Ex 1.2.1

a) $7^{x+6} = 7^{3x+4} \iff x+6 = 3x+4$

$\iff -2x = -2$

$\iff x = 1 \implies \underline{S = \{1\}}$

b) $6^{7-x} = 6^{2x+1} \iff 7-x = 2x+1$

$\iff -3x = -6$

$\iff x = 2 \implies \underline{S = \{2\}}$

c) $3^{2x+3} = 3^{x^2} \iff 2x+3 = x^2$

$\iff x^2-2x-3 = 0$

$\iff (x-3)(x+1) = 0$

$\iff x=3$ ou $x=-1 \implies \underline{S = \{-1; 3\}}$

d) $9^{x^2} = 3^{3x+2} \iff (3^2)^{x^2} = 3^{3x+2}$

$\iff 3^{2x^2} = 3^{3x+2}$

$\iff 2x^2 = 3x+2$

$\iff 2x^2 - 3x - 2 = 0 \qquad \Delta = 9+16 = 25$

$\iff x_{1,2} = \dfrac{3 \pm 5}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases} \implies \underline{S = \{-\tfrac{1}{2}; 2\}}$

e) $2^{-100x} = 0{,}5^{x-4} \iff 2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$

$\iff 2^{-100x} = (2^{-1})^{x-4}$

$\iff 2^{-100x} = 2^{-x+4}$

$\iff -100x = -x+4$

$\iff -99x = 4$

$\iff x = -\frac{4}{99} \implies \underline{S = \{-\tfrac{4}{99}\}}$

f) $\left(\frac{1}{4}\right)^{6-x} = 4 \Leftrightarrow (4^{-1})^{6-x} = 4$

$\Leftrightarrow 4^{-6+x} = 4$

$\Leftrightarrow -6+x = 1$

$\Leftrightarrow x = 7 \quad \Rightarrow \quad S = \{7\}$

g) $27^{x-1} = 9^{2x-3} \Leftrightarrow (3^3)^{x-1} = (3^2)^{2x-3}$

$\Leftrightarrow 3^{3x-3} = 3^{4x-6}$

$\Leftrightarrow 3x-3 = 4x-6$

$\Leftrightarrow -x = -3 \quad \Rightarrow \quad S = \{3\}$

h) $2^x \cdot 4^x = -5 \Leftrightarrow 2^x \cdot 2^{2x} = -5$

$\Leftrightarrow \underbrace{2^{3x}}_{>0} = -5 < 0$ ↯ impossible $\Rightarrow S = \emptyset$

i) $(5^{x-2})^4 = 125 \cdot 5^{5x-3} \Leftrightarrow 5^{4x-8} = 5^3 \cdot 5^{5x-3}$

$\Leftrightarrow 5^{4x-8} = 5^{5x}$

$\Leftrightarrow 4x-8 = 5x$

$\Leftrightarrow -x = 8 \quad \Rightarrow \quad S = \{-8\}$

j) $(3^{x-1})^3 = 9 \cdot 3^{x-2} \Leftrightarrow 3^{3x-3} = 3^2 \cdot 3^{x-2}$

$\Leftrightarrow 3^{3x-3} = 3^x$

$\Leftrightarrow 3x-3 = x$

$\Leftrightarrow 2x = 3 \quad \Rightarrow \quad S = \left\{\frac{3}{2}\right\}$

b) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$3^{4x+2} - 36 \cdot 3^{2x+1} + 243 = 0$

$(3^{2x+1})^2 - 36 \cdot 3^{2x+1} + 243 = 0$

Changement de variable : on pose $y = 3^{2x+1}$ ⊛

$\Rightarrow \quad y^2 - 36y + 243 = 0$

$(y-9)(y-27) = 0$

$y = 9 \overset{⊛}{\Rightarrow} 3^{2x+1} = 9 \Leftrightarrow 3^{2x+1} = 3^2 \Leftrightarrow 2x+1 = 2 \Leftrightarrow x = \frac{1}{2}$

$y = 27 \overset{⊛}{\Rightarrow} 3^{2x+1} = 27 \Leftrightarrow 3^{2x+1} = 3^3 \Leftrightarrow 2x+1 = 3 \Leftrightarrow x = 1$

$\Rightarrow \quad S = \left\{ \frac{1}{2} ; 1 \right\}$

l) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$5^{1+4x-7} - 120 \cdot 5^{2x-3} - 625 = 0$

$5^{4x-6} - 120 \cdot 5^{2x-3} - 625 = 0$

$(5^{2x-3})^2 - 120 \cdot 5^{2x-3} - 625 = 0$

Chgmt de variable : $y = 5^{2x-3}$

$\Rightarrow \quad y^2 - 120y - 625 = 0$

$(y-125)(y+5) = 0$

$y = 125 \Rightarrow 5^{2x-3} = 125 \Leftrightarrow 5^{2x-3} = 5^3 \Leftrightarrow 2x-3 = 3 \Leftrightarrow x = 3$

$y = -5 \Rightarrow \underbrace{5^{2x-3}}_{\text{positif}} = -5$ ↯ impossible

$\Rightarrow \quad S = \{3\}$

Ex 1.2.2

a) $\log_3(1) = \underline{0} \iff 3^0 = 1$

b) $\log_2(8) = \underline{3} \iff 2^3 = 8$

c) $\log_2(64) = \underline{6} \iff 2^6 = 64$

d) $\log_2(1024) = \underline{10} \iff 2^{10} = 1024$

e) $\log_5(5) = \underline{1} \iff 5^1 = 5$

f) $\log_3(\sqrt{3}) = \underline{\frac{1}{2}} \iff 3^{1/2} = \sqrt{3}$

g) $\log_{243}\left(\frac{1}{243}\right) = \underline{-1} \iff 243^{-1} = \frac{1}{243}$

h) $\log_3(27) = \underline{3} \iff 3^3 = 27$

i) $\log(1000) = \underline{3} \iff 10^3 = 1000$

j) $\log_4(\sqrt{2}) = \underline{\frac{1}{4}} \iff 4^x = \sqrt{2} \iff 2^{2x} = 2^{1/2} \iff 2x = \frac{1}{2} \iff x = \frac{1}{4}$

k) $\log_{\frac{1}{8}}(64) = \underline{-2} \iff \left(\frac{1}{8}\right)^x = 64 \iff 8^{-x} = 8^2 \iff -x = 2$

l) $\log_5(0{,}04) = \underline{-2} \iff 5^x = 0{,}04 = \frac{1}{25} \iff 5^x = 5^{-2} \iff x = -2$

m) $\log_3(\sqrt[4]{27}) = \underline{\frac{3}{4}} \iff 3^x = \sqrt[4]{27} = \sqrt[4]{3^3} = 3^{3/4} \iff x = \frac{3}{4}$

n) $\ln(e^2) = \underline{2} \iff e^2 = e^2$

o) $\log_a(a) = \underline{1} \iff a^1 = a$

p) $\log_a(a^3) = \underline{3} \iff a^3 = a^3$

q) $\log(10'000) = \underline{4} \iff 10^4 = 10'000$

r) $\ln(e) = \underline{1} \iff e^1 = e$

s) $\log_2\left(\frac{1}{8}\right) = \underline{-3} \iff 2^{-3} = \frac{1}{8}$

t) $\log_3(\sqrt[4]{3}) = \underline{\frac{1}{4}} \iff 3^{1/4} = \sqrt[4]{3}$

u) $\log(200) - \log(2) = \log\left(\frac{200}{2}\right) = \log(100) = \underline{2} \iff 10^2 = 100$

v) $\log_6(4) + \log_6(9) = \log_6(4 \cdot 9) = \log_6(36) = \underline{2} \iff 6^2 = 36$

w) $\log_5(1) = \underline{0} \iff 5^0 = 1$

x) $\log(-1)$ <u>non défini</u> $(u > 0)$

y) $\log(0{,}0001) = \underline{-4} \iff 10^{-4} = 0{,}0001$

z) $\ln(0)$ <u>non défini</u> $(u > 0)$

Ex 1.2.3

$$\log(2) \cong 0{,}3010 \quad \text{et} \quad \log(3) \cong 0{,}4771$$

a) $\log(6) = \log(2\cdot 3) = \log(2) + \log(3) \cong 0{,}3010 + 0{,}4771 = \underline{0{,}7781}$

b) $\log(16) = \log(2^4) = 4\cdot\log(2) \cong 4\cdot 0{,}3010 = \underline{1{,}204}$

c) $\log(\sqrt{2}) = \log(2^{1/2}) = \frac{1}{2}\log(2) \cong \frac{1}{2}\cdot 0{,}3010 = \underline{0{,}1505}$

d) $\log(0{,}5) = \log\left(\frac{1}{2}\right) = -\log(2) \cong \underline{-0{,}3010}$

e) $\log(36) = \log(6^2) = 2\cdot\log(6) \cong 2\cdot 0{,}7781 = \underline{1{,}5562}$

f) $\log\left(\frac{8}{27}\right) = \log\left(\left(\frac{2}{3}\right)^3\right) = 3\cdot\log\left(\frac{2}{3}\right) = 3(\log(2)-\log(3))$

$$\cong 3(0{,}3010 - 0{,}4771) = \underline{-0{,}5283}$$

Ex 1.2.4

a) $\log(16) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(9)$

$= \log(16) + \log(3^2) - \log(2^2) - \log(9^{1/2})$

$= \log(16) + \log(9) - \log(4) - \log(3)$

$= \log(16\cdot 9) - (\log(4) + \log(3))$

$= \log(16\cdot 9) - \log(4\cdot 3) = \log\left(\frac{16\cdot 9}{4\cdot 3}\right) = \underline{\log(12)}$

b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

$= \log(15) + \log(10^3) - (\log(30) + \log(5))$

$= \log(15\cdot 10^3) - \log(30\cdot 5) = \log\left(\frac{15\cdot 10^3}{30\cdot 5}\right) = \log(100) = \underline{2}$

c) $4\log(5) + \log\left(\frac{1}{5}\right) - 3\log(3) + \frac{1}{3}\log(27)$

$= \log(5^4) - \log(5) - \log(3^3) + \log(\underbrace{27^{1/3}}_{3})$

$= \log\left(\frac{5^4 \cdot 3}{5 \cdot 3^3}\right) = \log\left(\frac{125}{9}\right)$

d) $\dfrac{\log(20) + \log(100) - \log(2)}{\log(5'000) - \log(5) + \log(0{,}1)} = \dfrac{\log\left(\frac{20 \cdot 100}{2}\right)}{\log\left(\frac{5000 \cdot 0{,}1}{5}\right)} = \dfrac{\log(1000)}{\log(100)} = \dfrac{3}{2}$

Ex 1.2.5

a) $x = \log_2(32) \Leftrightarrow 2^x = 32 = 2^5 \Leftrightarrow x = 5 \Rightarrow S = \{5\}$

b) $2^x = 100 \Leftrightarrow x = \log_2(100) \Rightarrow S = \{\log_2(100)\}$

c) $\log_x(256) = 4 \Leftrightarrow x^4 = 256 = 4^4 \Leftrightarrow x = 4 \Rightarrow S = \{4\}$

d) $\log_2(x) = 4 \Leftrightarrow 2^4 = x \Leftrightarrow x = 16 \Rightarrow S = \{16\}$

e) $10^x = 5 \Leftrightarrow x = \log(5) \Rightarrow S = \{\log(5)\}$

f) $e^{2x-1} = 27$

$2x - 1 = \ln(27)$

$2x = \ln(27) + 1$

$x = \dfrac{\ln(27)+1}{2} \Rightarrow S = \left\{\dfrac{\ln(27)+1}{2}\right\}$

g) $\log_x(1'000) = 3 \Leftrightarrow x^3 = 1000 = 10^3 \Leftrightarrow x = 10 \Rightarrow S = \{10\}$

h) $\underbrace{12^x}_{>0} = -49$ ↯ $\Rightarrow S = \emptyset$

Ex 1.2.6

a) $\log_{11}(x+1) = \log_{11}(7) \Leftrightarrow x+1 = 7 \Leftrightarrow x = 6$

vérif : $\log_{11}(7) = \log_{11}(7)$ ✓ $\Rightarrow S = \{6\}$

b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$

$\Leftrightarrow \log_6(2x-3) = \log_6\left(\frac{12}{3}\right)$

$\Leftrightarrow 2x-3 = 4$

$\Leftrightarrow x = \frac{7}{2}$

vérif : $\log_6(7-3) = \log_6(12) - \log_6(3)$ ✓ $\Rightarrow S = \left\{\frac{7}{2}\right\}$

c) $\log(x) - \log(x+1) = 3\log(4)$

$\Leftrightarrow \log\left(\frac{x}{x+1}\right) = \log(4^3)$

$\Leftrightarrow \frac{x}{x+1} = 64 \quad \Big| \cdot (x+1)$

$\Leftrightarrow x = 64(x+1)$

$\Leftrightarrow x = 64x + 64$

$\Leftrightarrow -63x = 64$

$\Leftrightarrow x = -\frac{64}{63}$

vérif : $\log\left(-\frac{64}{63}\right)$ ϟ $\Rightarrow S = \emptyset$

d) $2\log_3(x) = 3\log_3(5)$

$\Leftrightarrow \log_3(x^2) = \log_3(5^3)$

$\Leftrightarrow x^2 = 125$

$\Leftrightarrow x = \pm\sqrt{125} = \pm 5\sqrt{5}$

vérif : $2\log_3(\sqrt{125}) = 3\log_3(5)$ ✓

$2\log_3(-\sqrt{125})$ ϟ

$\Rightarrow S = \{5\sqrt{5}\}$

e) $\ln(x) + \ln(x-2) = 0{,}5\ln(9)$

$\Leftrightarrow \ln\big(x(x-2)\big) = \ln\left(9^{0{,}5}\right) \qquad \left(9^{1/2} = 3\right)$

$\Leftrightarrow x(x-2) = 3$

$\Leftrightarrow x^2 - 2x = 3$

$\Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow (x-3)(x+1) = 0$

$\Leftrightarrow x = 3$ or $x = -1$

vérif : $\ln(3) + \ln(1) = 0{,}5\ln(9)$ ✓

$\ln(-1)$ ϟ

$\Rightarrow S = \{3\}$

f) $\log_8(x+4) = 1 - \log_8(x-3)$

$\Leftrightarrow \log_8(x+4) + \log_8(x-3) = 1$

$\Leftrightarrow \log_8\big((x+4)(x-3)\big) = 1$

$\Leftrightarrow (x+4)(x-3) = 8$

$\Leftrightarrow x^2 + x - 12 = 8$

$\Leftrightarrow x^2 + x - 20 = 0$

$\Leftrightarrow (x-4)(x+5) = 0$

$\Leftrightarrow x = 4$ or $x = -5$

vérif : $\log_8(8) = 1 - \log_8(1)$ ✓

$\log_8(-1)$ ϟ

$\Rightarrow S = \{4\}$

Variante :

$\Leftrightarrow \log_8(x+4) = \log_8(8) - \log_8(x-3)$

$\Leftrightarrow \log_8(x+4) = \log_8\left(\frac{8}{x-3}\right)$

$\Leftrightarrow x+4 = \frac{8}{x-3}$

Ex 1.2.14

$y = 79{,}041 + 6{,}39x - e^{3{,}261 - 0{,}933x}$ taille en cm en fct de l'âge x en années

si $x = 1$ $\Rightarrow$ $y = 79{,}041 + 6{,}39 \cdot 1 - e^{3{,}261 - 0{,}933 \cdot 1} \hat{=} 75{,}77$

La taille d'un enfant d'une année est d'environ 75,8 cm.

Ex 1.2.15

$\ln(m) = \ln(2{,}4) + 1{,}84h$ avec h taille en m et m masse en kg

a) $\ln(21{,}8) = \ln(2{,}4) + 1{,}84h$

$\ln(21{,}8) - \ln(2{,}4) = 1{,}84h$

$\dfrac{\ln(21{,}8) - \ln(2{,}4)}{1{,}84} = h \cong 1{,}199$

L'enfant mesure environ 1,20 m.

b) $\ln(m) = \ln(2{,}4) + 1{,}84 \cdot 1{,}5$

$\ln(m) = \underbrace{\ln(2{,}4) + 2{,}76}_{\text{nbre} \ \cong 3{,}64}$

$m = e^{\ln(2{,}4) + 2{,}76} \cong 37{,}92$

L'enfant pèse environ 37,9 kg.

Ex 1.2.16

$v = 0{,}0151 + 0{,}258 \log(P)$ vitesse moyenne en m/s d'un piéton en fct de la population P.

a) $v = 0{,}0151 + 0{,}258 \log(130'000) \cong 1{,}33$

La vitesse moyenne d'un piéton à Lausanne est de 1,33 m/s.

b) $0{,}0151 + 0{,}258 \log(P) = 1{,}5$

$$0{,}258 \log(P) = 1{,}5 - 0{,}0151$$

$$\log(P) = \frac{1{,}5 - 0{,}0151}{0{,}258}$$

$$P = 10^{\frac{1{,}5 - 0{,}0151}{0{,}258}} \cong 569'411{,}66$$

La population doit être d'environ 569'411 habitants.

Ex 1.2.17

$m = 2600(1 - 0{,}51e^{-0{,}057t})^3$ masse en kg en fct de l'âge en année.

a) $t = 0 \Rightarrow m = 2'600(1 - 0{,}51e^{0})^3 = 2'600(1 - 0{,}51)^3 \cong 305{,}89$

L'éléphanteau pèse environ 306 kg.

b) $2'600(1 - 0{,}51e^{-0{,}057t})^3 = 1'800$

$$(1 - 0{,}51e^{-0{,}057t})^3 = \frac{1'800}{2'600} = \frac{9}{13}$$

$$1 - 0{,}51e^{-0{,}057t} = \sqrt[3]{\frac{9}{13}}$$

$$-0{,}51e^{-0{,}057t} = \sqrt[3]{\frac{9}{13}} - 1$$

$$e^{-0{,}057t} = \frac{\sqrt[3]{\frac{9}{13}} - 1}{-0{,}51}$$

$$-0{,}057t = \ln\left(\frac{\sqrt[3]{\frac{9}{13}} - 1}{-0{,}51}\right)$$

$$t = \frac{\ln\left(\frac{\sqrt[3]{\frac{9}{13}} - 1}{-0{,}51}\right)}{-0{,}057} \cong 26{,}08$$

L'éléphante a 26 ans.

Ex 1.2.18

$T = 37 \cdot e^{-0{,}02t}$ Température T en degré en fct du temps t en min

a) $T = 37 \cdot e^{-0{,}02 \cdot 45} \cong 15{,}04$

Sa température sera d'environ 15,04°

Si $t = 5'$:

$T = 37 \cdot e^{-0{,}02 \cdot 5} \cong 33{,}48°$

b)
$$37 e^{-0{,}02t} = 25$$
$$e^{-0{,}02t} = \frac{25}{37}$$
$$-0{,}02t = \ln\left(\frac{25}{37}\right)$$
$$t = \frac{\ln\left(\frac{25}{37}\right)}{-0{,}02} \cong 19{,}602$$

Il faut le secourir avant 19,6 minutes.

Ex 1.2.19

a) $N(0) = 10'000$

$N(12) = 10'000 \cdot 2$ $\quad \cdot 2$

$N(24) = 10'000 \cdot 2^2$ $\quad \cdot 2$

$\vdots$

$N(12t) = 10'000 \cdot 2^t \quad \Rightarrow \quad N(t) = 10'000 \cdot 2^{t/12}$

b) 1 semaine $= 7 \cdot 24h = 168h$.

$N(168) = 10'000 \cdot 2^{168/12} = 10'000 \cdot 2^{14} = 163'840'000$

Au bout d'une semaine il y aura environ $1{,}6384 \cdot 10^8$ bactéries.

c) $N(t) = 30'000 \qquad t = ?$

$10'000 \cdot 2^{t/12} = 30'000$

$2^{t/12} = 3$

$t/12 = \log_2(3)$

$t = 12 \cdot \log_2(3) = 12 \cdot \dfrac{\log(3)}{\log(2)} \cong 19{,}02$

Le nombre de bactéries aura triplé après 19h environ.

Ex 1.2.20

a) $N(0) = 1'000$

$N(3) = 600$ $\quad \cdot \frac{600}{1000} = \frac{3}{5} = 0{,}6$

$N(6) = 360$ $\quad \cdot 0{,}6$

$\left(\begin{aligned} 1000 \cdot a &= 600 \\ \Leftrightarrow \quad a &= \frac{600}{1000} = \frac{3}{5} = 0{,}6 \end{aligned} \right)$

$N(3t) = 1'000 \cdot 0{,}6^t \quad \Rightarrow \quad N(t) = 1'000 \cdot 0{,}6^{t/3}$

b) $N(12) = 1'000 \cdot 0{,}6^{12/3} = 1000 \cdot 0{,}6^4 = 129{,}6$

Après une année il y aura 129 huîtres environ

c) $N(t) = 80 \qquad t = ?$

$$1'000 \cdot 0{,}6^{t/3} = 80$$
$$0{,}6^{t/3} = 0{,}08$$
$$\frac{t}{3} = \log_{0{,}6}(0{,}08)$$
$$t = 3 \cdot \log_{0{,}6}(0{,}08) = 3 \cdot \frac{\log(0{,}08)}{\log(0{,}6)} \cong 14{,}83$$

Il n'y aura plus que 80 huîtres après 14,8 mois soit 14 mois et 24 jours

Ex 1.2.21

a) $Q(0) = 10$

$Q(1) = 8$ $\quad \cdot 0{,}8$

$Q(2) = 10 \cdot 0{,}8^2$

$Q(t) = 10 \cdot 0{,}8^t$

b) $Q(8) = 10 \cdot 0{,}8^8 \cong 1{,}68$

le patient a 1,68 mg de médicament dans le corps 8h après l'absorption.

c) $Q(t) = 1$ $\qquad t = ?$

$10 \cdot 0{,}8^t = 1$

$0{,}8^t = 0{,}1$

$$t = \log_{0{,}8}(0{,}1) = \frac{\log(0{,}1)}{\log(0{,}8)} \cong 10{,}32 \text{ h} = 10\text{h}19'$$

Le patient n'aura plus qu'1 mg après environ 10h et 19 min.

Ex 1.2.22

1) $C_n = C_{12} = 4'720\,(1+0{,}035)^{12} = 4'720 \cdot 1{,}035^{12} \cong \underline{7'132{,}24 \quad \text{CHF}}$

2) $5'888{,}65 = C_0\,(1+0{,}035)^{24}$

$$\frac{5'888{,}65}{1{,}035^{24}} = C_0 \cong \underline{2'360 \quad \text{CHF}}$$

3) $11'604{,}17 = 9'440\,(1+0{,}035)^n$

$$\frac{11'604{,}17}{9'440} = 1{,}035^n \qquad \Big|\ \log_{1{,}035}$$

$$\log_{1{,}035}\left(\frac{11'604{,}17}{9440}\right) = n$$

$$n = \frac{\log\left(\frac{11'604{,}17}{9440}\right)}{\log(1{,}035)} \cong \underline{6 \text{ ans}}$$

4) $9'404{,}43 = 790\,(1+i)^{72}$

$$\frac{9'404{,}43}{790} = (1+i)^{72}$$

$$\sqrt[72]{\frac{9'404{,}43}{790}} = 1+i \cong 1{,}035 \Rightarrow i \cong 0{,}035 = \underline{3{,}5\,\%}$$

Ex 1.2.23

$C_0 = 7'200'000 \qquad i = 3\% = 0{,}03 \qquad n = 2000 - 1867 = 133$

$C_{133} = 7'200'000 \cdot 1{,}03^{133} \cong 367'014'634$

Sa valeur aurait été de $\underline{367'014'634 \ \$}$

Ex 1.2.24

$C_0 = 10'000 \qquad i = 11\% = 0{,}11 \qquad C_n = 20'000$

$20'000 = 10'000 \cdot 1{,}11^n$

$$2 = 1{,}11^n \qquad \Big|\ \log_{1{,}11}$$

$$\log_{1{,}11}(2) = n$$

$$n = \frac{\log(2)}{\log(1{,}11)} \cong 6{,}64 \Rightarrow \text{Il faudra au minimum } \underline{7 \text{ ans}}$$

Ex 1.2.25

a) $v(0) = 18'000$

$v(1) = 18'000 - 18'000 \cdot 0{,}25 = 18'000(1-0{,}25) = 18'000 \cdot 0{,}75$ $\cdot 0{,}75$

$\vdots$

$v(t) = 18'000 \cdot 0{,}75^t$

b) $v(8) = 18'000 \cdot 0{,}75^8 \cong 1'802$ CHF

c) $v(100) \cong 0$

Lorsque t devient très grand, la voiture ne vaut plus rien.

Ex 1.2.26

a) $\left.\begin{array}{l} Q(0) = 50 \\ Q(30) = 43 \end{array}\right\} \cdot a = \frac{43}{50} = 0{,}86$ $\left(\text{car } 50 \cdot a = 43 \Leftrightarrow a = \frac{43}{50}\right)$

$Q(30t) = 50 \cdot 0{,}86^t \Rightarrow Q(t) = 50 \cdot 0{,}86^{t/30}$

b) 3 semaines = 21 jours

$Q(21) = 50 \cdot 0{,}86^{21/30} \cong 44{,}99$

Après 3 semaines, il restera un peu moins de 45 mg

c) On cherche le temps nécessaire à ce qu'il ne reste que la moitié de la quantité initiale, ici 25 mg. (demi-vie)

$$Q(t) = 25$$
$$50 \cdot 0{,}86^{t/30} = 25$$
$$0{,}86^{t/30} = \frac{1}{2} = 0{,}5$$
$$\frac{t}{30} = \log_{0{,}86}(0{,}5)$$
$$t = 30 \cdot \log_{0{,}86}(0{,}5) = 30 \cdot \frac{\log(0{,}5)}{\log(0{,}86)} \cong 137{,}87$$

La demi-vie est d'environ 138 jours

Ex 1.2.27

a) $Q(0) = 100$

$Q(30) = 50$ $\cdot \frac{1}{2} = 0{,}5$

$Q(30t) = 100 \cdot 0{,}5^{t} \Rightarrow Q(t) = 100 \cdot 0{,}5^{t/30}$

b) $Q(5) = 100 \cdot 0{,}5^{5/30} = 100 \cdot 0{,}5^{1/6} \cong 89{,}09$

Après 5 ans il reste environ 89 tonnes de césium

Ex 1.2.28

demi-vie : ~ 5730 ans (± 30 ans)

$Q(0) = 100$

$Q(5730) = 50$ $\cdot 0{,}5$

$Q(t) = 100 \cdot 0{,}5^{t/5730}$

Perte de 83% donc reste 17 % de C^{14}

$\Rightarrow Q(t) = 17$

$100 \cdot 0{,}5^{t/5730} = 17 \Leftrightarrow 0{,}5^{t/5730} = 0{,}17$

$$\frac{t}{5730} = \log_{0,5}(0{,}17)$$

$$t = 5730 \cdot \log_{0,5}(0{,}17) \cong 14'648{,}13$$

$\Rightarrow$ les peintures dateraient de ~14'648 - 1940 = 12'708 av J.C.

(en 2024 : ~ 14'648 + 84 = 14'732 ans)

Ex 1.2.29

$$h(t) = \frac{40}{1+200e^{-0{,}2t}}$$

a) $h(30) = \dfrac{40}{1+200e^{-0{,}2\cdot 30}} = \dfrac{40}{1+200e^{-6}} \hat{=} 26{,}74 \text{ m}$

b) $16 = \dfrac{40}{1+200e^{-0{,}2\cdot t}}$ on cherche à isoler $e^{-0{,}2t}$

$$16\left(1+200e^{-0{,}2\cdot t}\right) = 40$$

$$1+200e^{-0{,}2t} = \frac{40}{16} = \frac{5}{2}$$

$$200e^{-0{,}2t} = \frac{5}{2} - 1 = \frac{3}{2}$$

$$e^{-0{,}2t} = \frac{3}{2}\cdot\frac{1}{200} = \frac{3}{400} \quad | \ln(\,)$$

car $e^{-0{,}2t}$ est isolé et $\ln(e^{-0{,}2t}) = -0{,}2t$

$$-0{,}2t = \ln\left(\frac{3}{400}\right)$$

$$t = \frac{\ln\left(\frac{3}{400}\right)}{-0{,}2} \cong 24{,}5 \text{ ans}$$

c) $h(100) \cong 39{,}99$ m

On remarque que quand t devient grand $e^{-0{,}2t}$ devient quasiment nul, donc $h = \dfrac{40}{1} = 40$ m est la hauteur maximale que l'arbre peut atteindre.

Ex 1.2.30

$$N(t) = \frac{1000}{1+999\cdot 10^{-0,17t}}$$

a) $N(20) = \dfrac{1000}{1+999\cdot 10^{-3,4}} \cong 715,46$

Après 20 jours environ 715 personnes ont été atteintes.

b) $600 = \dfrac{1000}{1+999\cdot 10^{-0,17t}}$ on cherche à isoler $10^{-0,17t}$

$$600\left(1+999\cdot 10^{-0,17t}\right) = 1000$$

$$1+999\cdot 10^{-0,17t} = \frac{1000}{600} = \frac{5}{3}$$

$$999\cdot 10^{-0,17t} = \frac{5}{3}-1 = \frac{2}{3}$$

$$10^{-0,17t} = \frac{2}{3\cdot 999} = \frac{2}{2997} \quad \Big| \log$$

car $10^{-0,17t}$ est isolé et $\log(10^{-0,17t}) = -0,17t$

$$-0,17t = \log\left(\frac{2}{2997}\right)$$

$$t = \frac{\log\left(\frac{2}{2997}\right)}{-0,17} \cong 18,68$$

Après environ 19 jours, 600 personnes ont été atteintes.

c) $N(100) \cong 1000$

On remarque que quand t devient grand $10^{-0,17t}$ devient quasiment nul, donc $N(t) = \dfrac{1000}{1} = 1000$, l'entier de la population a été atteinte par le virus.