Etude de fonction

1. ED(f)
2. signe
3. asymptotes
4. croissance
5. graphe

Exemples :

a) $f(x) = \dfrac{x^3}{x^2-3} = \dfrac{x^3}{(x+\sqrt{3})(x-\sqrt{3})}$ ← zéro : 0 ; ← v.i. : $\pm\sqrt{3}$

1. $ED(f) = \mathbb{R} - \{\pm\sqrt{3}\}$

2. signe :

x		$-\sqrt{3}$		0		$\sqrt{3}$	
sign(f)	−	‖	+	0	−	‖	+

$f(1000)$: $\dfrac{+}{+}$

3. asymptotes :

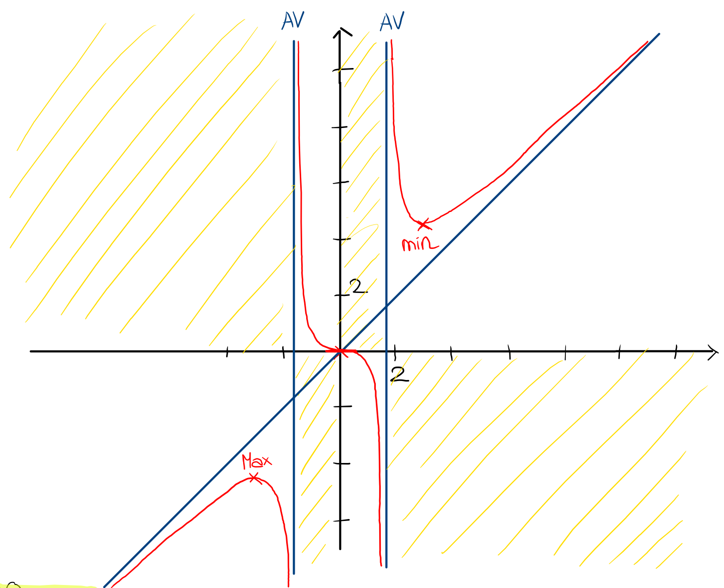
AV/trou : $\lim_{x\to\sqrt{3}} f(x) = \text{"}\dfrac{(\sqrt{3})^3}{0}\text{"} = \infty \Rightarrow$ AV en $x=\sqrt{3}$

$\lim_{x\to-\sqrt{3}} f(x) = \text{"}\dfrac{(-\sqrt{3})^3}{0}\text{"} = \infty \Rightarrow$ AV en $x=-\sqrt{3}$

AH/AO : AO car $\deg(N) = 3 = 2+1 = \deg(D)+1$

$$\begin{array}{r|l} x^3 & x^2-3 \\ \hline -x^3+3x & x \\ \hline 3x & \end{array}$$

$\Rightarrow f(x) = x + \dfrac{3x}{x^2-3} \Rightarrow$ AO : $y = x$



4. croissance : $u = x^3$, $u' = 3x^2$; $v = x^2-3$, $v' = 2x$; $\left(\dfrac{u}{v}\right)' = \dfrac{u'v-uv'}{v^2}$

$f'(x) = \dfrac{3x^2(x^2-3) - 2x^4}{(x^2-3)^2} = \dfrac{x^2\left[3(x^2-3)-2x^2\right]}{(x^2-3)^2} = \dfrac{x^2(x^2-9)}{(x^2-3)^2}$ ← zéros de f' : 0 (2), −3 et 3 ; ← v.i : $\pm\sqrt{3}$ (2)

x		−3		$-\sqrt{3}$		0		$\sqrt{3}$		3	
f'	+	0	−	‖ (2)	−	0 (2)	−	‖ (2)	−	0	+
f	↗	Max	↘	‖	↘	palier	↘	‖	↘	min	↗

$f'(1000)$: $\dfrac{+}{+}$

Max : $f(-3) = \dfrac{(-3)^3}{(-3)^2-3} = \dfrac{-27}{6} = -\dfrac{9}{2} \Rightarrow$ Max$\left(-3 ; -\frac{9}{2}\right)$

palier$(0 ; 0)$

min : $f(3) = \dfrac{3^3}{3^2-3} = \dfrac{27}{6} = \dfrac{9}{2} \Rightarrow$ min$\left(3 ; \frac{9}{2}\right)$

ex 2.8.10 a) d) f)

Rappel pour calculer l'AH d'une fonction rationnelle

Limites à l'infini de fonctions rationnelles

$$\lim_{x\to\pm\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0} = \lim_{x\to\pm\infty} \frac{a_n x^n}{b_m x^m}$$

formulaire p. 15

Exples : 1) $f(x) = \dfrac{3x^3+2x-1}{2x^4+5}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3\cancel{x^3}}{2\cancel{x^4}} = \lim_{x\to\infty} \frac{3}{2x} = \text{"}\frac{3}{\infty}\text{"} = 0 \Rightarrow AH : y = 0$$

2) $g(x) = \dfrac{1-x^4}{2x^4+5}$

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{-\cancel{x^4}}{2\cancel{x^4}} = \lim_{x\to\infty} \frac{-1}{2} = -\frac{1}{2} \Rightarrow AH : y = -\frac{1}{2}$$

3) $h(x) = \dfrac{3x^3+2x-1}{x-1}$

$$\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{3x^{\cancel{3}\,2}}{\cancel{x}} = \lim_{x\to\infty} \frac{3x^2}{1} = \frac{\infty}{1} = \infty \Rightarrow \text{pas d'AH}$$