Ex 2.2.19

a) $\displaystyle\int_1^2 \frac{x}{x+6}\,dx = \int_1^2 \left(1 - \frac{6}{x+6}\right)dx$ $\qquad$ $\deg(N) = \deg(D)$: $\begin{array}{r|l} x & x+6 \\ -x-6 & 1 \\ \hline -6 & \end{array}$

$$= x - 6\ln(|x+6|)\Big|_1^2$$

$$= \left(2 - 6\ln(8)\right) - \left(1 - 6\ln(7)\right) = 1 - 6\ln\left(\frac{8}{7}\right) = 1 + 6\ln\left(\frac{7}{8}\right)$$

b) $\displaystyle\int_0^4 \sqrt{x}\,(x+2)\,dx = \int_0^4 x^{1/2}(x+2)\,dx = \int_0^4 \left(x^{3/2} + 2x^{1/2}\right)dx = \frac{1}{5/2}x^{5/2} + 2\cdot\frac{1}{3/2}x^{3/2}\Big|_0^4$

$$= \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3}\Big|_0^4 = \frac{2}{5}\cdot 32 + \frac{4}{3}\cdot 8 - 0 = \frac{352}{15}$$

variante : par partie : $f' = \sqrt{x} = x^{1/2}$ $\quad$ $f = \frac{2}{3}x^{3/2} = \frac{2}{3}\sqrt{x^3}$

$g = x+2$ $\quad$ $g' = 1$

$$I = \frac{2}{3}\sqrt{x^3}(x+2)\Big|_0^4 - \int \frac{2}{3}x^{3/2}\,dx = \frac{2}{3}\sqrt{x^3}(x+2) - \frac{2}{3}\cdot\frac{2}{5}\sqrt{x^5}\Big|_0^4$$

$$= \frac{2}{3}\cdot 8\cdot 6 - \frac{2}{3}\cdot\frac{2}{5}\cdot 32 - 0 = \frac{352}{15}$$

c) $\displaystyle\int_0^{\pi/2} \underbrace{\sin^5(x)}_{u^5}\cdot\underbrace{\cos(x)}_{u'}\,dx = \frac{1}{6}\sin^6(x)\Big|_0^{\pi/2} = \frac{1}{6} - 0 = \frac{1}{6}$

d) $\displaystyle\int_2^{\sqrt{20}} 3x\sqrt{x^2+5}\,dx = \frac{3}{2}\int_2^{\sqrt{20}} 2x\,(x^2+5)^{1/2}\,dx = \frac{3}{2}\cdot\frac{1}{3/2}(x^2+5)^{3/2}\Big|_2^{\sqrt{20}} = \sqrt{(x^2+5)^3}\Big|_2^{\sqrt{20}}$

$u = x^2+5$

$u' = 2x$ $\qquad$ $= 5^3 - 3^3 = 98$

e) ex 2.2.17 b) avec chgmt de variable

f) $\displaystyle\int_2^3 \frac{5x-2}{x^2-x}$

$\deg(N) < \deg(D)$ et $D = x^2-x = x(x-1) \Rightarrow$ décomp. en élts simples.

$$\frac{5x-2}{x^2-x} = \frac{A}{x} + \frac{B}{x-1} = \frac{A(x-1)+Bx}{x(x-1)} = \frac{(A+B)x-A}{x(x-1)}$$

$$\Rightarrow \begin{cases} A+B = 5 \\ -A = -2 \end{cases} \Leftrightarrow \begin{cases} A = 2 \\ B = 3 \end{cases}$$

$$\Rightarrow I = \int_2^3 \left(\frac{2}{x} + \frac{3}{x-1}\right) dx = 2\ln(|x|) + 3\ln(|x-1|)\Big|_2^3$$

$$= 2\ln(3) + 3\ln(2) - 2\ln(2) - \underbrace{3\ln(1)}_{=0}$$

$$= 2\ln(3) + \ln(2) = \ln(3^2 \cdot 2) = \ln(18)$$

g) $$\int_{-1}^{0} \frac{1}{\underbrace{x^2+2x+2}_{\Delta=-4<0}} dx = \int_{-1}^{0} \frac{1}{(x+1)^2+1} dx$$

chgmt de variable : $t = x+1 \Leftrightarrow x = t-1$

$dt = dx$

bornes :

x	t
-1	0
0	1

$$= \int_0^1 \frac{1}{t^2+1} dt$$

$$= \operatorname{Arctan}(t)\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

h) $$\int_0^4 x\sqrt{4-x}\, dx = -\frac{2}{3}x\sqrt{(4-x)^3}\Big|_0^4 + \int_0^4 \frac{2}{3}\sqrt{(4-x)^3}\, dx$$

$f' = \sqrt{4-x} = (4-x)^{1/2}$ $\qquad f = -\frac{2}{3}(4-x)^{3/2}$

$g = x$ $\qquad g' = 1$

$$= -\frac{2}{3}\left[x\sqrt{(4-x)^3} - \left(-\frac{2}{5}\sqrt{(4-x)^5}\right)\right]\Big|_0^4$$

$$= -\frac{2}{3}\left(0 + 0 - \left(0 + \frac{2}{5}\cdot 2^5\right)\right) = \frac{4}{15}\cdot 32 = \frac{128}{15}$$