x+3} = \frac{2(x+3)}{x+3} = \underline{2}$$

$$b) \frac{x}{x+3} - \frac{x+6}{x+3} = \frac{x-(x+6)}{x+3} = \underline{\underline{-6}}$$

$$c) \frac{6}{x^2-4} - \frac{3x}{x^2-4} = \frac{6-3x}{x^2-4} = \frac{-3(2+x)}{(x+2)(x-2)} = \underline{\underline{\frac{-3}{x+2}}}$$

$$d) \frac{2}{3x+1} + \frac{9}{(3x+1)^2} = \frac{2(3x+1)}{(3x+1)^2} + \frac{9}{(3x+1)^2} = \frac{6x+2+9}{(3x+1)^2} = \underline{\underline{\frac{6x+11}{(3x+1)^2}}}$$

$$e) \frac{5}{a} - \frac{2a-1}{a^2} + \frac{a+5}{a^3} = \frac{5a^2}{a^3} - \frac{a(2a-1)}{a^3} + \frac{a+5}{a^3} = \frac{5a^2 - a(2a-1) + a+5}{a^3}$$

$$= \frac{5a^2 - 2a^2 + a + a + 5}{a^3} = \underline{\underline{\frac{3a^2 + 2a + 5}{a^3}}}$$

$$f) \frac{x}{x+1} + \frac{1}{x-1} - \frac{2x}{x^2-1} = \frac{x(x-1)}{(x+1)(x-1)} + \frac{x+1}{(x+1)(x-1)} - \frac{2x}{(x+1)(x-1)}$$

$$= \frac{x^2 - x + x + 1 - 2x}{(x+1)(x-1)} = \frac{x^2 - 2x + 1}{(x+1)(x-1)} = \frac{(x-1)^2}{(x+1)(x-1)} = \underline{\underline{\frac{x-1}{x+1}}}$$

$$g) \frac{x-3}{x+3} - \frac{2x}{x^2+5x+6} = \frac{(x-3)(x+2)}{(x+3)(x+2)} - \frac{2x}{(x+3)(x+2)} = \frac{x^2-x-6-2x}{(x+3)(x+2)} = \frac{x^2-3x-6}{(x+3)(x+2)} \quad \Delta=33$$

$$h) \frac{1}{m} - \frac{m}{m^2-1} - \frac{2m+1}{m-m^3} = \frac{m^2-1}{m(m+1)(m-1)} - \frac{m^2}{m(m+1)(m-1)} + \frac{2m+1}{m(m+1)(m-1)}$$

ppmc : $m(m+1)(m-1)$

$$= \frac{m^2-1-m^2+2m+1}{m(m+1)(m-1)} = \underline{\underline{\frac{2m}{m(m+1)(m-1)}}}$$

$$\begin{aligned}
 \text{i) } \frac{2y+1}{\underbrace{y^2+4y+4}_{(y+2)^2}} - \frac{6y}{\underbrace{y^2-4}_{(y+2)(y-2)}} + \frac{3}{y-2} &= \frac{(2y+1)(y-2)}{(y+2)^2(y-2)} - \frac{6y(y+2)}{(y+2)^2(y-2)} + \frac{3(y+2)^2}{(y+2)^2(y-2)} \\
 &= \frac{2y^2-4y+y-2 - (6y^2+12y) + 3(y^2+4y+4)}{(y+2)^2(y-2)} \\
 \text{ppmc: } (y+2)^2(y-2) & \quad \Bigg| \\
 &= \frac{2y^2-4y+y-2 - 6y^2-12y + 3y^2+12y+12}{(y+2)^2(y-2)} \\
 &= \frac{-y^2-3y+10}{(y+2)^2(y-2)} = \frac{-(y+5)(y-2)}{(y+2)^2(y-2)} = \underline{\underline{-\frac{y+5}{(y+2)^2}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{j) } \frac{13-5x}{\underbrace{6x^2-6}_{6(x+1)(x-1)}} + \frac{3x}{x+1} - \frac{3x-5}{\underbrace{3x-3}_{3(x-1)}} &= \frac{13-5x + 18x(x-1) - 2(3x-5)(x+1)}{6(x+1)(x-1)} \\
 &= \frac{13-5x + 18x^2-18x - 6x^2+4x+10}{6(x+1)(x-1)} \\
 &= \underline{\underline{\frac{12x^2-19x+23}{6(x+1)(x-1)}}}
 \end{aligned}$$

Ex 2.4.4

$$\begin{aligned} d) \frac{4}{\underbrace{x^2-y^2}_{(x+y)(x-y)}} + \frac{3y}{\underbrace{x^2y-x^3}_{-x^2(-y+x) \\ -x^2(x-y)}} - \frac{x-3y}{\underbrace{x^3-xy^2}_{x(x+y)(x-y)}} &= \frac{-4x^2 + 3y(x+y) + x(x-3y)}{-x^2(x+y)(x-y)} \\ &= \frac{-4x^2 + 3xy + 3y^2 + x^2 - 3xy}{-x^2(x+y)(x-y)} \\ &= \frac{-3x^2 + 3y^2}{-x^2(x+y)(x-y)} = \frac{-3(x^2-y^2)}{-x^2(x+y)(x-y)} = \underline{\underline{\frac{3}{x^2}}} \end{aligned}$$

$$\begin{aligned} f) \frac{x-3}{x+3} - \frac{4x-6y}{\underbrace{xy+3y+2x+6}_{y(x+3)+2(x+3) \\ (x+3)(y+2)}} + \frac{y+6}{y+2} &= \frac{(x-3)(y+2) - (4x-6y) + (y+6)(x+3)}{(x+3)(y+2)} \\ &= \frac{\underline{xy} + \underline{2x} - \underline{3y} - 6 - \underline{4x} + \underline{6y} + \underline{xy} + \underline{3y} + \underline{6x} + 18}{(x+3)(y+2)} \\ &= \frac{4x + 6y + 2xy + 12}{(x+3)(y+2)} = \frac{2(2x + 3y + xy + 6)}{(x+3)(y+2)} \\ &= \frac{2(x+3)(y+2)}{(x+3)(y+2)} = \underline{\underline{2}} \end{aligned}$$

2.4.5 Effectuer et réduire:

$$\begin{aligned}
 \text{a)} \quad \left(\frac{z+2}{z} - \frac{2}{\underbrace{z^2+z}_{z(z+1)}} \right) \left(\frac{1}{z} + 1 \right) &= \frac{(z+2)(z+1)-2}{z(z+1)} \cdot \frac{1+z}{z} \\
 &= \frac{z^2+3z}{z(z+1)} \cdot \frac{1+z}{z} = \frac{\cancel{z}(z+3)}{\cancel{z}(z+1)} \cdot \frac{\cancel{z+1}}{z} = \frac{z+3}{z}
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad \left(\frac{1}{u} - \frac{2}{u^2} - \frac{3}{u^3} \right) \div \left(\frac{1}{u^2} - 1 \right) &= \frac{u^2-2u-3}{u^3} \div \frac{1-u^2}{u^2} \\
 &= \frac{(u-3)\cancel{(u+1)}}{\cancel{u^3}u} \cdot \frac{\cancel{u^2}}{\cancel{(1+u)}(1-u)} = \frac{u-3}{u(1-u)}
 \end{aligned}$$

$$\begin{aligned}
 \text{d)} \quad \frac{2x-1}{x+y} \cdot \frac{x^2-y^2}{4x^2-1} \cdot \left(1 - \frac{2x}{2x-1} + \frac{1}{2x+1} \right) \\
 &= \frac{2x-1}{x+y} \cdot \frac{(x+y)(x-y)}{(2x+1)(2x-1)} \cdot \frac{(2x+1)(2x-1) - 2x(2x+1) + (2x-1)}{(2x-1)(2x+1)} \\
 &= \frac{2x-1}{x+y} \cdot \frac{(x+y)(x-y)}{(2x+1)(2x-1)} \cdot \frac{4x^2-1 - 4x^2 - 2x + 2x - 1}{(2x-1)(2x+1)} \\
 &= \frac{\cancel{2x-1}}{\cancel{x+y}} \cdot \frac{\cancel{(x+y)}(x-y)}{\cancel{(2x+1)}\cancel{(2x-1)}} \cdot \frac{-2}{(2x-1)(2x+1)} = \frac{-2(x-y)}{(2x+1)^2(2x-1)}
 \end{aligned}$$