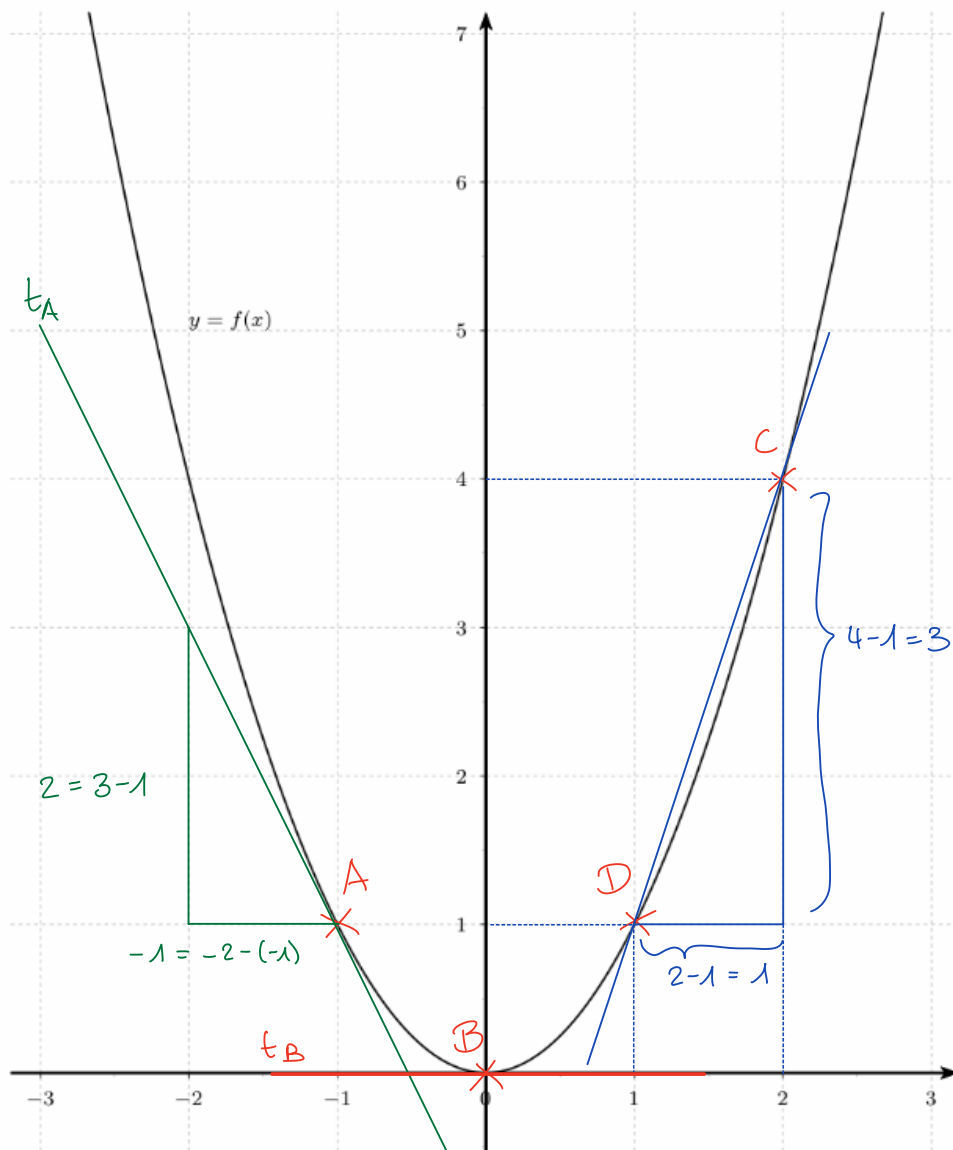


2.7.1 On donne une représentation du graphique de la fonction $f(x) = x^2$.



Soit les points $A(-1; 1)$, $B(0; 0)$, $C(2; 4)$ et $D(1; 1)$.

- Déterminer la pente de la droite qui passe par les points D et C .
- Déterminer graphiquement la pente de la tangente à la courbe $y = f(x)$ aux points $A(-1; 1)$, $B(0; 0)$, $C(2; 4)$ et D .
- Donner ensuite une équation de la tangente en ces points.

$$a) \quad m_{DC} = \frac{4-1}{2-1} = \frac{3}{1} = 3$$

$$b) \quad m_A = \frac{2}{-1} = -2 \quad m_B = 0 \quad m_C = 4 \quad m_D = 2$$

$$c) \quad t_A: \quad y = -2x - 1$$

↙
↘

pente
ordonnée à l'origine

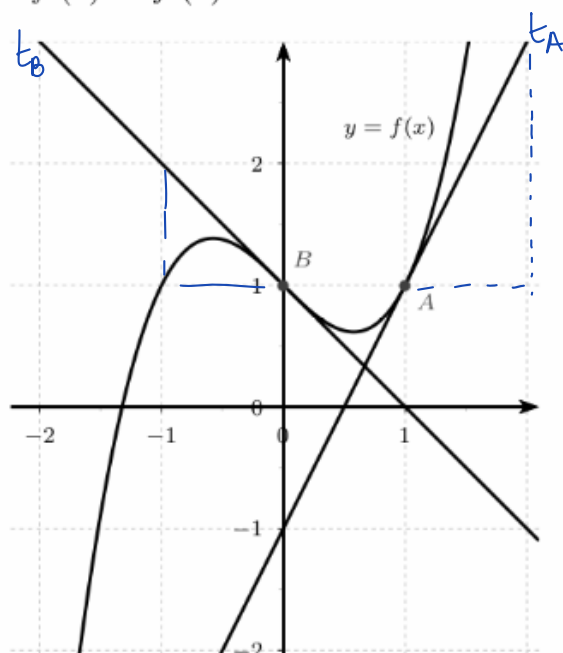
$$t_C: y = 4x - 4$$

$$t_D: y = 2x - 1$$

$$t_B: y = 0$$

2.7.2 Voici la représentation graphique d'une fonction $f(x)$. Les tangentes en A et B sont représentées.

Déterminer graphiquement $f'(0)$ et $f'(1)$.



$$f'(0) = m_B = -1$$

$$f'(1) = m_A = \frac{2}{1} = 2$$

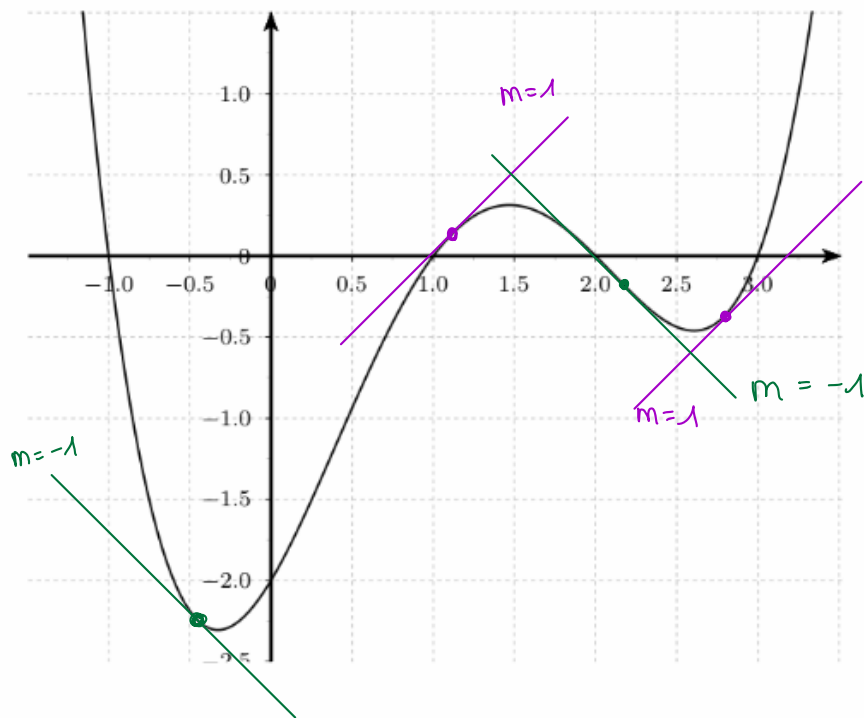
2.7.3 Sur le graphe de la fonction $f(x)$ ci-dessous, indiquer les valeurs approximatives de x pour lesquelles :

a) $f(x) = 0$

b) $f'(x) = 0$

c) $f'(x) = 1$

d) $f'(x) = -1$



a) $x \approx -0.4$
 $x \approx 1.1$
 $x \approx 2.8$

zeros def.

b) $x \approx -0.3$
 $x \approx 1.4$
 $x \approx 2.6$

extrema def.

c) $x \approx 1.1$
 $x \approx 2.6$

d) $x \approx -0.4$
 $x \approx 2.2$

Ex 2.7.12

$$f(x) = -x^2 + x + 2$$

a) $f'(x) = -2x + 1$

b) • coupe l'axe Oy : ord. à l'O. : $f(0) = 2 \Rightarrow A(0; 2)$

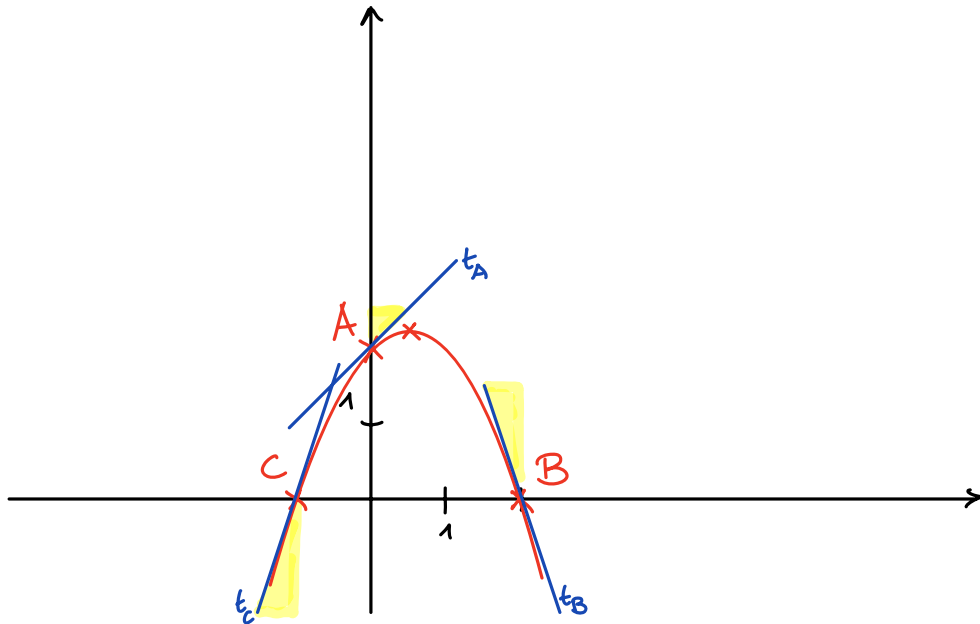
$\Rightarrow$ pente : $m_A = f'(0) = -2 \cdot 0 + 1 = \underline{1}$

• coupe l'axe Ox : zéros de f : $f(x) = 0 \Leftrightarrow -x^2 + x + 2 = 0$
 $\Leftrightarrow -(x^2 - x - 2) = 0$
 $\Leftrightarrow -(x-2)(x+1) = 0$
 $\begin{matrix} \downarrow & \downarrow \\ 2 & -1 \end{matrix}$
 $\Rightarrow B(2; 0) \text{ et } C(-1; 0)$

$\Rightarrow$ pente : $m_B = f'(2) = -2 \cdot 2 + 1 = \underline{-3}$

$m_C = f'(-1) = -2 \cdot (-1) + 1 = \underline{3}$

c)



sommet : $x = \frac{-1}{-2} = \frac{1}{2}$

$$y = f\left(\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + 2 = \frac{9}{4}$$

Ex 2.7.17

a) $f(x) = 47$ $f'(x) = 0$

b) $f(x) = 3x$ $f'(x) = 3$

c) $f(x) = x^5$ $f'(x) = 5x^4$

d) $f(x) = 8x^7$ $f'(x) = 56x^6$

e) $f(x) = 5x^0 = 5 \Rightarrow f'(x) = 0$

f) $f(x) = \frac{1}{3}x^3$ $f'(x) = \frac{1}{3} \cdot 3x^2 = x^2$

g) $f(x) = x^3 + x^2 + x + 1$

$f'(x) = 3x^2 + 2x + 1$

h) $f(x) = 7x^4 - 3x + 8$

$f'(x) = 28x^3 - 3$

i) $f(x) = x^2 + 5x - 6$

$f'(x) = 2x + 5$

j) $f(x) = x^3 + 5x^2 - 2x + 4$

$f'(x) = 3x^2 + 10x - 2$

k) $f(x) = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 2x + 4$

$f'(x) = \frac{2}{3} \cdot 3x^2 - \frac{5}{2} \cdot 2x + 2 = 2x^2 - 5x + 2$

l) $f(x) = 2x^5 - \frac{7}{6}x^3 + \frac{3}{4}x^2 - x + \sqrt{2}$

$f'(x) = 10x^4 - \frac{7}{6} \cdot 3x^2 + \frac{3}{4} \cdot 2x - 1 + 0$

$= 10x^4 - \frac{7}{2}x^2 + \frac{3}{2}x - 1$

Ex 2.7.18

$$a) \left((x+1)(x-3) \right)' = 1 \cdot (x-3) + 1 \cdot (x+1) = x-3+x+1 = \underline{2x-2}$$

$$\begin{array}{ll} u = x+1 & v = x-3 \\ u' = 1 & v' = 1 \end{array}$$

$$\text{variante: } (x^2 - 3x + x - 3)' = (x^2 - 2x - 3)' = 2x - 2$$

$$b) \left(x(x^2+5) \right)' = 1 \cdot (x^2+5) + x \cdot 2x = x^2+5+2x^2 = \underline{3x^2+5}$$

$$\begin{array}{ll} u = x & v = x^2+5 \\ u' = 1 & v' = 2x \end{array}$$

$$\text{variante: } (x^3+5x)' = 3x^2+5$$

$$c) \left((7x^2-4x+3)(5-2x) \right)' = (14x-4)(5-2x) + (7x^2-4x+3) \cdot (-2)$$

$$\begin{array}{ll} u = 7x^2-4x+3 & v = 5-2x \\ u' = 14x-4 & v' = -2 \end{array} \quad \left| \right.$$

$$= 70x - 28x^2 - 20 + 8x - 14x^2 + 8x - 6$$

$$= \underline{-42x^2 + 86x - 26}$$

$$\begin{aligned} d) \left((2x-1)(2-2x)(1+x) \right)' &= 2(2-2x)(1+x) + (2x-1) \cdot (-2)(1+x) + (2x-1)(2-2x) \cdot 1 \\ &= (4-4x)(1+x) + (2x-1)(-2-2x) + 4x-4x^2-2+2x \\ &= \underbrace{4+4x-4x-4x^2}_{-4x^2} - \underbrace{4x-4x^2}_{-4x^2} + \underbrace{2+2x}_{2x} + \underbrace{4x-4x^2}_{-4x^2} - \underbrace{2+2x}_{2x} \\ &= \underline{-12x^2 + 4x + 4} \end{aligned}$$

$$e) \left(\frac{4-3x}{2x-1} \right)' = \frac{-3(2x-1) - (4-3x) \cdot 2}{(2x-1)^2} = \frac{-6x+3-8+6x}{(2x-1)^2} = \underline{\underline{\frac{-5}{(2x-1)^2}}}$$

$$\begin{array}{ll} u = 4-3x & v = 2x-1 \\ u' = -3 & v' = 2 \end{array} \quad \left| \right.$$

$$f) \left(\frac{x-2}{3-x} \right)' = \frac{1(3-x) - (x-2) \cdot (-1)}{(3-x)^2} = \frac{3-x+x-2}{(3-x)^2} = \frac{1}{(3-x)^2}$$

$$g) \left(\frac{5}{2x^2-1} \right)' = 5 \cdot \left(\frac{1}{2x^2-1} \right)' = 5 \cdot \frac{-4x}{(2x^2-1)^2} = \underline{\underline{-\frac{20x}{(2x^2-1)^2}}}$$

$$\begin{array}{l} v = 2x^2-1 \\ v' = 4x \end{array}$$

$$\begin{aligned}
 h) \left(\frac{x^3 - 10x^2}{1-x} \right)' &= \frac{(3x^2 - 20x)(1-x) - (x^3 - 10x^2)(-1)}{(1-x)^2} \\
 &= \frac{3x^2 - 3x^3 - 20x + 20x^2 + x^3 - 10x^2}{(1-x)^2} \\
 &= \frac{-2x^3 + 13x^2 - 20x}{(1-x)^2}
 \end{aligned}$$

$$\begin{aligned}
 u &= x^3 - 10x^2 \\
 u' &= 3x^2 - 20x
 \end{aligned}$$

$$\begin{aligned}
 v &= 1-x \\
 v' &= -1
 \end{aligned}$$

$$\begin{aligned}
 i) \left(\frac{8x^2 - 8x + 3}{4x^2 - 1} \right)' & \quad \begin{aligned} u &= 8x^2 - 8x + 3 \\ u' &= 16x - 8 \end{aligned} \quad \begin{aligned} v &= 4x^2 - 1 \\ v' &= 8x \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(16x-8)(4x^2-1) - (8x^2-8x+3) \cdot 8x}{(4x^2-1)^2} \\
 &= \frac{64x^3 - 16x - 32x^2 + 8 - 64x^3 + 64x^2 - 24x}{(4x^2-1)^2} = \frac{32x^2 - 40x + 8}{(4x^2-1)^2}
 \end{aligned}$$

$$j) \left(\frac{x^3}{x+1} \right)' = \frac{3x^2(x+1) - x^3 \cdot 1}{(x+1)^2} = \frac{3x^3 + 3x^2 - x^3}{(x+1)^2} = \frac{2x^3 + 3x^2}{(x+1)^2}$$

$$k) \left(1 + \frac{1}{x} - \frac{2}{x^2} \right)' = 0 - \frac{1}{x^2} - 2 \cdot \left(-\frac{2}{x^4} \right) = -\frac{1}{x^2} + \frac{4}{x^3} = \frac{-x+4}{x^3}$$

$$\begin{aligned}
 l) \left(\frac{x^3-4}{3x} + x \right)' &= \frac{3x^2 \cdot 3x - (x^3-4) \cdot 3}{9x^2} + 1 = \frac{9x^3 - 3x^3 + 12}{9x^2} + 1 \\
 &= \frac{6x^3 + 12}{9x^2} + \frac{9x^2}{9x^2} = \frac{\overbrace{6x^3 + 9x^2 + 12}^{3(2x^3 + 3x^2 + 4)}}{9x^2} = \frac{2x^3 + 3x^2 + 4}{3x^2}
 \end{aligned}$$

$$\begin{aligned}
 m) \quad \left(\frac{x(x+5)}{x^2+x} \right)' &= \left(\frac{x^2+5x}{x^2+x} \right)' = \frac{(2x+5)(x^2+x) - (x^2+5x)(2x+1)}{(x^2+x)^2} \\
 &= \frac{2x^3 + \underline{2x^2} + \underline{5x^2} + 5x - 2x^3 - \underline{x^2} - \underline{10x^2} - 5x}{(x^2+x)^2} \\
 &= \frac{-4x^2}{(x^2+x)^2} = \frac{-4x^2}{x^2(x+1)^2} = \underline{\underline{\frac{-4}{(x+1)^2}}}
 \end{aligned}$$

$$\begin{aligned}
 n) \quad \left(\left(x + \frac{2}{x^2} \right) x \right)' & \quad \text{(erreur rép. brochure)} \quad \left| \begin{array}{ll} u = x + \frac{2}{x^2} & v = x \\ u' = 1 + 2 \cdot \left(-\frac{2x}{x^4} \right) & v' = 1 \\ & = 1 - \frac{4}{x^3} \end{array} \right. \\
 &= \left(1 - \frac{4}{x^3} \right) x + \left(x + \frac{2}{x^2} \right) \cdot 1 \\
 &= x - \frac{4}{x^2} + x + \frac{2}{x^2} = -\frac{2}{x^2}
 \end{aligned}$$

$$o) \quad \left(\frac{2x+3(x^2-1)}{3} \right)' = \frac{1}{3} (2x+3x^2-3)' = \frac{1}{3} (2+6x) = \underline{\underline{\frac{6x+2}{3}}}$$

$$\begin{aligned}
 p) \quad \left(\frac{x - \frac{1}{x}}{x + \frac{1}{x}} \right)' &= \frac{\left(1 + \frac{1}{x^2} \right) \left(x + \frac{1}{x} \right) - \left(x - \frac{1}{x} \right) \left(1 - \frac{1}{x^2} \right)}{\left(x + \frac{1}{x} \right)^2} \\
 \begin{array}{ll} u = x - \frac{1}{x} & v = x + \frac{1}{x} \\ u' = 1 + \frac{1}{x^2} & v' = 1 - \frac{1}{x^2} \end{array} & \left| \begin{array}{l} = \frac{x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^3} - x + \frac{1}{x} + \frac{1}{x} - \frac{1}{x^3}}{\left(x + \frac{1}{x} \right)^2} \\ = \frac{\frac{4}{x}}{\left(x + \frac{1}{x} \right)^2} = \frac{4}{x} \div \left(x + \frac{1}{x} \right)^2 = \frac{4}{x} \div \left(\frac{x^2+1}{x} \right)^2 \\ = \frac{4}{x} \cdot \frac{x^2}{(x^2+1)^2} = \underline{\underline{\frac{4x}{(x^2+1)^2}}} \end{array} \right.
 \end{aligned}$$

Ex 2.7.20

Rappel :

$$(u^n)' = n \cdot u^{n-1} \cdot u'$$

↑
dérivée
interne

a) $f(x) = (2x+3)^4$

$$f'(x) = 4(2x+3)^3 \cdot \underline{2} = \underline{8(2x+3)^3}$$

dérivée
interne

b) $f(x) = (3-x)^5$

$$f'(x) = 5(3-x)^4 \cdot \underline{(-1)} = \underline{-5(3-x)^4}$$

dérivée
interne

c) $f(x) = (x^2+5x+1)^3$

$$f'(x) = \underline{3(x^2+5x+1)^2(2x+5)}$$

dérivée
interne

d) $f(x) = (x^3-2x)^7$

$$f'(x) = \underline{7(x^3-2x)^6(3x^2-2)}$$

dérivée
interne

Rappel :

$$(u \cdot v)' = u'v + uv'$$

e) $f(x) = x^2(5x+2)^3$

$$u = x^2 \quad v = (5x+2)^3$$

$$u' = 2x \quad v' = 3(5x+2)^2 \cdot \underline{5} = 15(5x+2)^2$$

$$\begin{aligned} f'(x) &= 2x(5x+2)^3 + 15x^2(5x+2)^2 && \text{(mise en évidence)} \\ &= x(5x+2)^2 \left[\overbrace{2(5x+2)}^{10x+4} + 15x \right] \\ &= \underline{x(5x+2)^2(25x+4)} \end{aligned}$$

$$f) f(x) = (2+x)^2(1-x)^3$$

$$\begin{aligned} u &= (2+x)^2 & v &= (1-x)^3 \\ u' &= 2(2+x) \cdot 1 & v' &= 3(1-x)^2 \cdot (-1) \\ &= 2(2+x) & &= -3(1-x)^2 \end{aligned}$$

$$\begin{aligned} f'(x) &= 2(2+x)(1-x)^3 - 3(2+x)^2(1-x)^2 && \text{(mee)} \\ &= (2+x)(1-x)^2 [2(1-x) - 3(2+x)] \\ &= (2+x)(1-x)^2 (2-2x-6-3x) \\ &= \underline{(2+x)(1-x)^2(-5x-4)} = \underline{-(2+x)(1-x)^2(5x+4)} \end{aligned}$$

$$g) f(x) = (2x+5)^3(3x-1)^4$$

$$\begin{aligned} u &= (2x+5)^3 & v &= (3x-1)^4 \\ u' &= 3(2x+5)^2 \cdot 2 & v' &= 4(3x-1)^3 \cdot 3 \\ &= 6(2x+5)^2 & &= 12(3x-1)^3 \end{aligned}$$

$$\begin{aligned} f'(x) &= 6(2x+5)^2(3x-1)^4 + 12(2x+5)^3(3x-1)^3 && \text{(mee)} \\ &= 6(2x+5)^2(3x-1)^3 \left[(3x-1) + \overbrace{2(2x+5)}^{4x+10} \right] \\ &= \underline{6(2x+5)^2(3x-1)^3(7x+9)} \end{aligned}$$

$$h) f(x) = (1-3x)^2(2-x)(x+3)^3$$

$$\begin{aligned} f'(x) &= -6(1-3x)(2-x)(x+3)^3 - (1-3x)^2(x+3)^3 + 3(1-3x)^2(2-x)(x+3)^2 \\ &= (1-3x)(x+3)^2 [-6(2-x)(x+3) - (1-3x)(x+3) + 3(1-3x)(2-x)] \\ &= (1-3x)(x+3)^2 (6x^2 + 6x - 36 + 3x^2 + 8x - 3 + 9x^2 - 21x + 6) \\ &= (1-3x)(x+3)^2 (18x^2 - 7x - 33) \end{aligned}$$

Rappel :

$$\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$$

$$i) f(x) = \frac{1}{(x^2+3)^2}$$

$$v = (x^2+3)^2$$

$$v' = 2(x^2+3) \cdot 2x = 4x(x^2+3)$$

$$f'(x) = -\frac{4x(x^2+3)}{(x^2+3)^4} = -\frac{4x}{(x^2+3)^3}$$

Rappel :

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$j) f(x) = \frac{x}{(3x+2)^2}$$

$$u = x$$

$$u' = 1$$

$$v = (3x+2)^2$$

$$v' = 2(3x+2) \cdot 3 = 6(3x+2)$$

$$f'(x) = \frac{(3x+2)^2 - 6x(3x+2)}{(3x+2)^4} \quad (\text{mee})$$

$$= \frac{\cancel{(3x+2)} [(3x+2) - 6x]}{(3x+2)^{4-1}} = \frac{-3x+2}{(3x+2)^3}$$

$$k) f(x) = \frac{(1-x)^3}{(1+x)^2}$$

$$u = (1-x)^3$$

$$u' = 3(1-x)^2 \cdot (-1)$$

$$= -3(1-x)^2$$

$$v = (1+x)^2$$

$$v' = 2(1+x)$$

$$f'(x) = \frac{-3(1-x)^2(1+x)^2 - 2(1-x)^3(1+x)}{(1+x)^4}$$

$$= \frac{-(1-x)^2(1+x) \left[\overbrace{3+3x+2-2x}^{3+3x+2-2x} \right]}{(1+x)^{4-1}} = -\frac{(1-x)^2(x+5)}{(1+x)^3}$$

$$1) f(x) = \frac{x(x-3)^2}{(x-2)^2}$$

$$u = x(x-3)^2$$

$$v = (x-2)^2$$

$$u' = 1 \cdot (x-3)^2 + x \cdot 2(x-3) \cdot 1$$

$$v' = 2(x-2)$$

$$= (x-3)^2 + 2x(x-3)$$

$$= (x-3)[(x-3) + 2x]$$

$$= (x-3)(3x-3)$$

$$= 3(x-3)(x-1)$$

$$f'(x) = \frac{3(x-3)(x-1)(x-2)^2 - 2x(x-3)^2(x-2)}{(x-2)^4}$$

$$= \frac{(x-3)(x-2) \left[\overbrace{3(x-1)(x-2) - 2x(x-3)}^{x^2-3x+2} \right]}{(x-2)^4}$$

$$= \frac{(x-3)(3x^2 - 9x + 6 - 2x^2 + 6x)}{(x-2)^3} = \frac{(x-3)(\overbrace{x^2 - 3x + 6}^{\Delta = 9 - 24 < 0})}{(x-2)^3}$$

Ex 2.7.21

Rappel : $\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}}$

$$d) f(x) = \sqrt{8x^2 - 5x + 3}$$

$$f'(x) = \frac{16x - 5}{2\sqrt{8x^2 - 5x + 3}}$$

$$e) f(x) = \sqrt{x^2 + 4}$$

$$f'(x) = \frac{2x}{2\sqrt{x^2 + 4}} = \frac{x}{\sqrt{x^2 + 4}}$$

$$i) f(x) = \frac{1}{\sqrt{x}}$$

$$\text{avec } \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$f'(x) = -\frac{\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = -\frac{\frac{1}{2\sqrt{x}}}{x} = -\frac{1}{2\sqrt{x}} \cdot \frac{1}{x} = \underline{\underline{-\frac{1}{2x\sqrt{x}}}}$$

$$\text{Variante : avec } (x^n)' = nx^{n-1}$$

$$f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$$f'(x) = -\frac{1}{2}x^{-1/2-1} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2} \cdot \frac{1}{\sqrt{x^3}} = \underline{\underline{-\frac{1}{2\sqrt{x^3}}}}$$

$$k) f(x) = (1+x)\sqrt{1-x}$$

$$\begin{array}{ll} u = 1+x & v = \sqrt{1-x} \\ u' = 1 & v' = \frac{-1}{2\sqrt{1-x}} \end{array}$$

$$f'(x) = \sqrt{1-x} + (1+x) \cdot \frac{-1}{2\sqrt{1-x}} = \sqrt{1-x} - \frac{1+x}{2\sqrt{1-x}}$$

$$= \frac{\sqrt{1-x} \cdot 2\sqrt{1-x}}{2\sqrt{1-x}} - \frac{1+x}{2\sqrt{1-x}}$$

$$= \frac{2(1-x) - (1+x)}{2\sqrt{1-x}} = \frac{2-2x-1-x}{2\sqrt{1-x}} = \underline{\underline{\frac{1-3x}{2\sqrt{1-x}}}}$$

$$l) \quad f(x) = \sqrt{\frac{3x-2}{x+1}}$$

$$u = \frac{3x-2}{x+1} \quad u' = \frac{3(x+1) - (3x-2) \cdot 1}{(x+1)^2} = \frac{3x+3-3x+2}{(x+1)^2} = \frac{5}{(x+1)^2}$$

$$f'(x) = \frac{\frac{5}{(x+1)^2}}{2\sqrt{\frac{3x-2}{x+1}}} = \frac{5}{(x+1)^2} \cdot \frac{1}{2\sqrt{\frac{3x-2}{x+1}}} = \underline{\underline{\frac{5}{2(x+1)^2} \sqrt{\frac{x+1}{3x-2}}}}$$