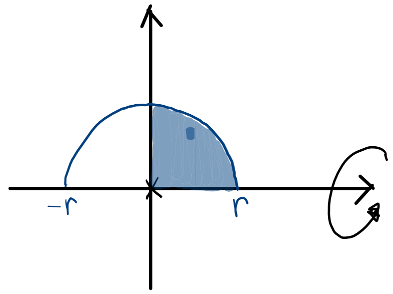
Volume d'une sphère de rayon r



équation d'un cercle de centre O(0,0) et de rayon r :

$$x^2+y^2=r^2$$

$$y=(\pm)\sqrt{r^2-x^2}$$

$$V=2\cdot\pi\int_0^r (r^2-x^2)\,dx = 2\cdot\pi\left(r^2x-\frac{1}{3}x^3\right)\Big|_0^r = 2\pi\left(r^3-\frac{1}{3}r^3-0\right)$$

$$= 2\pi\cdot\frac{2}{3}r^3 = \frac{4\pi r^3}{3}$$

Aire d'un disque de rayon r

$$\mathcal{A}=4\cdot\int_0^r\sqrt{r^2-x^2}\,dx$$

$x=r\sin(t) \iff t=\operatorname{Arcsin}\left(\frac{x}{r}\right)$

$dx=r\cos(t)\,dt$

bornes : $0=r\sin(t) \iff t=\operatorname{Arcsin}(0)=0$

$r=r\sin(t) \iff t=\operatorname{Arcsin}(1)=\frac{\pi}{2}$

$$=4\int_0^{\pi/2}\sqrt{\underbrace{r^2-r^2\sin^2(t)}_{=r^2(1-\sin^2(t))}}\cdot r\cos(t)\,dt$$

$$=\underbrace{r^2\cos^2(t)}_{r\cos(t)}$$

$$=4\int_0^{\pi/2}r^2\cos^2(t)\,dt = 4r^2\int_0^{\pi/2}\cos^2(t) = 4r^2\cdot\frac{1}{2}\left(t+\sin(t)\cos(t)\right)\Big|_0^{\pi/2}$$

$$=2r^2\left(\frac{\pi}{2}+0-(0+0)\right) = \pi r^2$$