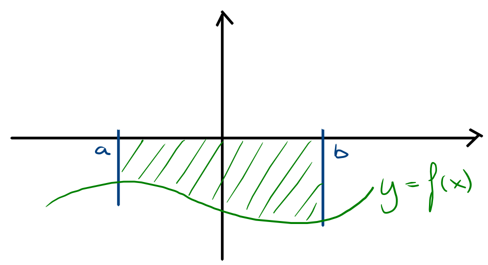
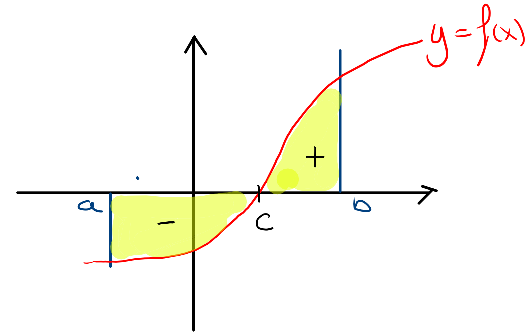
Calcul d'aire

1. Domaine délimité par $y = f(x)$, Ox, $x = a$ et $x = b$

- si f est positive sur $[a;b]$ alors $\mathcal{A} = \int_a^b f(x)\,dx$
- si f est négative sur $[a;b]$ alors $\mathcal{A} = -\int_a^b f(x)\,dx = \left| \int_a^b f(x)\,dx \right|$



- si f est pos. et nég. sur $[a;b]$



$$\mathcal{A} = \left| \int_a^c f(x)\,dx \right| + \int_c^b f(x)\,dx$$

avec c zéro de f sur $[a;b]$ et f nég. sur $[a;c]$ et pos. sur $[c;b]$

Exple : $f(x) = x^3 + 1$, $x = -2$ et $x = 2$

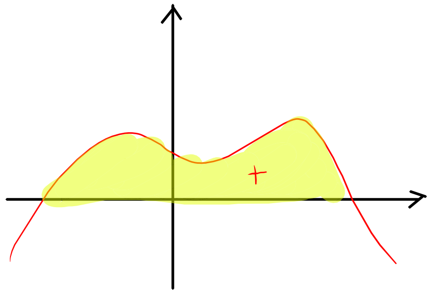
zéro de f : $x^3 + 1 = 0 \iff (x+1)(\underbrace{x^2 - x + 1}_{\Delta < 0}) = 0 \iff x = -1$

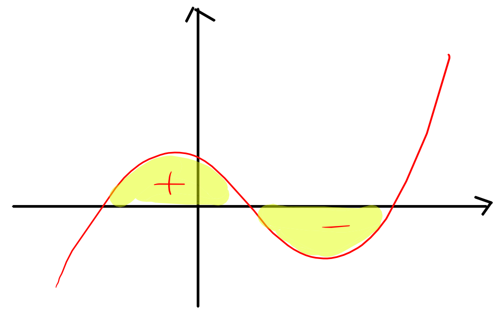
signe de f :

x		-1	
sgn(f)	$-$	0	$+$

$$\mathcal{A} = \left| \int_{-2}^{-1} (x^3 + 1)\,dx \right| + \int_{-1}^{2} (x^3 + 1)\,dx$$

2. Domaine délimité par $y = f(x)$ et Ox.





<u>Exple</u> $f(x) = -x^2 + 5x + 6$

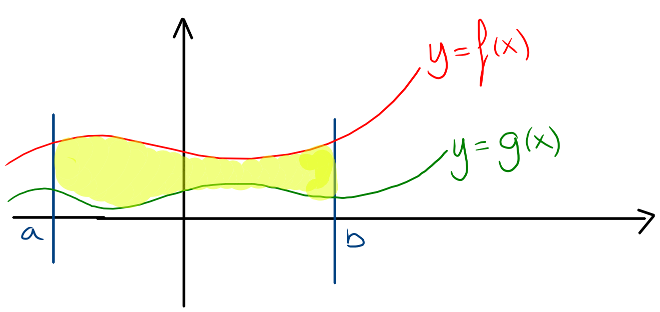
<u>zéros</u> = bornes d'intégration : $-(x^2 - 5x - 6) = 0$

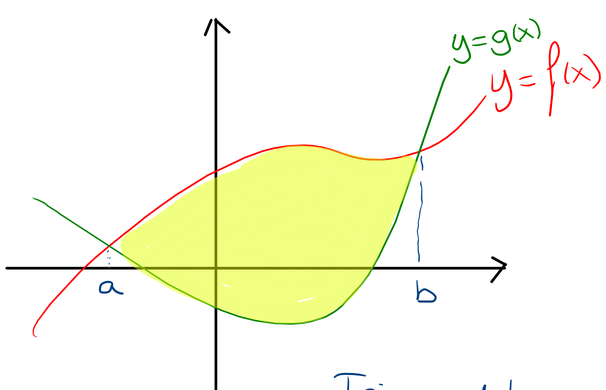
$$-(x-6)(x+1) = 0$$

$$6 \qquad -1$$

$$\mathcal{A} = \int_{-1}^{6} (-x^2 + 5x + 6)\, dx$$

3. Domaine délimité par deux courbes





Ici a et b sont les abscisses des points d'$\cap$ de f et g.

$$\mathcal{A} = \int_a^b (f(x) - g(x))\, dx \quad \text{avec } f > g \text{ sur } [a;b]$$

$$\mathcal{A} = \left| \int_a^b (f(x) - g(x))\, dx \right|$$

Ex 2.2.27

d) $f(x) = x(6-2x^2) = -2x^3+6x$

$g(x) = x(2-x^2) = -x^3+2x$

$f \cap g: \quad -2x^3+6x = -x^3+2x \iff x^3-4x=0 \iff x(x+2)(x-2)=0$

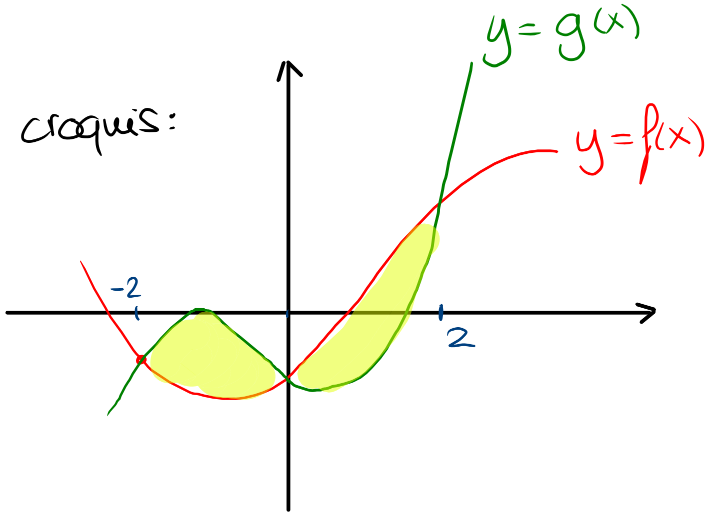
$x = 0, \quad -2, \quad 2$

$$I_1 = \int_{-2}^{0} \left(\left(-2x^3+6x\right)-\left(-x^3+2x\right)\right) dx = \int_{-2}^{0} \left(-x^3+4x\right) dx = -\frac{1}{4}x^4+2x^2 \Big|_{-2}^{0} = 0-\left(-4+8\right) = -4$$

$$I_2 = \int_{0}^{2} (-x^3+4x)\,dx = -\frac{1}{4}x^4+2x^2 \Big|_{0}^{2} = -4+8 = 4$$

$$A = |-4|+4 = \underline{8\ u^2}$$

croquis:



ex 2.2.24 b) d)

2.2.25 a) b)

2.2.27 c)

2.2.29

Ex 2.2.24

b) $f(x) = \frac{4}{x^2} - 1 \qquad a = 1 \quad b = 4$

zéros : $\frac{4}{x^2} - 1 = 0 \iff \frac{4 - x^2}{x^2} = 0 \iff x = \pm 2$

$$I_1 = \int_1^2 \left(\frac{4}{x^2} - 1\right) dx = \int_1^2 (4x^{-2} - 1)\, dx = -4x^{-1} - x \Big|_1^2 = -\frac{4}{x} - x \Big|_1^2$$

$$= -2 - 2 - (-4 - 1) = 1$$

$$I_2 = -\frac{4}{x} - x \Big|_2^4 = -1 - 4 - (-2 - 2) = -1$$

$$\Rightarrow \quad \mathcal{A} = 1 + |-1| = \underline{\underline{2u^2}}$$