

Règles de dérivation (suite)

$$5) \quad f(x) = (u \circ v)(x) = u(v(x))$$

$$\Rightarrow f'(x) = u'(v(x)) \cdot \underbrace{v'(x)}_{\substack{\uparrow \\ \text{dérivée} \\ \text{interne}}} \quad \Rightarrow \quad (u(v))' = u'(v) \cdot \underbrace{v'}_{\substack{\uparrow \\ \text{dérivée} \\ \text{interne}}}$$

En particulier

$$\boxed{(u^n)' = n \cdot u^{n-1} \cdot \underbrace{u'}_{\substack{\text{dérivée} \\ \text{interne}}}}$$

$$\text{exple : } f(x) = \underbrace{(3x^2 + 2x + 5)}_u^3 \Rightarrow f'(x) = 3(3x^2 + 2x + 5)^2 \cdot \underbrace{(6x + 2)}_{u'}$$

$$\boxed{(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot \underbrace{u'}_{\substack{\text{dérivée} \\ \text{interne}}} = \frac{\underbrace{u'}_{\substack{\text{dérivée} \\ \text{interne}}}}{2\sqrt{u}}}$$

$$\text{exple : } f(x) = \sqrt{x^2 + 3} \Rightarrow f'(x) = \frac{\cancel{2}x}{2\sqrt{x^2 + 3}} = \frac{x}{\sqrt{x^2 + 3}}$$

$u = x^2 + 3$
 $u' = 2x$

ex 2.7.20 a) $\rightarrow$ d)

2.7.21 d) $\rightarrow$ e)

Ex 2.7.20 f) $f(x) = (2+x)^2 \cdot (1-x)^3$

$$u = (2+x)^2 \quad v = (1-x)^3$$

$$\begin{aligned} u' &= 2(2+x) \cdot \underline{1} & v' &= 3(1-x)^2 \cdot \underline{(-1)} \\ &= 2(2+x) & &= -3(1-x)^2 \end{aligned}$$

$$\begin{aligned} f'(x) &= 2(2+x)(1-x)^3 + (-3)(2+x)^2(1-x)^2 \\ &= 2(2+x)(1-x)^3 - 3(2+x)^2(1-x)^2 \\ &= (2+x)(1-x)^2 [2(1-x) - 3(2+x)] \\ &= (2+x)(1-x)^2 (2 - 2x - 6 - 3x) \\ &= (2+x)(1-x)^2 (-5x - 4) \end{aligned}$$

2.7.20

a) $\rightarrow$ d) règle 5)

e) $\rightarrow$ g) règle 3) + 5)

a) $f(x) = (2x+3)^4$

$$f'(x) = 4(2x+3)^3 \cdot 2 = \underline{8(2x+3)^3}$$

b) $f(x) = (3-x)^5$

$$f'(x) = 5(3-x)^4 \cdot (-1) = \underline{-5(3-x)^4}$$

c) $f(x) = (x^2+5x+1)^3$

$$f'(x) = \underline{3(x^2+5x+1)^2(2x+5)}$$

d) $f(x) = (x^3-2x)^7$

$$f'(x) = \underline{7(x^3-2x)^6(3x^2-2)}$$

e) $f(x) = x^2(5x+2)^3$

$$u = x^2 \quad v = (5x+2)^3$$

$$u' = 2x \quad v' = 3(5x+2)^2 \cdot 5$$

$$= 15(5x+2)^2$$

$$f'(x) = 2x(5x+2)^3 + 15x^2(5x+2)^2$$

$$= x(5x+2)^2 [2(5x+2) + 15x]$$

$$= \underline{x(5x+2)^2(25x+4)}$$

+ ex 2.7.20 g)

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad \text{et} \quad (u^n)' = n \cdot u^{n-1} \cdot u'$$

Exemples

$$1) \quad f(x) = \frac{(x-1)^3}{x+2}$$

$$\begin{array}{l|l} \begin{array}{l} u = (x-1)^3 \\ u' = 3(x-1)^2 \cdot 1 \\ \quad = 3(x-1)^2 \end{array} & \begin{array}{l} v = x+2 \\ v' = 1 \end{array} \end{array} \quad \Rightarrow \quad f'(x) = \frac{3(x-1)^2(x+2) - (x-1)^3}{(x+2)^2}$$

$$= \frac{(x-1)^2 [3(x+2) - (x-1)]}{(x+2)^2}$$

$$= \frac{(x-1)^2 (2x+7)}{(x+2)^2}$$

$$2) \quad f(x) = \frac{(2x-5)^2}{x^3}$$

$$\begin{array}{l|l} \begin{array}{l} u = (2x-5)^2 \\ u' = 2(2x-5) \cdot 2 \\ \quad = 4(2x-5) \end{array} & \begin{array}{l} v = x^3 \\ v' = 3x^2 \end{array} \end{array}$$

$$f'(x) = \frac{4(2x-5)x^3 - 3x^2(2x-5)^2}{(x^3)^2} = \frac{\cancel{x^4}(2x-5) [4x - 3(2x-5)]}{\cancel{x^6} x^4}$$

$$= \frac{(2x-5)(-2x+15)}{x^4}$$

ex 2.7.20 i) j) k) (l)

2.7.21 i) k) l)