Ex 2.7.25

a) $f(x) = 1 + 2x - x^3$ $\quad a = 1$ $\quad t: y = mx + h$

1) dérivée : $f'(x) = -3x^2 + 2$

pente de la tgte : $m = f'(1) = -3+2 = -1$ $\Rightarrow$ $t: y = -x + h$

2) $f(1) = 1 + 2 - 1 = 2$ $\Rightarrow$ $T(1;2)$

$T(1;2) \in t \Rightarrow 2 = -1 + h \Leftrightarrow h = 3$

$\Rightarrow$ $t: y = -x + 3$

b) $f(x) = \dfrac{x+3}{x}$ $\quad a = 3$ $\quad t: y = mx + h$

1) dérivée : $f'(x) = \dfrac{1\cdot(x+3) - x\cdot 1}{x^2} = \dfrac{x+3-x}{x^2} = \dfrac{3}{x^2}$

pente de la tgte : $m = f'(3) = \dfrac{3}{9} = \dfrac{1}{3}$ $\Rightarrow$ $t: y = \dfrac{1}{3}x + h$

2) $f(3) = \dfrac{3+3}{3} = \dfrac{6}{3} = 2$ $\Rightarrow$ $T(3;2)$

$T(3;2) \in t \Rightarrow 2 = \dfrac{1}{3}\cdot 3 + h \Leftrightarrow 2 = 1 + h \Leftrightarrow h = 1$

$\Rightarrow$ $t: y = \dfrac{1}{3}x + 1$

c) $f(x) = \sqrt{2x+1}$, $a = 4$

1) dérivée : $f'(x) = \dfrac{2}{2\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}}$

pente de la tgte : $m = f'(4) = \dfrac{1}{\sqrt{8+1}} = \dfrac{1}{3}$ $\Rightarrow$ $t: y = \dfrac{1}{3}x + h$

2) $f(4) = \sqrt{8+1} = 3$ $\Rightarrow$ $T(4;3)$

$T(4;3) \in t \Rightarrow 3 = \dfrac{1}{3}\cdot 4 + h \Leftrightarrow h = 3 - \dfrac{4}{3} = \dfrac{5}{3}$

$\Rightarrow$ $t: y = \dfrac{1}{3}x + \dfrac{5}{3}$

Ex 2.7.26

$y = x^2$ et $m = -3$

$f'(x) = 2x \Rightarrow 2x = -3 \Leftrightarrow x = -\frac{3}{2}$

$f\left(-\frac{3}{2}\right) = \frac{9}{4} \Rightarrow \underline{P\left(-\frac{3}{2} ; \frac{9}{4}\right)}$

Ex 2.7.27

$m = \frac{14-2}{1-(-3)} = 3 \Rightarrow f'(x) = 3x^2 - 2x - 5 = 3 \Leftrightarrow 3x^2 - 2x - 8 = 0$

$\Delta = 100 \Rightarrow x_{1,2} = \frac{2 \pm 10}{6} = \begin{cases} \underline{-4/3} \\ \underline{2} \end{cases}$

Ex 2.7.28

$m = 0 \Rightarrow f'(x) = \frac{1(x^2+9) - x \cdot 2x}{(x^2+9)^2} = \frac{-x^2+9}{(x^2+9)^2} = 0$

$\Rightarrow -x^2 + 9 = 0 \Leftrightarrow -(x^2-9) = 0 \Leftrightarrow -(-x+3)(x+3) = 0$

$\Rightarrow x = \begin{cases} -3 \Rightarrow f(-3) = -1/6 \Rightarrow \underline{P_1(-3 ; -1/6)} \\ 3 \Rightarrow f(3) = 1/6 \Rightarrow \underline{P_2(3 ; 1/6)} \end{cases}$

Ex 2.7.31

$$f(x) = \frac{x^2+ax+b}{cx^2+dx-2}$$

- Pas d'AH $\Rightarrow$ $c=0$
- AV $x=2$ $\Rightarrow$ $dx-2 = x-2 \Leftrightarrow d=1 \Rightarrow f(x) = \frac{x^2+ax+b}{x-2}$
- $P(1;-2) \in y=f(x) \Rightarrow -2 = \frac{1+a+b}{-1} \Leftrightarrow 2 = 1+a+b \Leftrightarrow a+b=1$ (1)

$$f'(x) = \frac{(2x+a)(x-2) - 1(x^2+ax+b)}{(x-2)^2}$$

$$\Rightarrow m = f'(1) = -5 \Rightarrow \frac{(2+a)\cdot(-1) - (1+a+b)}{1} = -2a-b-3 = -5$$

$$\Leftrightarrow -2a-b = -2 \quad (2)$$

(1) et (2) $\Rightarrow$ $\begin{cases} a+b=1 \\ -2a-b=-2 \end{cases} \left| \begin{matrix} 1 \\ 1 \end{matrix} \right. \Leftrightarrow \begin{cases} a+b=1 \\ -a=-1 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=0 \end{cases}$

Ex 2.7.32

$f(x) = x^3+ax^2+bx$ $\quad T(-1;\ldots)$ $\quad t: y = x+4$

$\Rightarrow m = f'(-1) = 1$ (1)

$T(-1;\ldots) \in t \Rightarrow y = -1+4 = 3 \Rightarrow T(-1;3) \Rightarrow f(-1) = 3$ (2)

$f'(x) = 3x^2+2ax+b$

(1) $\Rightarrow$ $3-2a+b=1 \Leftrightarrow$

(2) $\Rightarrow$ $-1+a-b=3 \Leftrightarrow$

$\begin{cases} -2a+b=-2 \\ a-b=4 \end{cases} \left| \begin{matrix} 1 \\ 1 \end{matrix} \right. \Leftrightarrow \begin{cases} -a=2 \\ a-b=4 \end{cases}$

$\Leftrightarrow \begin{cases} a=-2 \\ b=-6 \end{cases}$