

Ex 2.5.2

a) $2x^2 - 7x - 4$

résoudre : $2x^2 - 7x - 4 = 0$ $\Delta = 49 - 4 \cdot 2 \cdot (-4) = 81$

$$x_{1,2} = \frac{7 \pm 9}{4} = \begin{cases} 4 \\ -1/2 \end{cases} \Rightarrow \underline{S = \{-1/2, 4\}}$$

factoriser : $2x^2 - 7x - 4 = 2(x-4)(x + \frac{1}{2}) = \underline{(x-4)(2x+1)}$

c) $6x^2 - 25x - 25$

résoudre : $6x^2 - 25x - 25 = 0$ $\Delta = 25^2 - 4 \cdot 6 \cdot (-25) = 1225 = 35^2$

$$x_{1,2} = \frac{25 \pm 35}{12} = \begin{cases} 5 \\ -\frac{10}{12} = -\frac{5}{6} \end{cases} \Rightarrow \underline{S = \{-5/6, 5\}}$$

factoriser : $6x^2 - 25x - 25 = 6(x-5)(x + \frac{5}{6}) = \underline{(x-5)(6x+5)}$

d) $6x^2 - 20x + 20$

résoudre : $6x^2 - 20x + 20 = 0$ $\Delta = 20^2 - 4 \cdot 6 \cdot 20 = -80 < 0$

factoriser : $6x^2 - 20x + 20$ n'est pas factorisable $\Rightarrow \underline{S = \emptyset}$

Ex 2.5.3

a) $x^2 - 9 = 0$

PR $(x+3)(x-3) = 0$
 $\downarrow \quad \downarrow$
 $x+3=0 \quad x-3=0$
 $x=-3 \quad x=3$

$\Rightarrow S = \{ \pm 3 \}$

b) $4x^2 - 1 = 0$

PR $(2x+1)(2x-1) = 0$
 $\downarrow \quad \downarrow$
 $2x+1=0 \quad \vdots$
 $2x=-1 \quad x=\frac{1}{2}$
 $x=-\frac{1}{2}$

$\Rightarrow S = \{ \pm \frac{1}{2} \}$

c) $(x-2)^2 - 9(x-2) = 0$

Mee $(x-2)[(x-2)-9] = 0$

$(x-2)(x-11) = 0$
 $\downarrow \quad \downarrow$
 $2 \quad 11$

$\Rightarrow S = \{ 2; 11 \}$

d) $(x^2 - x - 6)(x+5) = 0$

SP $(x+2)(x-3)(x+5) = 0$
 $\downarrow \quad \downarrow \quad \downarrow$
 $-2 \quad 3 \quad -5$

$\Rightarrow S = \{ -5; -2; 3 \}$

e) $x^4 - 5x^2 + 4 = 0$

$y = x^2 \Rightarrow y^2 - 5y + 4 = 0$

SP $(y-4)(y-1) = 0$

$\Rightarrow (x^2-4)(x^2-1) = 0$

PR $(x+2)(x-2)(x+1)(x-1) = 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $-2 \quad 2 \quad -1 \quad 1$

$\Rightarrow S = \{ \pm 1; \pm 2 \}$

$$f) \quad (x-1)(x^2+1) = 0$$

\downarrow
 1

$\underbrace{\hspace{10em}}$ pas factorisable, pas de zéro

$\Rightarrow S = \{1\}$

$$g) \quad x^3 + x^2 = 4x + 4$$

GR $x^3 + x^2 - 4x - 4 = 0$

$$x^2(x+1) - 4(x+1) = 0$$

PR $(x+1)(x^2-4) = 0$

$$(x+1)(x+2)(x-2) = 0$$

\downarrow \downarrow \downarrow
 -1 -2 2

$\Rightarrow S = \{-1, -2, 2\}$

$$h) \quad x^2 - 9 - 4(x-3) = 0$$

$$\underbrace{(x+3)(x-3)} - 4(x-3) = 0$$

GR $(x-3)[(x+3)-4] = 0$

$$(x-3)(x-1) = 0$$

\downarrow \downarrow
 3 1

$\Rightarrow S = \{1, 3\}$

$$j) \quad x^3 + 2x^2 - x - 2 = 0$$

GR $x^2(x+2) - (x+2) = 0$

PR $(x+2)(x^2-1) = 0$

$$(x+2)(x+1)(x-1) = 0$$

\downarrow \downarrow \downarrow
 -2 -1 1

$\Rightarrow S = \{-2, -1, 1\}$

Ex 2.5.4

$$a) (x^2 - 8x + 12)(x+2)^3 = 0$$

$$\text{SP } (x-6)(x-2)(x+2)^3 = 0$$
$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 6 & 2 & -2 \\ & & \text{(valeur triple)} \end{array}$$

$$\Rightarrow S = \underline{\{-2; 2; 6\}}$$

$$b) (x-3)(x^2-4) = 0$$

$$\text{PR } (x-3)(x+2)(x-2) = 0$$
$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 3 & -2 & 2 \end{array}$$

$$\Rightarrow S = \underline{\{-2; 2; 3\}}$$

$$c) x^3 + 2x^2 - 4x = 8$$

$$x^3 + 2x^2 - 4x - 8 = 0$$

$$\text{GR } x^2(x+2) - 4(x+2) = 0$$

$$(x+2)(x^2-4) = 0$$

$$(x+2)(x+2)(x-2) = 0$$

$$(x+2)^2(x-2) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ -2 & & 2 \\ \text{(double)} & & \end{array}$$

$$\Rightarrow S = \underline{\{\pm 2\}}$$

$$d) (2x^2 + 3x + 1)^2 - (2x^2 - 4x - 1)^2 = 0$$

$$\text{PR } [(2x^2 + 3x + 1) + (2x^2 - 4x - 1)][(2x^2 + 3x + 1) - (2x^2 - 4x - 1)] = 0$$
$$(4x^2 - x)(2x^2 + 3x + 1 - 2x^2 + 4x + 1)$$

$$\text{NEE } x(4x-1)(7x+2) = 0$$
$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 1/4 & -2/7 \end{array}$$

$$\Rightarrow S = \underline{\{0; 1/4; -2/7\}}$$

$$e) \quad x(x-2) + (x-3)(x-2) = 0$$

$$\text{HEE} \quad (x-2)[x + (x-3)] = 0$$

$$(x-2)(2x-3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 2 & 3/2 \end{array}$$

$$\Rightarrow \underline{S = \{2; \frac{3}{2}\}}$$

$$f) \quad 6x^2 = 3x^3 - 72x$$

$$-3x^3 + 6x^2 + 72x = 0$$

$$3x^3 - 6x^2 - 72x = 0$$

$$\text{HEE} \quad 3x(x^2 - 2x - 24) = 0$$

$$\text{SP} \quad 3x(x-6)(x+4) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 6 & -4 \end{array}$$

$$\Rightarrow \underline{S = \{0; 6; -4\}}$$

$$g) \quad x^3 + 3x^2 = 9x + 27$$

$$x^3 + 3x^2 - 9x - 27 = 0$$

$$\text{GR} \quad x^2(x+3) - 9(x+3) = 0$$

$$(x+3)(x^2-9) = 0$$

$$\text{PR} \quad (x+3)(x+3)(x-3) = 0$$

$$(x+3)^2(x-3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ -3 & 3 \\ \text{(double)} & \end{array}$$

$$\Rightarrow \underline{S = \{\pm 3\}}$$

$$h) \quad (x-1)(x-2)(x-3) = x(x^2-9)$$

$$(x-1)(x-2)(x-3) - x(x^2-9) = 0$$

$$\text{PR} \quad (x-1)(x-2)(x-3) - x(x+3)(x-3) = 0$$

$$\text{HEE} \quad (x-3)[(x-1)(x-2) - x(x+3)] = 0$$

$$(x-3)(x^2 - 2x - x + 2 - x^2 - 3x) = 0$$

$$\text{HEE} \quad (x-3)(-6x+2) = 0$$

$$-2(x-3)(3x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 3 & 1/3 \end{array}$$

$$\Rightarrow \underline{S = \{\frac{1}{3}; 3\}}$$

Ex 2.5.6

- a) $x^4 - 13x^2 + 36 = 0 \Leftrightarrow (x^2 - 9)(x^2 - 4) = 0$
 $\Leftrightarrow (x+3)(x-3)(x+2)(x-2) = 0 \Rightarrow S = \{-3; -2; 2; 3\}$
- b) $x^4 - 1 = 0 \Leftrightarrow \underbrace{(x^2+1)}_{\Delta < 0} (x+1)(x-1) = 0 \Rightarrow S = \{\pm 1\}$
(Σ de 2 casés)
- c) $x^4 + 2x^2 + 1 = 0 \Leftrightarrow \underbrace{(x^2+1)^2}_{\Delta < 0} = 0 \Rightarrow S = \emptyset$
(Σ de 2 casés)
- d) $x^6 - 7x^3 - 8 = 0 \Leftrightarrow (x^3 - 8)(x^3 + 1) = 0$
 $\Leftrightarrow (x-2) \underbrace{(x^2+2x+4)}_{\Delta < 0} (x+1) \underbrace{(x^2-x+1)}_{\Delta < 0} = 0 \Rightarrow S = \{-1; 2\}$

Ex 2.5.7

- b) $\frac{3x-7}{5} + \frac{x^2-9}{7} = 2 \quad | \cdot 35$
 $\Leftrightarrow 21x - 49 + 5x^2 - 45 = 70$
 $\Leftrightarrow 5x^2 + 21x - 164 = 0 \quad \Delta = 3721 = 61^2$
 $\Rightarrow x_{1,2} = \frac{-21 \pm 61}{10} = \begin{cases} -\frac{41}{5} \\ 4 \end{cases} \Rightarrow S = \{-\frac{41}{5}; 4\}$
- d) $\frac{x^2-3}{2} - \frac{x^2+1}{3} = \frac{x^2-11}{6} \quad | \cdot 6$
 $\Leftrightarrow 3x^2 - 9 - 2x^2 - 2 = x^2 - 11$
 $\Leftrightarrow 0 = 0 \quad \checkmark \text{ tjs vrai } \forall x \in \mathbb{R} \Rightarrow S = \mathbb{R}$

$$e) \quad \frac{3x+1}{8} - \frac{x^2+5}{4} = \frac{55}{2} \quad | \cdot 8$$

$$\Leftrightarrow 3x+1 - 2x^2-10 = 220$$

$$\Leftrightarrow 2x^2 - 3x + 229 = 0 \quad \Delta = -1823 \quad \Rightarrow \quad \underline{S = \emptyset}$$

$$j) \quad \frac{5-4x}{2} + \frac{3x^2-1}{3} = \frac{2x^2+5}{6} \quad | \cdot 6$$

$$\Leftrightarrow 15-12x+6x^2-2 = 2x^2+5$$

$$\Leftrightarrow 4x^2-12x+8 = 0 \quad | : 4$$

$$\Leftrightarrow x^2-3x+2 = 0 \quad \Leftrightarrow (x-2)(x-1) = 0 \quad \Rightarrow \quad \underline{S = \{1, 2\}}$$

Ex 2.5.11

a) $\frac{x-1}{2x-1} = \frac{3x-5}{\underbrace{4x-2}_{2(2x-1)}} \quad | \cdot 2(2x-1)$ v.i.: $2x-1 \neq 0 \Leftrightarrow 2x \neq 1 \Leftrightarrow x \neq \frac{1}{2}$
ppmc: $2(2x-1)$

$$2x-2 = 3x-5$$

$$x = 3 \quad \checkmark \quad \Rightarrow \quad \underline{S = \{3\}}$$

b) $\frac{x^2+x+1}{2x+2} = x \quad | \cdot 2x+2$ v.i.: $2x+2 \neq 0 \Leftrightarrow 2x \neq -2 \Leftrightarrow x \neq -1$

$$x^2+x+1 = 2x^2+2x$$

$$x^2+x-1 = 0 \quad \Delta = 5$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \quad \checkmark \quad \Rightarrow \quad \underline{S = \left\{ \frac{-1 \pm \sqrt{5}}{2} \right\}}$$

c) $\frac{1}{x+1} + \frac{1}{x+3} + \frac{3}{4} = 0 \quad | \cdot 4(x+1)(x+3)$ v.i.: -1 et -3
ppmc: $4(x+1)(x+3)$

$$4x+12 + 4x+4 + 3(x+1)(x+3) = 0$$

$$8x+16 + 3x^2+12x+9 = 0$$

$$3x^2+20x+25 = 0 \quad \Delta = 100$$

$$x_{1,2} = \frac{-20 \pm 10}{6} = \begin{cases} -5 & \checkmark \\ -\frac{5}{3} & \checkmark \end{cases} \quad \Rightarrow \quad \underline{S = \left\{ -5, -\frac{5}{3} \right\}}$$

d) $\frac{x}{x-1} = \frac{3x-4}{(x-1)(x-2)} \quad | (x-1)(x-2)$ v.i.: 1 et 2
ppmc: $(x-1)(x-2)$

$$x(x-2) = 3x-4$$

$$x^2-2x = 3x-4$$

$$x^2-5x+4 = 0$$

$$(x-4)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 4 & 1 \\ \checkmark & \alpha \end{array}$$

$$\Rightarrow \quad \underline{S = \{4\}}$$

$$f) \quad \frac{x}{x-6} - \frac{1}{2} = \frac{x}{6} + \frac{x+6}{\underbrace{6-x}_{-(x-6)}} \quad | \cdot 6(x-6)$$

v.i. : 6

ppmc : $6(x-6)$

$$6x - 3(x-6) = x(x-6) - 6(x+6)$$

$$6x - 3x + 18 = x^2 - 6x - 6x - 36$$

$$x^2 - 15x - 54 = 0$$

$$(x-18)(x+3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 18 & -3 \\ \checkmark & \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{-3; 18\}}$$

Ex 2.5.12

$$a) \quad \frac{x-4}{x+8} = 0 \quad | \cdot (x+8)$$

v.i. : -8

$$x-4 = 0$$

$$x = 4 \quad \checkmark$$

$$\Rightarrow \underline{S = \{4\}}$$

$$d) \quad \frac{x+1}{x} - 2x = \frac{x-1}{x} \quad | \cdot x$$

v.i. : 0

$$x+1 - 2x^2 = x-1$$

$$-2x^2 + 2 = 0$$

$$-2(x^2 - 1) = 0$$

$$-2(x+1)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ -1 & 1 \\ \checkmark & \checkmark \end{array}$$

$$\Rightarrow S = \{\pm 1\}$$

$$e) \quad \frac{z}{z-3} - \frac{2}{\underbrace{2-z}_{-(z-2)}} = \frac{3}{\underbrace{z^2-5z+6}_{(z-3)(z-2)}} \quad | \cdot (z-3)(z-2)$$

v.i. : 2 et 3

$$z(z-2) + 2(z-3) = 3$$

$$z^2 - 2z + 2z - 6 = 3$$

$$z^2 - 9 = 0$$

$$(z+3)(z-3) = 0 \quad \Leftrightarrow z = \begin{array}{l} -3 \quad \checkmark \\ 3 \quad \times \end{array}$$

$$\Rightarrow \underline{S = \{-3\}}$$

$$f) \quad \frac{4x}{x+3} - \frac{x}{x-3} = -\frac{12}{\underbrace{x^2-9}_{(x+3)(x-3)}} \quad | \cdot (x+3)(x-3)$$

v.i. : ± 3

ppmc : $(x+3)(x-3)$

$$4x(x-3) - x(x+3) = -12$$

$$4x^2 - 12x - x^2 - 3x = -12$$

$$3x^2 - 15x + 12 = 0$$

$$3(x^2 - 5x + 4) = 0$$

$$3(x-4)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 4 & 1 \\ \checkmark & \checkmark \end{array}$$

$$\Rightarrow S = \{1, 4\}$$

$$h) \quad \frac{x+4}{x} - \frac{1}{x+4} = \frac{4}{\underbrace{x^2+4x}_{x(x+4)}} \quad | \cdot x(x+4)$$

v.i. : 0 et -4

$$(x+4)^2 - x = 4$$

$$x^2 + 8x + 16 - x = 4$$

$$x^2 + 7x + 12 = 0$$

$$(x+4)(x+3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ -4 & -3 \\ \checkmark & \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{-3\}}$$

$$i) \quad \frac{1}{\underbrace{x^2-x}_{x(x-1)}} + \frac{5}{\underbrace{x^2+x}_{x(x+1)}} = \frac{4}{\underbrace{x^2-1}_{(x+1)(x-1)}} \quad | \cdot x(x+1)(x-1)$$

v.i. : 0, 1 et -1

$$x+1 + 5(x-1) = 4x$$

$$x+1 + 5x-5 = 4x$$

$$2x-4 = 0$$

$$x = 2 \quad \checkmark$$

$$\Rightarrow \underline{S = \{2\}}$$

Ex 2.5.13

$$a) \quad 1 = \frac{3x}{\underbrace{x^2-9}_{(x+3)(x-3)}} - \frac{x}{\underbrace{2x-6}_{2(x-3)}} \quad | \cdot 2(x+3)(x-3) \quad \text{v.i.: } \pm 3$$

$$2(x+3)(x-3) = 3x \cdot 2 - x(x+3)$$

$$2x^2 - 18 = 6x - x^2 - 3x$$

$$3x^2 - 3x - 18 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 3 & -2 \end{array} \quad \begin{array}{l} \times \\ \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{-2\}}$$

$$b) \quad \frac{x}{x+3} = \frac{x-1}{2x} + \frac{1}{4} \quad | \cdot 4x(x+3)$$

$$\text{v.i.: } -3 \text{ et } 0$$

$$\text{ppmc: } (x+3) \cdot 2x \cdot 2 = 4x(x+3)$$

$$4x^2 = 2(x+3)(x-1) + x(x+3)$$

$$4x^2 = 2x^2 + 4x - 6 + x^2 + 3x$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 6 & 1 \end{array} \quad \begin{array}{l} \checkmark \\ \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{1, 6\}}$$

$$c) \quad \frac{2-x}{x+1} - \frac{5}{3} = \frac{2x+1}{3-2x} \quad | \cdot 3(x+1)(3-2x) \quad \text{v.i.: } -1, \frac{3}{2}$$

$$3(2-x)(3-2x) - 5(x+1)(3-2x) = 3(2x+1)(x+1)$$

$$6x^2 - 21x + 18 + 10x^2 - 5x - 15 = 6x^2 + 9x + 3$$

$$10x^2 - 35x = 0$$

$$5x(2x-7) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 0 & \frac{7}{2} \end{array} \quad \begin{array}{l} \checkmark \\ \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{0, \frac{7}{2}\}}$$

$$d) \frac{10x-2}{6x-3} + \frac{3x+5}{4x^2-1} = \frac{x-1}{2x+1} \quad | \cdot 3(2x-1)(2x+1) \quad \text{v.i. } \pm \frac{1}{2}$$

$$\underbrace{3(2x-1)} \quad \underbrace{(2x-1)(2x+1)}$$

$$(10x-2)(2x+1) + 3(3x+5) = 3(x-1)(2x-1)$$

$$20x^2 + 6x - 2 + 9x + 15 = 6x^2 - 9x + 3$$

$$14x^2 + 24x + 10 = 0$$

$$7x^2 + 12x + 5 = 0 \quad \Delta = 4$$

$$x_{1,2} = \frac{-12 \pm 2}{14} = \begin{cases} -1 & \checkmark \\ -\frac{5}{7} & \checkmark \end{cases} \Rightarrow \underline{S = \left\{ -1; -\frac{5}{7} \right\}}$$

Ex 2.5.14

$$a) \sqrt{7-x} = x-5 \quad | ()^2$$

$$7-x = x^2 - 10x + 25$$

$$x^2 - 9x + 18 = 0$$

$$(x-3)(x-6) = 0$$

$$x = \begin{cases} 3 \\ 6 \end{cases} \quad \text{verif: } \begin{array}{l} \sqrt{4} \stackrel{?}{=} -2 \Leftrightarrow 2 \neq -2 \quad \times \\ \sqrt{1} \stackrel{?}{=} 1 \Leftrightarrow 1 = 1 \quad \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{6\}}$$

$$b) x = 4 + \sqrt{4x-19}$$

$$x-4 = \sqrt{4x-19} \quad | ()^2$$

$$x^2 - 8x + 16 = 4x - 19$$

$$x^2 - 12x + 35 = 0$$

$$(x-5)(x-7) = 0$$

$$x = \begin{cases} 5 \\ 7 \end{cases} \quad \text{verif: } \begin{array}{l} 5 \stackrel{?}{=} 4 + \sqrt{1} \Leftrightarrow 5 = 5 \quad \checkmark \\ 7 \stackrel{?}{=} 4 + \sqrt{9} \Leftrightarrow 7 = 7 \quad \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{5; 7\}}$$

$$c) \sqrt{x+1} - x = x+2$$

$$\sqrt{x+1} = 2x+2 \quad |(\)^2$$

$$x+1 = 4x^2+8x+4$$

$$4x^2+7x+3 = 0 \quad \Delta = 1$$

$$x_{1,2} = \frac{-7 \pm 1}{8} = \begin{cases} -1 \\ -\frac{3}{4} \end{cases}$$

$$\text{vérif: } \sqrt{0} + 1 \stackrel{?}{=} 1 \quad \checkmark$$

$$\sqrt{\frac{1}{4}} + \frac{3}{4} \stackrel{?}{=} \frac{5}{4}$$

$$\frac{1}{2} + \frac{3}{4} \stackrel{?}{=} \frac{5}{4} \quad \checkmark$$

$$\Rightarrow \underline{S = \{-1, -\frac{3}{4}\}}$$

$$d) x - \sqrt{-7x-24} = -2$$

$$x+2 = \sqrt{-7x-24} \quad |(\)^2$$

$$x^2+4x+4 = -7x-24$$

$$x^2+11x+28 = 0$$

$$(x+4)(x+7) = 0$$

$$x = \begin{cases} -4 \\ -7 \end{cases}$$

$$\text{vérif: } \begin{matrix} -4 - \sqrt{28-24} \stackrel{?}{=} -2 \\ -4 - 2 \neq -2 \end{matrix}$$

K

$$\text{vérif: } \begin{matrix} -7 - \sqrt{49-24} \stackrel{?}{=} -2 \\ -7 - 5 \neq -2 \end{matrix}$$

K

$$\Rightarrow \underline{S = \emptyset}$$

$$g) x + \sqrt{x} = 20$$

$$\sqrt{x} = 20-x \quad |(\)^2$$

$$x = 400 - 40x + x^2$$

$$0 = x^2 - 41x + 400$$

$$\Delta = 41^2 - 4 \cdot 400 = 81$$

$$x_{1,2} = \frac{41 \pm 9}{2} = \begin{cases} 25 \\ 16 \end{cases}$$

$$\text{vérif: } \begin{matrix} 25 + \sqrt{25} \stackrel{?}{=} 20 \\ 25 + 5 \neq 20 \end{matrix} \quad K$$

$$\text{vérif: } \begin{matrix} 16 + \sqrt{16} \stackrel{?}{=} 20 \\ 16 + 4 = 20 \end{matrix} \quad \checkmark$$

$$\Rightarrow \underline{S = \{16\}}$$

Ex 2.5.19

a)
$$\begin{cases} (1) & x+y = 1 & | & 1 \\ (2) & x-y = 0 & | & 1 \end{cases}$$

$$2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\begin{cases} \frac{1}{2} + y = 1 \\ y = \frac{1}{2} \end{cases}$$

$$\Rightarrow S = \left\{ \left(\frac{1}{2}; \frac{1}{2} \right) \right\}$$

b)
$$\begin{cases} (1) & 5x-2y = 5 & | & -1 \\ (2) & 3x-y = 10 & | & 2 \end{cases}$$

$$\begin{array}{r} -5x+2y = -5 \\ + \\ 6x-2y = 20 \\ \hline x = 15 \end{array}$$

$$\begin{aligned} (2) \Rightarrow 45 - y &= 10 \\ 35 &= y \end{aligned}$$

$$\Rightarrow S = \left\{ (15; 35) \right\}$$

c)
$$\begin{cases} (1) & 6x+4 = -6y \\ (2) & 1-x = 6y \end{cases}$$

$$\Leftrightarrow \begin{cases} 6x+6y = -4 & | & 1 \\ -x-6y = -1 & | & 1 \end{cases}$$

$$\begin{aligned} 5x &= -5 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} (2) \Rightarrow 1+1 &= 6y \\ \frac{2}{6} &= \frac{1}{3} = y \end{aligned}$$

$$\Rightarrow S = \left\{ (-1; \frac{1}{3}) \right\}$$

d)
$$\begin{cases} 2x-4y = 2 & | & -1 \\ x-2y = 1 & | & 2 \end{cases}$$

$$\Rightarrow \begin{array}{r} -2x+4y = -2 \\ + \\ 2x-4y = 2 \\ \hline 0 = 0 \end{array} \quad \checkmark$$

$$\Rightarrow S = \left\{ (x; y) \mid x-2y=1 \right\}$$

e)
$$\begin{cases} 2x+4y = 5 & | & -1 \\ x+2y = 2 & | & 2 \end{cases}$$

$$\Rightarrow \begin{array}{r} -2x-4y = -5 \\ + \\ 2x+4y = 4 \\ \hline 0 = -1 \end{array}$$

⚡ impossible

$$\Rightarrow S = \emptyset$$

$$f) \begin{cases} (1) & 2x+3y = 4 & | & -2 \\ (2) & 5x+6y = 10 & | & 1 \end{cases} \Rightarrow \begin{array}{r} -4x-6y = -8 \\ + \quad 5x+6y = 10 \\ \hline x = 2 \end{array}$$

$$\begin{array}{l} (1) \\ \Rightarrow 4+3y = 4 \\ \quad 3y = 0 \\ \quad y = 0 \end{array} \Rightarrow \underline{S = \{(2; 0)\}}$$

Ex 2.5.21

$$a) \begin{cases} (1) & 12x-5y = 29 & | & 1 \\ (2) & 4x-3y = 11 & | & -3 \end{cases} \Rightarrow \begin{array}{r} 12x-5y = 29 \\ + \quad -12x+9y = -33 \\ \hline 4y = -4 \\ y = -1 \end{array}$$

$$\begin{array}{l} (2) \\ \Rightarrow 4x+3 = 11 \\ \quad 4x = 8 \\ \quad x = 2 \end{array} \Rightarrow \underline{S = \{(2; -1)\}}$$

$$b) \begin{cases} x+y = 19 & | & 3 & | & 2 \\ 2x-3y = 11 & | & 1 & | & -1 \end{cases}$$

$$3x+3y = 57$$

$$2x-3y = 11$$

$$\hline 5x = 68$$

$$x = \frac{68}{5}$$

$$2x+2y = 38$$

$$-2x+3y = -11$$

$$\hline 5y = 27$$

$$y = \frac{27}{5}$$

$$\Rightarrow S = \left\{ \left(\frac{68}{5}; \frac{27}{5} \right) \right\}$$

$$c) \begin{cases} 12x + 11y = 6 \\ 3y - 2x = 24 \end{cases} \Leftrightarrow \begin{cases} 12x + 11y = 6 & | 1 & | -3 \\ -2x + 3y = 24 & | 6 & | 11 \end{cases}$$

$$\begin{array}{r} 12x + 11y = 6 \\ + \quad -12x + 18y = 144 \\ \hline 29y = 150 \\ y = \frac{150}{29} \end{array}$$

$$\begin{array}{r} -36x - 33y = -18 \\ -22x + 33y = 264 \\ \hline -58x = 246 \\ x = -\frac{246}{58} = -\frac{123}{29} \end{array}$$

$$\Rightarrow S = \left\{ \left(-\frac{123}{29}, \frac{150}{29} \right) \right\}$$

$$j) \begin{cases} (1) & x + y + z = 25 \\ (2) & x - y + z = 5 \\ (3) & x + 2z = 2y - 10 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z = 25 \\ x - y + z = 5 \\ x - 2y + 2z = -10 \end{cases} \begin{array}{l} | 1 \\ -1 \\ | \end{array} \begin{array}{l} | \\ -2 \\ 1 \end{array} \quad \text{pour éliminer } z$$

$$\begin{array}{r} x + y + z = 25 \\ -x + y - z = -5 \\ \hline 2y = 20 \\ y = 10 \end{array}$$

$$\begin{array}{r} -2x + 2y - 2z = -10 \\ x - 2y + 2z = -10 \\ \hline -x = -20 \\ x = 20 \end{array}$$

$$\begin{array}{l} (1) \\ \Rightarrow 20 + 10 + z = 25 \\ z = -5 \end{array}$$

$$\Rightarrow S = \{ (20, 10, -5) \}$$

$$k) \begin{cases} (1) & x - y - z = 6 \\ (2) & x - 2y - 3z = 10 \\ (3) & 5x + 6y + z = 2 \end{cases} \begin{array}{l} | 1 \\ | \\ | 1 \end{array} \begin{array}{l} | -3 \\ | 1 \\ | \end{array}$$

$$6x + 5y = 8$$

$$\begin{array}{r} -3x + 3y + 3z = -18 \\ x - 2y - 3z = 10 \\ \hline -2x + y = -8 \end{array}$$

$$\begin{array}{l}
 (4) \\
 (5)
 \end{array}
 \left\{ \begin{array}{l}
 6x + 5y = 8 \\
 -2x + y = -8
 \end{array} \right. \begin{array}{l}
 | 1 \\
 | 3
 \end{array} \Rightarrow \begin{array}{l}
 6x + 5y = 8 \\
 \underline{-6x + 3y = -24} \\
 8y = -16 \\
 y = -2
 \end{array}$$

$$\begin{array}{l}
 (5) \\
 \Rightarrow \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{l}
 -2x - 2 = -8 \\
 -2x = -6 \\
 x = 3
 \end{array}$$

$$\begin{array}{l}
 (1) \\
 \Rightarrow
 \end{array}
 \begin{array}{l}
 3 + 2 - z = 6 \\
 z = -1
 \end{array}$$

$$z = -1 \Rightarrow \underline{S = \{3; -2; -1\}}$$

m) $\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \left\{ \begin{array}{l} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ x - y + z = 17 \end{array} \right. \begin{array}{l} | 1 \\ | 1 \\ | 3 \end{array}$

pour éliminer y

$$\begin{array}{r}
 3x - y + z = 29 \\
 \underline{-x + y - z = -17} \\
 2x = 12 \\
 x = 6
 \end{array}$$

$$\begin{array}{r}
 x + 3y + 30z = 6 \\
 + \\
 3x - 3y + 3z = 51 \\
 \underline{4x + 33z = 57} \quad (4)
 \end{array}$$

$$\begin{array}{l}
 (4) \\
 \Rightarrow \\
 \\
 \\
 \end{array}
 \begin{array}{l}
 24 + 33z = 57 \\
 33z = 33 \\
 z = 1
 \end{array}$$

$$\begin{array}{l}
 (3) \\
 \Rightarrow
 \end{array}
 \begin{array}{l}
 6 - y + 1 = 17 \\
 y = -10
 \end{array}$$

$$\Rightarrow \underline{S = \{6; -10; 1\}}$$

o) $\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \left\{ \begin{array}{l} 2x + y - z = 1 \\ x + 2y + z = 8 \\ 3x - y + 2z = 7 \end{array} \right. \begin{array}{l} | 1 \\ | 1 \\ | 1 \end{array} \begin{array}{l} | 2 \\ | 1 \\ | 1 \end{array}$

$$\begin{array}{l}
 3x + 3y = 9 \quad | :3 \\
 \Rightarrow x + y = 3
 \end{array}$$

$$\begin{array}{r}
 4x + 2y - 2z = 2 \\
 + \\
 3x - y + 2z = 7 \\
 \underline{7x + y = 9}
 \end{array}$$

$$\Rightarrow \begin{cases} (4) & x+y = 3 & | & 1 \\ (5) & 7x+y = 9 & | & -1 \end{cases}$$

$$\begin{array}{r} x+y = 3 \\ -7x-y = -9 \\ \hline -6x = -6 \\ x = 1 \end{array}$$

$$\begin{array}{l} (4) \\ \Rightarrow 1+y = 3 \\ y = 2 \end{array}$$

$$\begin{array}{l} (1) \\ \Rightarrow 2+2-z = 1 \\ z = 3 \end{array}$$

$$\Rightarrow \underline{S = \{1; 2; 3\}}$$

facultativ:

$$p) \begin{cases} (1) & 2x-y+3z = 4 & | & 1 \\ (2) & 3x+4y-z = -5 & | & 3 & 4 \\ (3) & x+5y-4z = -9 & | & -1 \end{cases}$$

$$\begin{array}{r} 2x-y+3z = 4 \\ 9x+12y-3z = -15 \\ \hline 11x+11y = -11 \quad | :11 \\ x+y = -1 \end{array}$$

$$\begin{array}{r} 12x+16y-4z = -20 \\ -x-5y+4z = 9 \\ \hline 11x+11y = -11 \\ x+y = -1 \end{array}$$

ceea ce în ecuații \Rightarrow sistem indeterminat

$$\Rightarrow y = -1-x$$

$$\begin{array}{l} (2) \\ \Rightarrow 3x+4(-1-x)-z = -5 \\ 3x-4-4x+z = -5 \\ -x+z = -1 \\ z = -x+1 \end{array}$$

$$\Rightarrow \underline{S = \{x; -1-x; -x+1\}}$$

Ex 2.5.22

$$b) \begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\ x + y = 30 \end{cases} \Rightarrow y = 30 - x$$

$$\stackrel{(1)}{\Rightarrow} \frac{x}{30-x} - \frac{30-x}{x} = \frac{5}{6} \quad | \cdot 6x(30-x) \quad \text{v.i. : 0 et 30}$$

$$6x^2 - 6(30-x)^2 = 5x(30-x)$$

$$6x^2 - 6(900 - 60x + x^2) = 150x - 5x^2$$

$$6x^2 - 5400 + 360x - 6x^2 = 150x - 5x^2$$

$$5x^2 + 210x - 5400 = 0 \quad | :5$$

$$x^2 + 42x - 1080 = 0 \quad \Delta = 6084 = 78^2$$

$$x_{1,2} = \frac{-42 \pm 78}{2} = \begin{cases} 18 \checkmark \stackrel{(2)}{\Rightarrow} y_1 = 30 - 18 = 12 \\ -60 \stackrel{(2)}{\Rightarrow} y_2 = 30 + 60 = 90 \end{cases}$$

$$\Rightarrow \underline{S = \{(18; 12); (-60; 90)\}}$$

$$c) \begin{cases} (1) \frac{1}{x} + \frac{1}{y} = \frac{11}{10} \\ (2) x + y = 11 \end{cases} \Rightarrow y = 11 - x$$

$$\stackrel{(1)}{\Rightarrow} \frac{1}{x} + \frac{1}{11-x} = \frac{11}{10} \quad | \cdot 10x(11-x) \quad \text{v.i. : 0 et 11}$$

$$10(11-x) + 10x = 11x(11-x)$$

$$121 - 10x + 10x = 121x - 11x^2$$

$$11x^2 - 121x + 110 = 0 \quad | : 11$$

$$x^2 - 11x + 10 = 0$$

$$(x-10)(x-1) = 0$$

$$x_{1,2} = \begin{cases} 1 \checkmark \stackrel{(2)}{\Rightarrow} y_1 = 11 - 1 = 10 \\ 10 \checkmark \Rightarrow y_2 = 11 - 10 = 1 \end{cases}$$

$$\Rightarrow \underline{S = \{(1; 10); (10; 1)\}}$$

$$h) \begin{cases} (1) & x+y = 9 \\ (2) & x^2-y^2 = 9 \end{cases} \Rightarrow y = 9-x$$

$$\stackrel{(2)}{\Rightarrow} x^2 - (9-x)^2 = 9$$

$$x^2 - (81 - 18x + x^2) = 9$$

$$-81 + 18x = 9$$

$$18x = 90$$

$$x = 5 \stackrel{(1)}{\Rightarrow} y = 9-5 = 4 \Rightarrow \underline{S = \{(5; 4)\}}$$

Ex suppl.

$$\begin{cases} (1) & x^2 + y^2 - 4x - 6y = 5 \\ (2) & x + y = 5 \end{cases} \Rightarrow y = 5-x$$

$$\stackrel{(1)}{\Rightarrow} x^2 + (5-x)^2 - 4x - 6(5-x) = 5$$

$$x^2 + 25 - 10x + x^2 - 4x - 30 + 6x = 5$$

$$2x^2 - 8x - 10 = 0 \quad | : 2$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$\Rightarrow x_{1,2} = \begin{cases} -1 \\ 5 \end{cases} \begin{matrix} \stackrel{(2)}{\Rightarrow} \\ \stackrel{(2)}{\Rightarrow} \end{matrix} \begin{matrix} y_1 = 5 - (-1) = 6 \\ y_2 = 5 - 5 = 0 \end{matrix} \Rightarrow \underline{S = \{(-1; 6); (5; 0)\}}$$