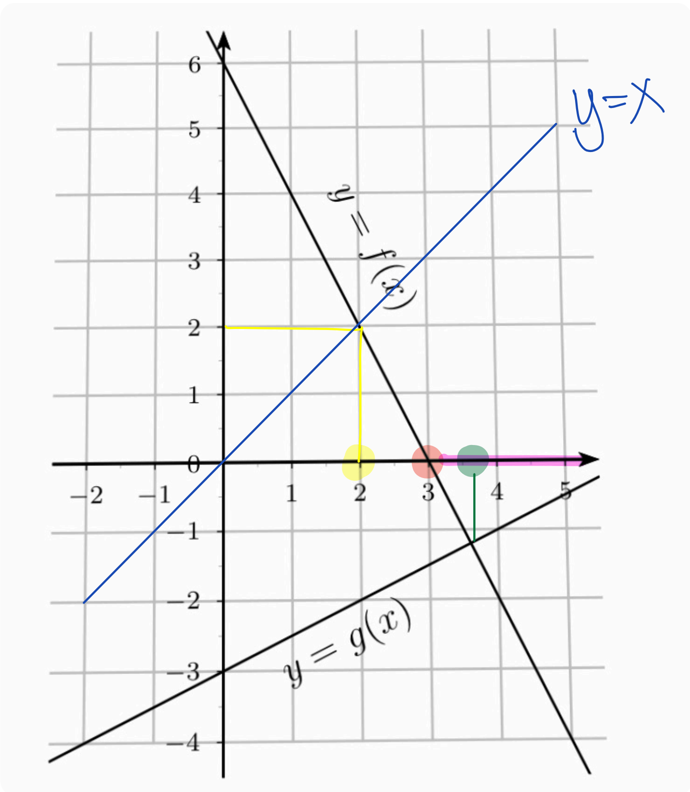
Ex 3.3.3



a) $f(x) = 0$

$-2x + 6 = 0$

$\Leftrightarrow \quad x = 3 \quad \Rightarrow S = \{3\}$

b) $f(x) = g(x)$

$-2x + 6 = \frac{1}{2}x - 3$

$\Leftrightarrow \quad -\frac{5}{2}x = -9$

$\Leftrightarrow \quad x = \frac{18}{5} \quad \Rightarrow S = \left\{\frac{18}{5}\right\}$

c) $f(x) = x$

$-2x + 6 = x$

$\Leftrightarrow \quad -3x = -6$

$\Leftrightarrow \quad x = 2 \quad \Rightarrow \quad S = \{2\}$

d) $f(x) < 0$

$-2x + 6 < 0$

$-2x < -6$

$x > 3 \quad \Rightarrow S =]3; +\infty[$

e) $f(x) > g(x)$

$-2x + 6 > \frac{1}{2}x - 3$

$-\frac{5}{2}x > -9$

$x < \frac{18}{5} \quad \Rightarrow S = \left]-\infty; \frac{18}{5}\right[$

f) $f(x) \geqslant x$

$-2x + 6 \geqslant x$

$-3x \geqslant -6$

$x \leqslant 2 \quad \Rightarrow S =]-\infty; 2]$

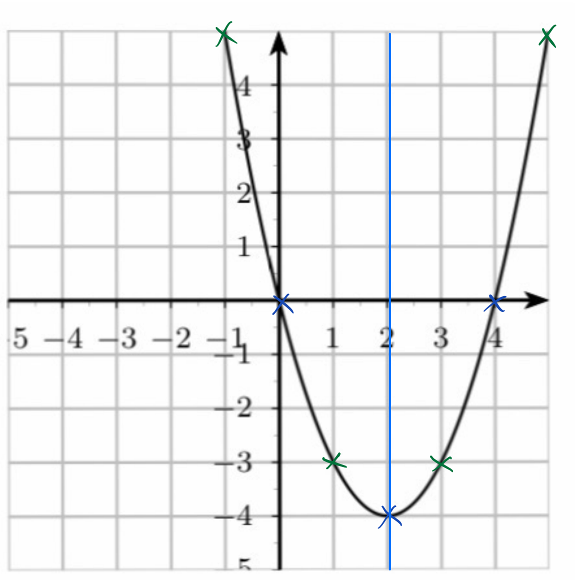
Ex 3.3.7

a) $f(x) = x^2 - 4x = x(x-4)$ $\quad a = 1 > 0 \Rightarrow \cup$

zéros : 0 et 4 $\quad$ oà O : 0

sommet : $x_S = 2$ et $f(2) = -4$ $\Rightarrow$ $S(2;-4)$

au milieu des deux zéros



autres points :

- $f(-1) = 1+4 = 5 \Rightarrow (-1;5)$
  et $(5;5)$ par sym.

- $f(1) = 1-4 = -3 \Rightarrow (1;-3)$
  et $(3;3)$ par sym.

b) $f(x) = 2x^2 - 4x - 2 = 2(x^2 - 2x - 1)$ $\quad a = 2 > 0 \Rightarrow \cup$

zéros : $\Delta = 8 \Rightarrow x_{1,2} = \frac{2 \pm \sqrt{8}}{2} \cong \begin{cases} 2{,}4 \\ -0{,}4 \end{cases}$

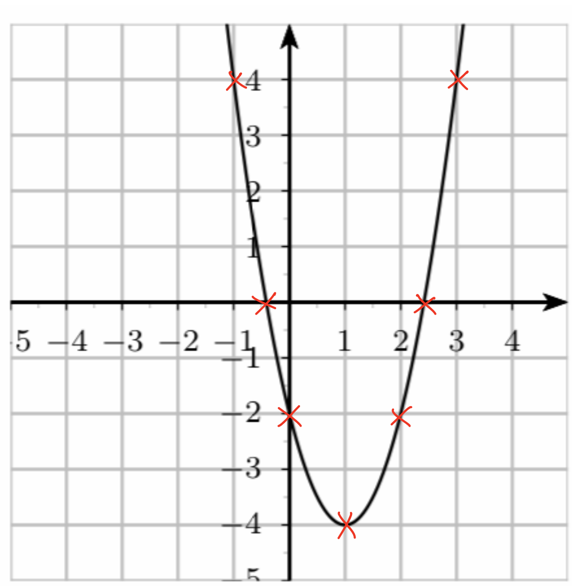
sommet : $x_S = \frac{2}{2} = 1$ et $f(1) = -4$ $\Rightarrow$ $S(1;-4)$

oà O : -2

autres points :

- par sym : $(2;-2)$
  (oà O)

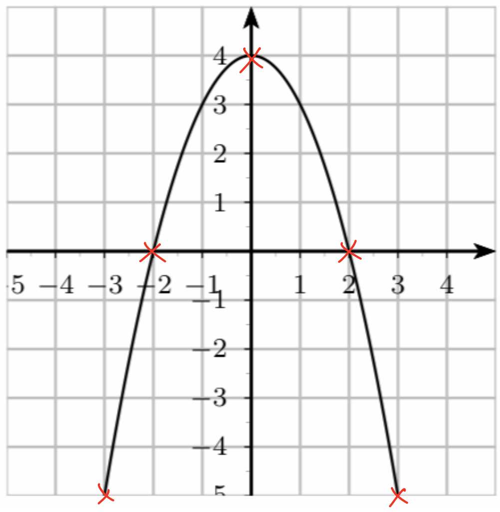
- $f(-1) = 2+4-2 = 4 \Rightarrow (-1;4)$
  et par sym : $(3;4)$



c) $f(x) = -x^2+4 = -(x+2)(x-2)$ $\quad a=-1<0 \Rightarrow \cap$

zéros : -2 et 2

sommet : $x_S = 0 \Rightarrow f(0)=4 \Rightarrow S(0;4)$ et oàO : 4



autres points :

- $f(3) = -9+4 = -5 \Rightarrow (3;-5)$

 et par sym : $(-3;-5)$

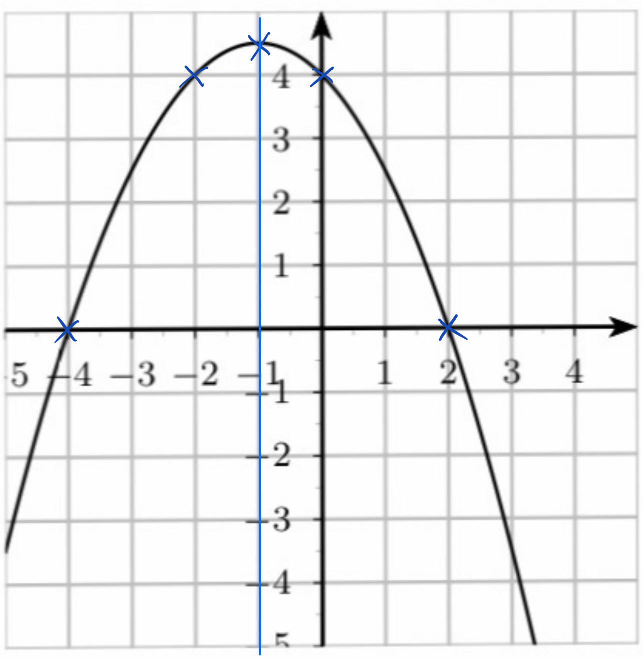
d) $f(x) = -\frac{1}{2}x^2 - x + 4 = -\frac{1}{2}(x^2+2x-8) = -\frac{1}{2}(x+4)(x-2)$

$a = -\frac{1}{2} < 0 \Rightarrow \cap$

zéros : -4 et 2

sommet : $x_S = -1 \Rightarrow f(-1) = -\frac{1}{2}+1+4 = \frac{9}{2} \Rightarrow S(-1;\frac{9}{2})$

oàO : 4



autre point : par sym : $(-2;4)$ (oàO)

Ex 3.3.8

a) $6x^2 - x - 2$

zéro(s) : $\Delta = 1 + 4 \cdot 6 \cdot (+2) = 49$

$$x_{1,2} = \frac{1 \pm 7}{12} = \begin{cases} \frac{8}{12} = \frac{2}{3} \\ -\frac{6}{12} = -\frac{1}{2} \end{cases}$$

signe :

x		$-1/2$		$2/3$	
$6x^2-x-2$	$+$	0	$-$	0	$+$

car $a = 6 > 0$

b) $8x^2 - 10x + 3$

zéro(s) : $\Delta = 100 - 4 \cdot 8 \cdot 3 = 4$

$$x_{1,2} = \frac{10 \pm 2}{16} = \begin{cases} \frac{12}{16} = \frac{3}{4} \\ \frac{8}{16} = \frac{1}{2} \end{cases}$$

signe :

x		$1/2$		$3/4$	
$8x^2-10x+3$	$+$	0	$-$	0	$+$

car $a = 8 > 0$

c) $-x^2 + 6x - 9 = -(x^2 - 6x + 9) = -(x-3)^2$

zéro(s) : 3

signe :

x		3	
$-x^2+6x+9$	$-$	0	$-$

car $a = -1 < 0$

d) $-2x^2 + 7x + 4$

zéro(s) : $\Delta = 49 + 4 \cdot (+2) \cdot 4 = 81$

$$x_{1,2} = \frac{-7 \pm 9}{-4} = \begin{cases} \frac{2}{-4} = -\frac{1}{2} \\ -\frac{16}{-4} = 4 \end{cases}$$

signe :

x		$-1/2$		4	
$-2x^2+7x+4$	$-$	0	$+$	0	$-$

car $a = -2 < 0$

e) $25x^2 - 30x + 34$

zéro(s) : $\Delta = 900 - 4 \cdot 25 \cdot 34 = -2500 \Rightarrow$ pas de zéro

signe : tjrs positif :

x	
sgn	$+$

car $a = 25 > 0$

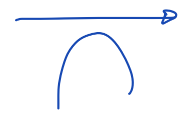
f) $-3x^2+24x-60$

zéro(s) : $24^2-4\cdot(-3)\cdot(-60) = -144$ $\Rightarrow$ pas de zéro

signe : tjrs négatif :

x	
sgn	$-$

$\leftarrow$ car $a=-3<0$

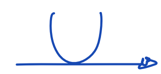


g) $6x^2$ zéro : 0

signe :

x		0	
sgn	$+$	0	$+$

car $a=6>0$



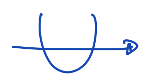
h) $6x^2+7x = x(6x+7)$

zéros : 0 et $-\frac{7}{6}$

signe :

x		$-7/6$		0	
sgn	$+$	0	$-$	0	$+$

$\leftarrow$ car $a=6>0$



i) $8x^2-25$

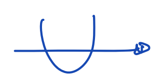
zéros : $\Delta = 0-4\cdot8\cdot(-25) = 800$

$$x_{1,2} = \frac{0\pm\sqrt{800}}{16} = \pm\frac{20\sqrt{2}}{16} = \pm\frac{5\sqrt{2}}{4}$$

signe :

x		$-\frac{5\sqrt{2}}{4}$		$\frac{5\sqrt{2}}{4}$	
sgn	$+$	0	$-$	0	$+$

$\leftarrow$ car $a=8>0$



j) $9x^2-42x+9$

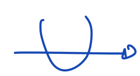
zéros : $\Delta = 42^2-4\cdot9\cdot9 = 1440$

$$x_{1,2} = \frac{42\pm\sqrt{1440}}{18} = \frac{42\pm12\sqrt{10}}{18} = \frac{7\pm2\sqrt{10}}{3} \simeq \begin{cases} 4{,}4 \\ 0{,}2 \end{cases}$$

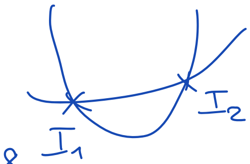
signe :

x		$0{,}2$		$4{,}4$	
sgn	$+$	0	$-$	0	$+$

$\leftarrow$ car $a=9>0$



Ex 3.3.14



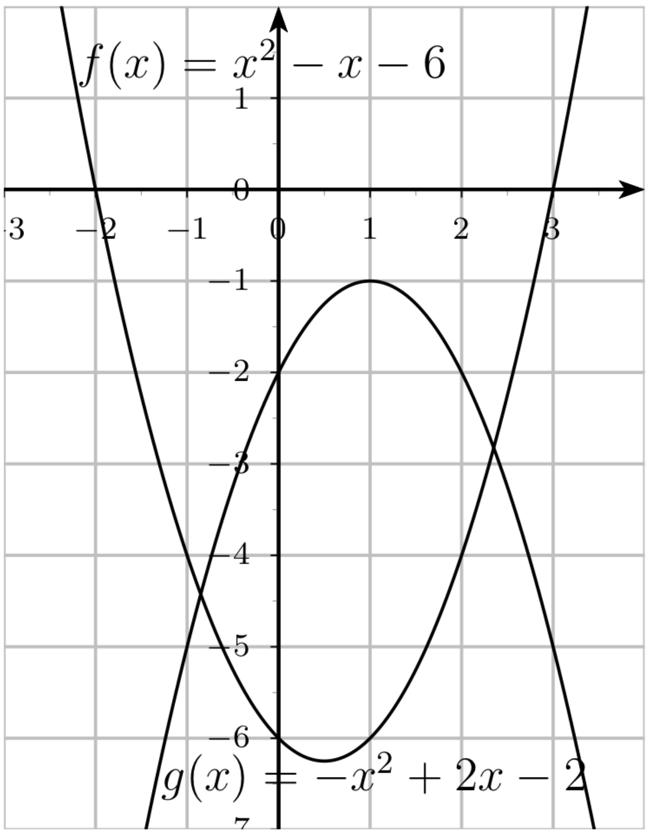
$$f \cap g \iff f(x) = g(x) \iff 2x^2 - 4x + 3 = 3x^2 - 12x + 18$$
$$\iff 0 = x^2 - 8x + 15$$
$$\iff (x-3)(x-5) = 0$$

$$\iff x = \begin{cases} 3 \Rightarrow f(3) = 2\cdot 9 - 4\cdot 3 + 3 = 9 \Rightarrow I_1(3;9) \\ 5 \Rightarrow f(5) = 2\cdot 25 - 4\cdot 5 + 3 = 33 \Rightarrow I_2(5;33) \end{cases}$$

variante :

$$x = \begin{cases} 3 \Rightarrow g(3) = 3\cdot 9 - 12\cdot 3 + 18 = 9 \Rightarrow I_1(3;9) \\ 5 \Rightarrow g(5) = 3\cdot 25 - 12\cdot 5 + 18 = 33 \Rightarrow I_2(5;33) \end{cases}$$

Ex 3.3.15



Pour f :

O à O : $f(0) = -6$

zéros : $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\swarrow \quad \searrow$

$3 \quad -2$

sommet : $x_S = \frac{3+(-2)}{2} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 6 = \frac{1}{4} - \frac{2}{4} - \frac{24}{4} = -\frac{25}{4}$

$\Rightarrow S_f\left(\frac{1}{2} ; -\frac{25}{4}\right) = (0{,}5 ; -6{,}25)$

Pour g :

O à O : $g(0) = -2$

zéro : $-x^2 + 2x - 2 = 0 \quad \Delta = -4 < 0$

$\Rightarrow$ pas de zéro

sommet : $x_S = \frac{-b}{2a} = \frac{-2}{-2} = 1$

$g(1) = -1 + 2 - 2 = -1 \Rightarrow S_g(1 ; -1)$

a) et b) voir zéros de f et g

c) $f(x) = g(x)$

$x^2 - x - 6 = -x^2 + 2x - 2$

$2x^2 - 3x - 4 = 0 \qquad \Delta = 9 + 32 = 41$

$x_{1,2} = \frac{3 \pm \sqrt{41}}{4} \cong \begin{cases} 2{,}4 \\ -0{,}9 \end{cases}$ (1e coord des pts d'$\cap$ de f et g)

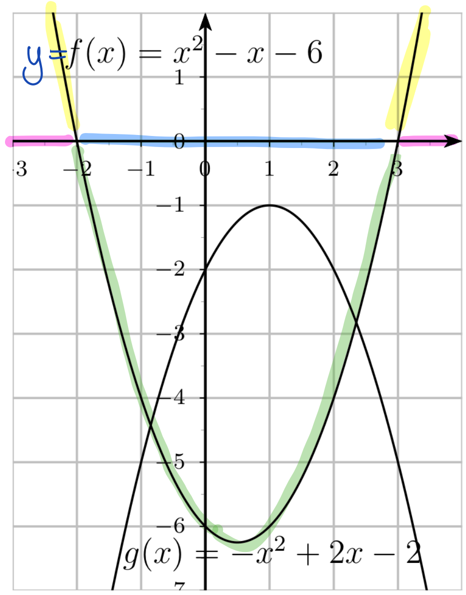
$S = \{2{,}4 ; -0{,}9\}$

d) $f(x) > 0$

$x \in]-\infty ; -2[\cup]3 ; +\infty[$

e) $f(x) < 0$

$x \in]-2 ; 3[$



f) $g(x) \geqslant 0$ faux car la parabole est en-dessous de l'axe Ox

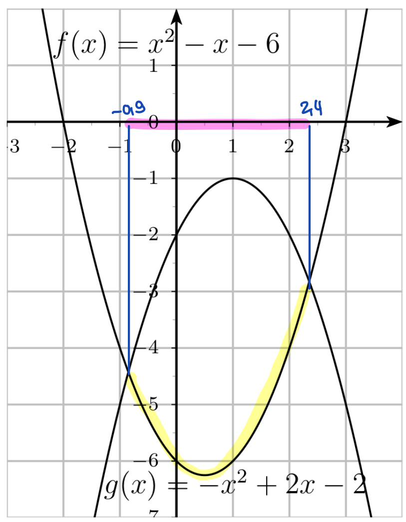
$\Rightarrow S = \emptyset$

g) $g(x) \leqslant 0$ tjrs vrai " " "

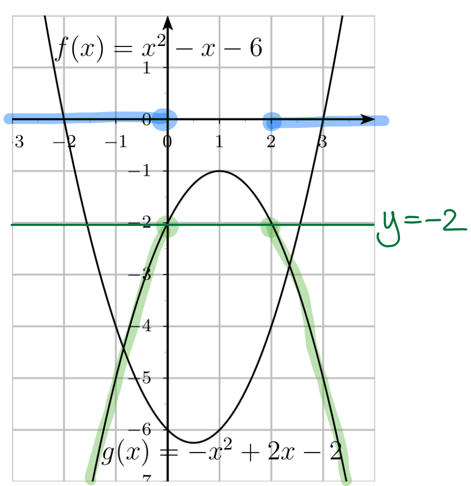
$\Rightarrow S = \mathbb{R}$

h) $f(x) < g(x)$ " f au-dessous de g "

$x \in]\sim -0{,}9 ; \sim 2{,}4[$



i)



$g(x) \leqslant -2$ " g au-dessous de $y=-2$ "

$x \in]-\infty ; 0] \cup [2 ; +\infty[$

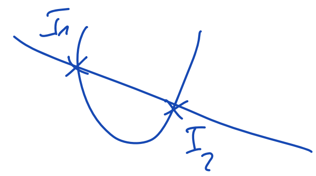
Ex 3.3.18

a) $f \cap g \Leftrightarrow f(x) = g(x)$

$\Leftrightarrow x^2 - 5x + 4 = -4x + 10$

$\Leftrightarrow x^2 - x - 6 = 0$

$\Leftrightarrow (x-3)(x+2) = 0$



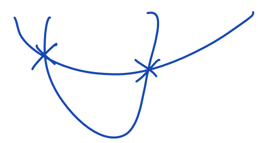
$\Leftrightarrow \begin{cases} x_1 = 3 \Rightarrow y_1 = f(x_1) = f(3) = 3^2 - 5\cdot 3 + 4 = -2 \Rightarrow \underline{I_1(3;-2)} \\ x_2 = -2 \Rightarrow y_2 = f(x_2) = f(-2) = (-2)^2 - 5\cdot(-2) + 4 = 18 \Rightarrow \underline{I_2(-2;18)} \end{cases}$

b) $f \cap g \Leftrightarrow f(x) = g(x)$

$\Leftrightarrow x^2 + x = 2x^2 - 6$

$\Leftrightarrow 0 = x^2 - x - 6$

$\Leftrightarrow (x-3)(x+2) = 0$



$\Leftrightarrow \begin{cases} x_1 = 3 \Rightarrow y_1 = f(x_1) = f(3) = 3^2 + 3 = 12 \Rightarrow \underline{I_1(3;12)} \\ x_2 = -2 \Rightarrow y_2 = f(x_2) = f(-2) = (-2)^2 + (-2) = 2 \Rightarrow \underline{I_2(-2;2)} \end{cases}$