Ex 2.2.22

a) $\displaystyle\int_1^{+\infty} \frac{2}{x^2}dx = \lim_{a\to+\infty} \int_1^a 2x^{-2}dx = \lim_{a\to+\infty} -\frac{2}{x}\Big|_1^a = \text{"}-\frac{2}{+\infty}\text{"} - (-2) = 0+2 = \underline{2}$

b) $\displaystyle\int_{-\infty}^{-2} \frac{1}{(x+1)^3}dx = \lim_{a\to-\infty} \int_a^{-2} (x+1)^{-3}dx = \lim_{a\to-\infty} -\frac{1}{2(x+1)^2}\Big|_a^{-2} = -\frac{1}{2} - \left(-\text{"}\frac{1}{+\infty}\text{"}\right) = -\frac{1}{2}+0 = \underline{-\frac{1}{2}}$

c) $\displaystyle\int_3^{+\infty} \frac{5+y}{y^3}dy = \lim_{a\to+\infty} \int_3^a \left(\frac{5}{y^3}+\frac{1}{y^2}\right)dy = \lim_{a\to+\infty} \left(-\frac{5}{2y^2}-\frac{1}{y}\right)\Big|_3^a$

$\displaystyle = \left(-\frac{5}{+\infty}-\frac{1}{+\infty}\right) - \left(-\frac{5}{18}-\frac{1}{3}\right) = 0+\frac{5}{18}+\frac{1}{3} = \underline{\frac{11}{18}}$

d) $\displaystyle\int_0^{+\infty} \frac{x^2+1}{x^2}dx = \lim_{a\to+\infty}\left(\lim_{b\to 0^+}\int_b^a \left(1+\frac{1}{x^2}\right)dx\right) = \lim_{a\to+\infty}\left(\lim_{b\to 0^+}\left(x-\frac{1}{x}\right)\Big|_b^a\right)$

$\displaystyle = +\infty - \text{"}\frac{1}{+\infty}\text{"} - \left(0-\text{"}\frac{1}{0_+}\text{"}\right) = +\infty - 0 - 0 + \infty = +\infty$

<u>l'intégrale généralisée diverge</u>

e) $\displaystyle\int_0^2 \frac{2}{x^2}dx = \lim_{a\to 0^+} \int_a^2 \frac{2}{x^2}dx = \lim_{a\to 0^+} -\frac{2}{x}\Big|_a^2 = -1 - \left(-\text{"}\frac{2}{0_+}\text{"}\right) = +\infty$

<u>l'intégrale généralisée diverge</u>

f) $\displaystyle\int_0^4 t^{-3/2}dt = \lim_{a\to 0^+} \int_a^4 t^{-3/2}dt = \lim_{a\to 0^+} -\frac{2}{\sqrt{t}}\Big|_a^4 = -1 - \left(-\text{"}\frac{2}{0_+}\text{"}\right) = +\infty$

<u>l'intégrale généralisée diverge</u>

g) $\displaystyle\int_{-\infty}^{+\infty} \underbrace{\frac{3}{z^2+1}}_{\text{fct paire}} dz = 2\cdot \lim_{a \to +\infty} \int_0^a \frac{3}{z^2+1} dz = 2 \cdot \lim_{a\to+\infty} 3\,\mathrm{Arctan}(z)\Big|_0^a$

car $f(-z) = \frac{3}{z^2+1} = f(z)$

$= 6\left(\mathrm{Arctan}(+\infty) - \mathrm{Arctan}(0)\right)$

$= 6\left(\frac{\pi}{2} - 0\right) = 3\pi$

h) $\displaystyle\int_0^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx = \lim_{a\to 0} \int_a^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}} dx \iff dx = 2t\cdot dt$

bornes : $x = a \to t = \sqrt{a}$
$x = \pi^2 \to t = \pi$

$\displaystyle = \lim_{a\to 0} \int_{\sqrt{a}}^{\pi} \frac{\sin(t)}{\cancel{t}} \cdot 2\cancel{t}\, dt = \lim_{a\to 0} \int_{\sqrt{a}}^{\pi} 2\sin(t) dt$

$\displaystyle = \lim_{a\to 0} -2\cos(t)\Big|_{\sqrt{a}}^{\pi} = -2\cos(\pi) + 2\cos(0) = 2+2 = \underline{4}$