Ex 1.1.1

a) $f(x) = e^{5x}$ $\qquad ED(f) = \mathbb{R}$

$f'(x) = 5e^{5x}$

b) $f(x) = e^{x^2}$ $\qquad ED(f) = \mathbb{R}$

$f'(x) = 2x\, e^{x^2}$

c) $f(x) = e^{1/x}$ $\qquad$ cond: $x \neq 0 \Rightarrow ED(f) = \mathbb{R}^*$

$f'(x) = -\frac{1}{x^2} e^{1/x} = -\frac{e^{1/x}}{x^2}$

d) $f(x) = e^{\sqrt{x^2+x}}$ $\qquad$ cond: $x^2+x \geq 0 \Leftrightarrow \underset{\downarrow\,0}{x}\,\underset{\downarrow\,-1}{(x+1)} \geq 0$

x		-1		0	
x^2+x	$+$	0	$-$	0	$+$

$\Rightarrow ED(f) =]-\infty;-1] \cup [0;+\infty[$

$$f'(x) = \frac{2x+1}{2\sqrt{x^2+x}}\, e^{\sqrt{x^2+x}}$$

e) $f(x) = \exp\left(\sqrt{\frac{1+x^2}{1-x^2}}\right) = e^{\sqrt{\frac{1+x^2}{1-x^2}}}$

cond: $\frac{1+x^2}{1-x^2} \geq 0 \Leftrightarrow \frac{1+x^2}{(1+x)(1-x)} \geq 0$

x		-1		1	
$\frac{1+x^2}{1-x^2}$	$-$	$\|$	$+$	$\|$	$-$

$\Rightarrow ED(f) =]-1;1[$

$$f'(x) = e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{\frac{2x(1-x^2)+2x(1+x^2)}{(1-x^2)^2}}{2\sqrt{\frac{1+x^2}{1-x^2}}}$$

$$= e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{\cancel{2}x(1-x^2+1+x^2)}{(1-x^2)^2} \cdot \frac{1}{\cancel{2}\sqrt{\frac{1+x^2}{1-x^2}}}$$

$$= e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{2x}{(1-x^2)^2} \cdot \sqrt{\frac{1-x^2}{1+x^2}}$$

f) $f(x) = e^{\sin(x)}$ $\qquad$ $\underline{ED(f) = \mathbb{R}}$

$\underline{f'(x) = \cos(x)\, e^{\sin(x)}}$

g) $f(x) = x^2 \cdot e^x$ $\qquad$ $\underline{ED(f) = \mathbb{R}}$ $\qquad$ $u = x^2 \quad v = e^x$

$u' = 2x \quad v' = e^x$

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x = \underline{xe^x(2+x)}$

h) $f(x) = e^{-x}\cos(x)$ $\qquad$ $\underline{ED(f) = \mathbb{R}}$ $\qquad$ $u = e^{-x} \quad v = \cos(x)$

$u' = -e^{-x} \quad v' = -\sin(x)$

$f'(x) = -e^{-x}\cos(x) - e^{-x}\sin(x) = \underline{-e^{-x}(\cos(x) + \sin(x))}$

g) suppl.

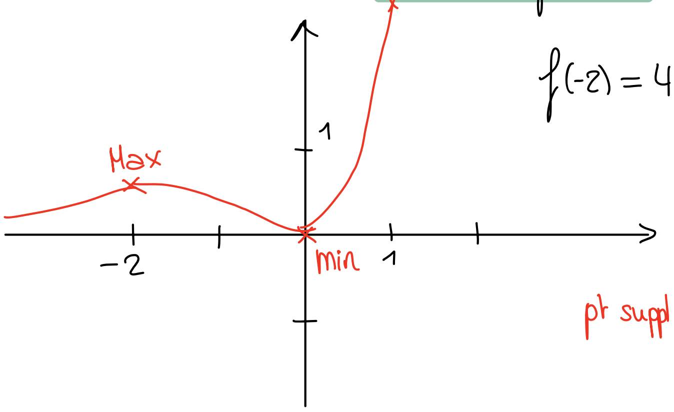
$*$ signe : zéro(s) : $\underbrace{x^2}_{0}\underbrace{e^x}_{>0} = 0$

x		0	
$\mathrm{sgn}(f)$	$+$	0	$+$

$*$ crssce : zéro(s) de f' : $\underbrace{x}_{0}\underbrace{e^x}_{>0}\underbrace{(2+x)}_{-2} = 0$

x		-2		0	
$\mathrm{sgn}(f')$	$+$	0	$-$	0	$+$
crssce(f)	$\nearrow$	Max	$\searrow$	min	$\nearrow$

$\underline{\mathrm{Max}(-2; f(-2))} \cong \underline{(-2; 0{,}54)}$ $\qquad$ $\underline{\min(0;0)}$



$f(-2) = 4e^{-2} = \frac{4}{e^2} \cong 0{,}54$

pt suppl : $f(1) = e \cong 2{,}7$

Ex 1.1.5

a) $$\lim_{x\to 2}\frac{e^x-e^2}{x-2} \overset{"\frac{0}{0}"\text{ B-H}}{\overbrace{=}} \lim_{x\to 2}\frac{e^x}{1} = e^2$$

b) $$\left(\lim_{x\to 0}\frac{e^x-1}{\sin(x)} \overset{"\frac{0}{0}"\text{ B-H}}{\overbrace{=}} \lim_{x\to 0}\frac{e^x}{\cos(x)} = 1\right)$$

c) $$\lim_{x\to 0}\frac{x\,e^x}{1-e^x} \overset{"\frac{0}{0}"\text{ B-H}}{\overbrace{=}} \lim_{x\to 0}\frac{(x+1)\,e^x}{-e^x} = -1$$

d) $$\lim_{x\to 0^+} x e^{1/x} = "0\cdot e^{\frac{1}{0_+}}" = "\overset{\text{f.i.}}{0\cdot\infty}" = \lim_{x\to 0^+}\frac{e^{1/x}}{1/x}$$
$$\underset{\text{B.H.}}{\overset{"\frac{\infty}{\infty}"}{=}} \lim_{x\to 0^+}\frac{-\frac{1}{x^2}e^{1/x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} e^{1/x} = e^{+\infty} = +\infty$$

e) $$\underbrace{\lim_{x\to -\infty}\frac{e^x}{2x}}_{=0} + \lim_{x\to -\infty}\frac{e^{-x}}{2x} \overset{"\frac{\infty}{\infty}"\text{ B-H}}{\overbrace{=}} \lim_{x\to -\infty}\frac{-e^{-x}}{2} = -\infty$$

f) $$\lim_{x\to +\infty}\frac{2\,e^x-1}{e^x+2} \overset{"\frac{\infty}{\infty}"\text{ B-H}}{\overbrace{=}} \lim_{x\to +\infty}\frac{2\,e^x}{e^x} = \lim_{x\to +\infty} 2 = 2$$

g) $$\lim_{x\to -\infty}(x^2+x)e^x = "\underset{\text{f.i.}}{\infty\cdot 0}" = \lim_{x\to -\infty}\frac{x^2+x}{e^{-x}} \underset{\text{B.H.}}{\overset{"\frac{\infty}{\infty}"}{=}} \lim_{x\to -\infty}\frac{2x+1}{-e^{-x}}$$
$$\underset{\text{B.H.}}{\overset{"\frac{\infty}{\infty}"}{=}} \lim_{x\to -\infty}\frac{2}{e^{-x}} = \frac{2}{\infty} = 0$$

h) $$\lim_{x\to +\infty}\frac{e^x}{x^2-2x+3} \overset{"\frac{\infty}{\infty}"\text{ B-H}}{\overbrace{=}} \lim_{x\to +\infty}\frac{e^x}{2x-2} \overset{"\frac{\infty}{\infty}"\text{ B-H}}{\overbrace{=}} \lim_{x\to +\infty}\frac{e^x}{2} = +\infty$$

Ex 1.1.6

a) $f(x) = \ln(5x)$ cond : $5x > 0 \Leftrightarrow x > 0 \Rightarrow ED(f) = \mathbb{R}_+^*$

$f'(x) = \dfrac{5}{5x} = \dfrac{1}{x}$

b) $f(x) = \ln(x-1)$ cond : $x-1>0 \Leftrightarrow x>1 \Rightarrow ED(f) =]1;+\infty[$

$f'(x) = \dfrac{1}{x-1}$

c) $f(x) = \ln(1-x)$ cond : $1-x>0 \Leftrightarrow 1>x \Rightarrow ED(f) =]-\infty;1[$

$f'(x) = \dfrac{-1}{1-x} = \dfrac{1}{x-1}$

(d) $f(x) = \ln(|1-x|)$ cond : $1-x \neq 0 \Leftrightarrow x \neq 1 \Rightarrow ED(f) = \mathbb{R}-\{1\}$

$$f(x) = \begin{cases} \ln(x-1) & \text{si } 1-x<0 \\ \ln(1-x) & \text{si } 1-x>0 \end{cases} \Rightarrow f'(x) = \begin{cases} \dfrac{1}{x-1} \\ \dfrac{-1}{1-x} \end{cases} \Rightarrow f'(x) = \dfrac{1}{x-1} \quad \forall x \in ED(f)$$

e) $f(x) = \ln(x^2-x)$ cond : $x^2-x>0 \Leftrightarrow x(x-1)>0$

x		0		1	
x^2-x	$+$	0	$-$	0	$+$

$\Rightarrow ED(f) =]-\infty;0[\cup]1;+\infty[$

$f'(x) = \dfrac{2x-1}{x^2-x}$

f) $f(x) = \ln(x-x^2)$ cond : $x-x^2>0 \Leftrightarrow x(1-x)>0$

x		0		1	
x^2-x	$-$	0	$+$	0	$-$

$\Rightarrow ED(f) =]0;1[$

$f'(x) = \dfrac{1-2x}{x-x^2} = \dfrac{2x-1}{x^2-x}$

(g) $f(x) = \ln(|x^2-x|)$ cond : $x^2-x \neq 0 \Leftrightarrow x(x-1) \neq 0 \Leftrightarrow x \neq \begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$

$\Rightarrow ED(f) = \mathbb{R}^* - \{1\}$

$f'(x) = \dfrac{2x-1}{x^2-x}$

h) $f(x) = \ln\left(\dfrac{x^2}{1-x}\right)$

cond. $\dfrac{x^2}{1-x} > 0$

x		0		1	
$\frac{x^2}{1-x}$	$+$	0	$+$	$\|\|$	$-$
		(2)			

$\Rightarrow ED(f) =]-\infty;1[- \{0\}$

$u = \dfrac{x^2}{1-x}$

$$u' = \frac{2x(1-x)+1\cdot x^2}{(1-x)^2} = \frac{2x-2x^2+x^2}{(1-x)^2} = \frac{2x-x^2}{(1-x)^2} = \frac{x(2-x)}{(1-x)^2}$$

$$f'(x) = \frac{\frac{-x(x-2)}{(1-x)^2}}{\frac{x^2}{1-x}} = \frac{x(2-x)}{(1-x)^2}\cdot\frac{1-x}{x^2} = \frac{2-x}{x(1-x)} = \frac{x-2}{x(x-1)}$$

i) $f(x) = \ln\left(\sqrt{3-x^2}\right)$

cond : $\underbrace{\sqrt{3-x^2} > 0 \quad \text{et} \quad 3-x^2 \geqslant 0}_{3-x^2>0}$

$\Leftrightarrow (\sqrt{3}+x)(\sqrt{3}-x) > 0$

x		$-\sqrt{3}$		$\sqrt{3}$	
$3-x^2$	$-$	0	$+$	0	$-$

$ED(f) =]-\sqrt{3};\sqrt{3}[$

$u = \sqrt{3-x^2}$ $\qquad u' = \dfrac{-2x}{2\sqrt{3-x^2}} = \dfrac{-x}{\sqrt{3-x^2}}$

$$f'(x) = \frac{\frac{-x}{\sqrt{3-x^2}}}{\sqrt{3-x^2}}$$

$$= \frac{-x}{\sqrt{3-x^2}}\cdot\frac{1}{\sqrt{3-x^2}} = \frac{-x}{3-x^2} = \frac{x}{x^2-3}$$

j) $f(x) = \ln(3x^5)$ $\qquad ED(f) = \mathbb{R}^*_+$

$$f'(x) = \frac{15x^4}{3x^5} = \frac{5}{x}$$

k) $f(x) = x\ln(x) - x$ $\qquad ED(f) = \mathbb{R}^*_+$

$$f'(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$$

l) $f(x) = \ln(|\cos(x)|)$ cond : $|\cos(x)| > 0 \Leftrightarrow \cos(x) \neq 0$

$ED(f) = \mathbb{R} - \left\{ \frac{\pi}{2} + k\cdot\pi \mid k \in \mathbb{Z} \right\}$

$$f(x) = \begin{cases} \ln(\cos(x)) & \text{si } \cos(x) > 0 \\ \ln(-\cos(x)) & \text{si } \cos(x) < 0 \end{cases}$$

$u = \cos(x)$, $u' = -\sin(x)$

$u = -\cos(x)$ $u' = \sin(x)$

$$f'(x) = \left\{ \begin{array}{lll} \frac{-\sin(x)}{\cos(x)} = -\tan(x) & \text{si } \cos(x) > 0 \\ \frac{\sin(x)}{-\cos(x)} = -\tan(x) & \text{si } \cos(x) < 0 \end{array} \right\} \Rightarrow f'(x) = -\tan(x)$$

m) $f(x) = \frac{x}{\ln(x)}$ cond : $\ln(x) \neq 0$ et $x > 0$

$x \neq 1$

$ED(f) = \mathbb{R}_+^* - \{1\}$

$u = x$ $v = \ln(x)$

$u' = 1$ $v' = \frac{1}{x}$

$$f'(x) = \frac{1 \ln(x) - x \cdot \frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$$

n) $f(x) = \frac{1}{x \ln(x)}$ cond : $x \ln(x) \neq 0$ et $x > 0$

0 1

$ED(f) = \mathbb{R}_+^* - \{1\}$

$u = 1$ $v = x\ln(x)$

$u' = 0$ $v' = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$

$$f'(x) = -\frac{\ln(x) + 1}{x^2 \ln^2(x)}$$

m) suppl.

* signe : zéro : $\frac{x}{\ln(x)} = 0 \Leftrightarrow x = 0$ mais $0 \notin ED(f) \Rightarrow$ pas de zéro

x	0		1	
sgn(f)	‖	$-$	‖	$+$

$f(0{,}5) \cong -0{,}7$ $f(2) \cong 2{,}9$

$*$ crssce : zéro de f' : $\dfrac{\ln(x)-1}{\ln^2(x)} = 0 \iff \ln(x) - 1 = 0$

$\iff \ln(x) = 1$

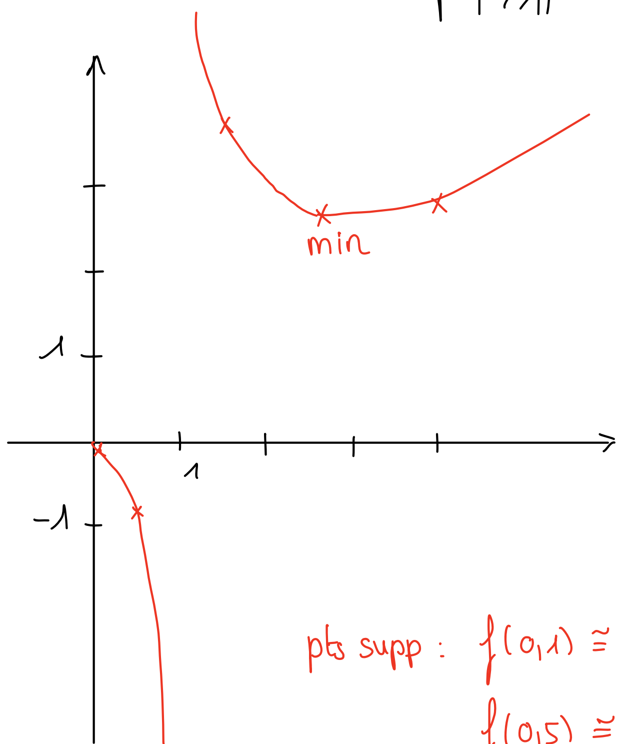
$\iff x = e$

$f'(0{,}5)<0$ $\quad f'(2)<0 \quad f'(3)>0$

x	0		1		e	
sgn(f')	‖	$-$	‖	$-$	0	$+$
crssce(f)	‖	↘	‖	↘	min	↗

$\min(e\,;f(e)) = (e\,;e) \cong (2{,}7\,;\,2{,}7)$

$f(e) = \dfrac{e}{1} = e \cong 2{,}7$



pts supp : $f(0{,}1) \cong -0{,}04$ $\qquad f(1{,}5) \cong 3{,}7$

$f(0{,}5) \cong -0{,}7$ $\qquad f(4) \cong 2{,}88$

Ex 1.1.9

a) $\lim_{x\to 0} \frac{\ln(x+1)}{x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x\to 0} \frac{\frac{1}{x+1}}{1} = 1$

$$\left(\text{b) } \lim_{x\to 0} \frac{\ln(\cos(x))}{x^2} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x\to 0} \frac{-\tan(x)}{2x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x\to 0} \frac{-1-\tan^2(x)}{2} = -\frac{1}{2} \right)$$

c) $\lim_{x\to e} \frac{\ln(x)-1}{x-e} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x\to e} \frac{\frac{1}{x}}{1} = \frac{1}{e}$

d) $\lim_{x\to 2} \frac{\ln(x^2-3)}{2-x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x\to 2} \frac{\frac{2x}{x^2-3}}{-1} = -4$

e) $\lim_{x\to -\infty} \frac{\ln(x^2+1)}{x} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim_{x\to -\infty} \frac{\frac{2x}{x^2+1}}{1} = 0$

f) $\lim_{x\to +\infty} \frac{\ln(x)+1}{1-\ln(x)} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim_{x\to +\infty} \frac{\frac{1}{x}}{-\frac{1}{x}} = -1$

g) $\lim_{x\to +\infty} \frac{\ln(x^2)}{\ln^2(x)} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim_{x\to +\infty} \frac{\frac{2}{x}}{2\ln(x)\cdot\frac{1}{x}} = \lim_{x\to +\infty} \frac{1}{\ln(x)} = 0$

h) $\lim_{x\to +\infty} \frac{\ln(x)}{e^x} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim_{x\to +\infty} \frac{\frac{1}{x}}{e^x} = 0$

Ex 1.1.13

a) $f(x) = e^{-x^2}$

1. $ED(f) = \mathbb{R}$

2. zéro : $\nexists$ signe :

x	
$f(x)$	$+$

3. AV-trou : $\nexists$

AH : $\lim\limits_{x \to -\infty} e^{-x^2} = \lim\limits_{x \to -\infty} \frac{1}{e^{x^2}} = 0$

idem pour $x \to +\infty$ $\Rightarrow$ AH en $y=0$

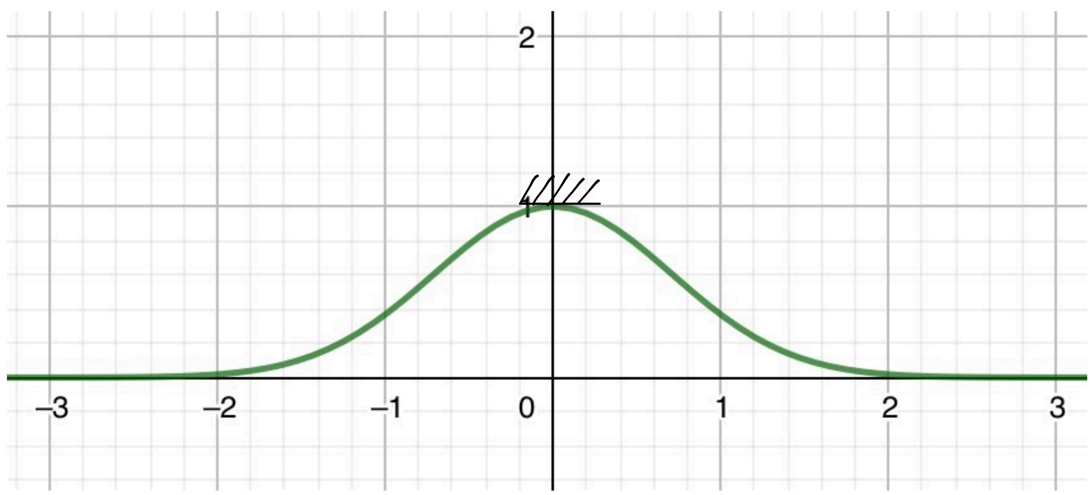
4. croissance : $f'(x) = -2xe^{-x^2}$

zéro de f' : $-2x\underbrace{e^{-x^2}}_{>0} = 0 \iff x = 0$

x		0	
sgn(f')	$+$	ϕ	$-$
croiss. f	$\nearrow$	Max	$\searrow$

Max $(0; f(0)) = (0;1)$

5. graphe :



b) $f(x) = e^{1/x}$

1. ED(f) : $ED(f) = \mathbb{R}^*$

2. zéro et signe : $e^{1/x} > 0$

x		0	
$f(x)$	$+$	‖	$+$

3. asymptotes : AV - trou : $\lim\limits_{x \to 0} e^{1/x} = \begin{cases} >0 : \text{“}e^{+\infty}\text{”} = +\infty & \Rightarrow \text{AV en } x = 0 \text{ (à droite)} \\ <0 : \text{“}e^{-\infty}\text{”} = 0 & \Rightarrow \text{trou en “}(0;0)\text{” (à gauche)} \end{cases}$

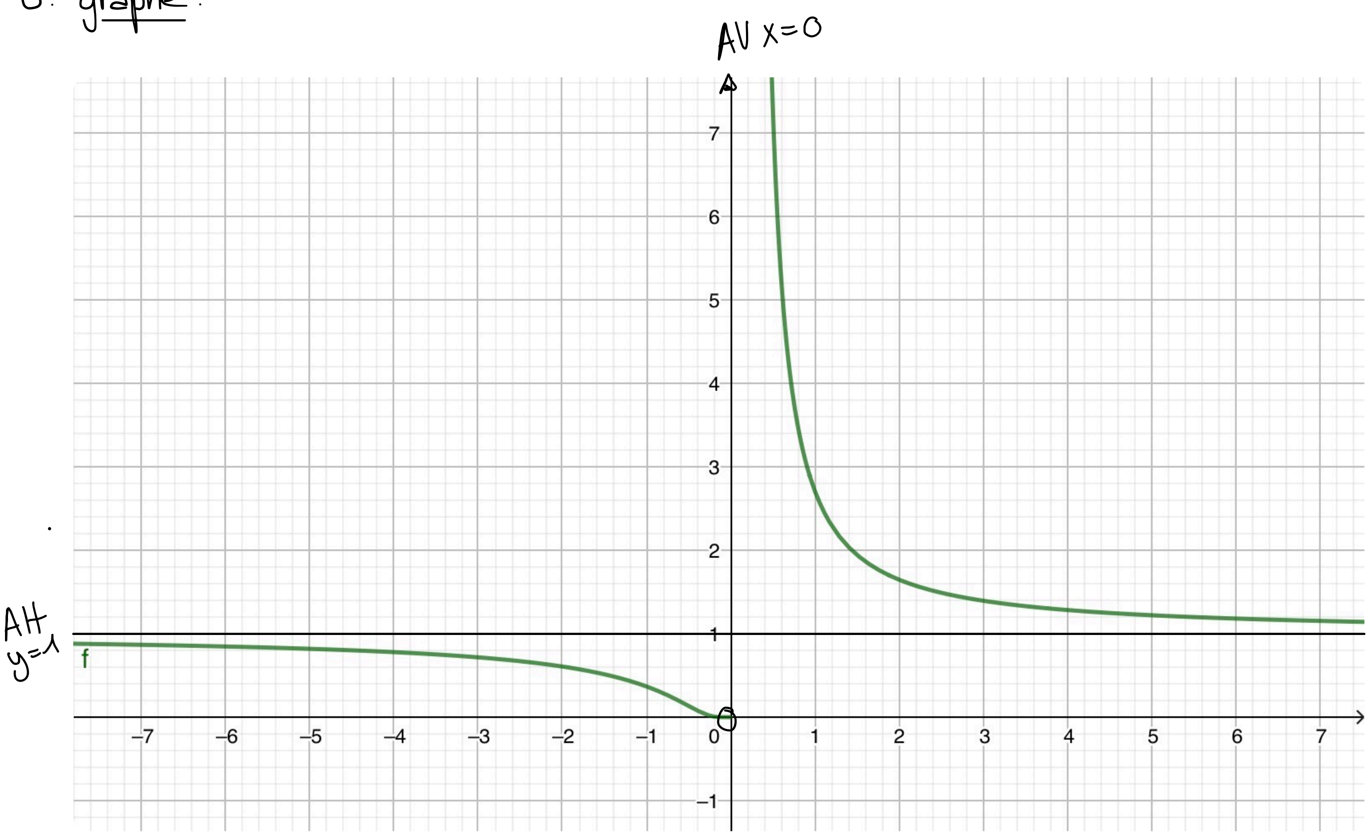
AH : $\lim\limits_{x \to \pm\infty} e^{1/x} = e^0 = 1 \Rightarrow$ AH en $y = 1$

4. croissance : $f'(x) = -\dfrac{1}{x^2} e^{1/x} = -\dfrac{e^{1/x}}{x^2}$ pas de zéro

v.i : 0

x		0	
sgn(f')	$-$	‖	$-$
f	↘	‖	↘

5. graphe :



c) $f(x) = (x^2-4x+4)e^x$

1. $ED(f) = \mathbb{R}$

2. zéro : $(x^2-4x+4)\underbrace{e^x}_{>0} = 0 \iff x^2-4x+4 = 0 \iff (x-2)^2 = 0 \iff x=2$ (2)

signe :

x		2	
sgn(f)	+	0 (2)	+

3. AV : $\not\exists$

AH : à gauche : $\lim\limits_{x\to-\infty} f(x) \overset{+\infty\cdot 0}{=} \lim\limits_{x\to-\infty} \frac{x^2-4x+4}{e^{-x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{B.H.}{=}} \lim\limits_{x\to-\infty} \frac{2x-4}{-e^{-x}} \overset{\text{"}\frac{-\infty}{-\infty}\text{"}}{\underset{B.H.}{=}} \lim\limits_{x\to-\infty} \frac{2}{e^{-x}}$

$= \frac{2}{+\infty} = 0 \Rightarrow$ AHG en $y=0$

à droite : $\lim\limits_{x\to+\infty} f(x) = +\infty\cdot(+\infty) = +\infty \Rightarrow$ pas d'AHD

4. croissance : $f'(x) = (2x-4)e^x + (x^2-4x+4)e^x = e^x(2x-4+x^2-4x+4) = e^x(x^2-2x)$

$= \underset{0}{x}\underbrace{e^x}_{>0}\underset{2}{(x-2)}$

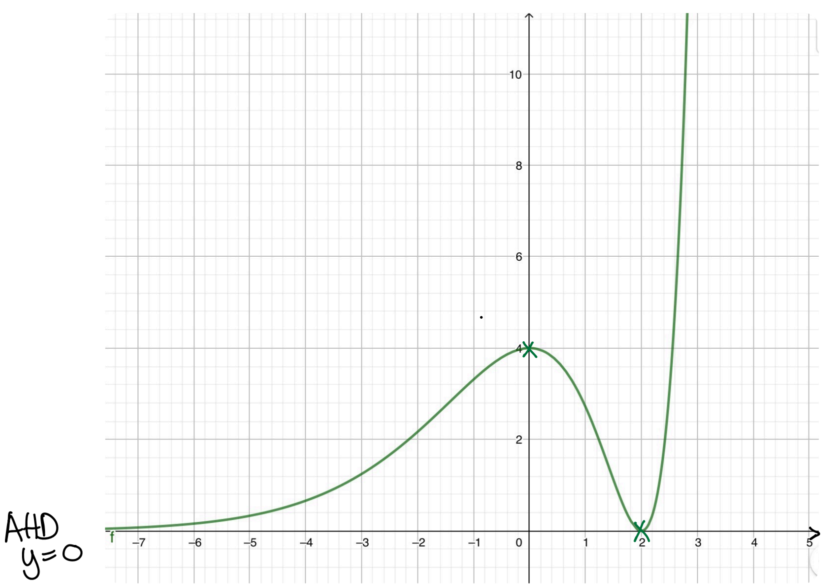
zéros de f' : 0 et 2

x		0		2	
sgn(f')	+	0	−	0	+
croiss. f	↗	Max	↘	min	↗

Max $(0; f(0)) = (0; 4)$

min $(2; f(2)) = (2; 0)$

5. graphe



e) $f(x) = \dfrac{e^x+2}{1-3e^x}$

1. ED : cond : $1 \neq 3e^x \Leftrightarrow e^x \neq \frac{1}{3} \Leftrightarrow x \neq \ln\left(\frac{1}{3}\right) = -\ln(3)$ $(\cong -1{,}1)$

$ED(f) = \mathbb{R} - \left\{\ln\left(\frac{1}{3}\right)\right\} = \mathbb{R} - \{-\ln(3)\}$

2. zéro : $\underbrace{e^x}_{>0} + 2 = 0$ pas de zéro

signe :

x		$-\ln(3)$	
sgn(f)	$+$	‖	$-$

$f(-2) \cong 3{,}6$ $f(0) = -\frac{3}{2}$

3. AV-trou : $\lim\limits_{x \to -\ln(3)} f(x) = \dfrac{\frac{1}{3}+2}{0} = \infty \Rightarrow$ AV en $x = -\ln(3)$

AH : $\lim\limits_{x \to -\infty} \dfrac{e^x+2}{1-3e^x} = \dfrac{2}{1} = 2 \Rightarrow$ AHG en $y = 2$

$\lim\limits_{x \to +\infty} f(x) \overset{\frac{+\infty}{-\infty}}{\underset{BH}{=}} \lim\limits_{x \to +\infty} \dfrac{\cancel{e^x}^{\,1}}{-3\cancel{e^x}} = -\dfrac{1}{3} \Rightarrow$ AHD en $y = -\frac{1}{3}$

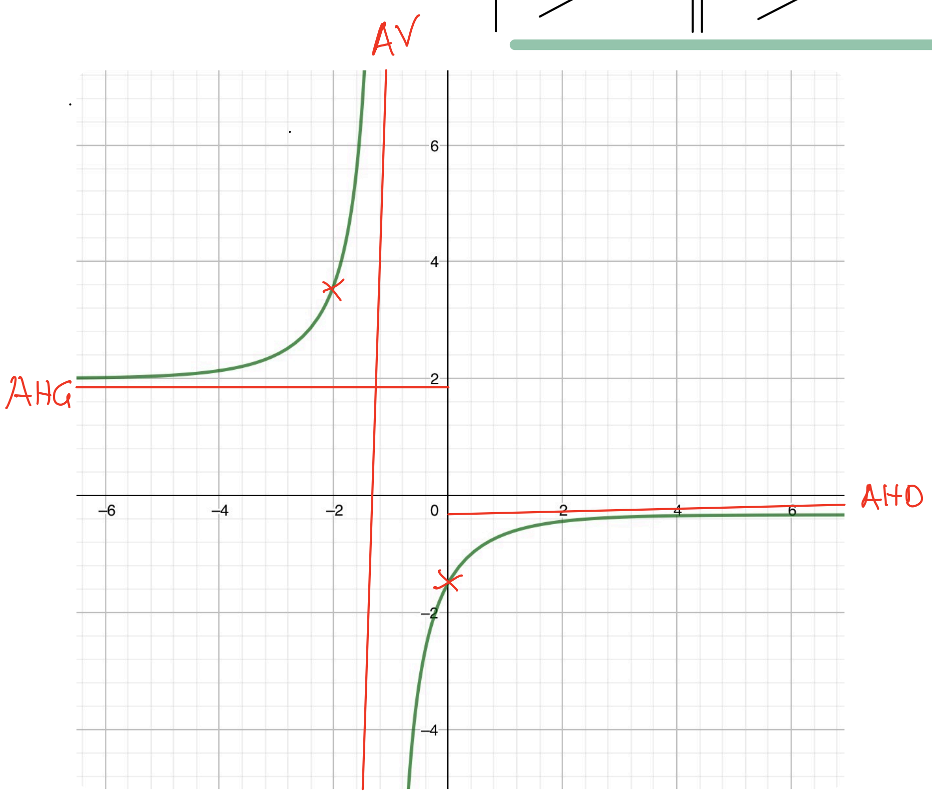
4. croissance : $f'(x) = \dfrac{e^x(1-3e^x) + 3e^x(e^x+2)}{(1-3e^x)^2} = \dfrac{e^x(1-3e^x+3e^x+6)}{(1-3e^x)^2} = \dfrac{7e^x}{(1-3e^x)^2}$

zéro de f' : aucun

v.i : $-\ln(3)$

x		$-\ln(3)$	
sgn(f')	$+$	‖	$+$
f	↗	‖	↗

5. graphe :



Ex 1.1.14

a) $f(x) = x^2 \ln(x)$

1. $ED(f) = \mathbb{R}_+^*$

2. zéro et signe : $x^2 \cdot \ln(x) = 0$

$x^2 = 0 \Rightarrow 0 \notin ED(f)$; $\ln(x) = 0 \Rightarrow 1$ ✓

x	0		1	
$f(x)$	$\|\|$	$-$	0	$+$

3. AV-trou : $\lim_{x\to 0_+} x^2 \ln(x) \overset{0\cdot(-\infty)}{=}$ f.i. $\lim_{x\to 0_+} \frac{\ln(x)}{\frac{1}{x^2}} \overset{\frac{-\infty}{+\infty}}{\underset{B.H.}{=}} \lim_{x\to 0_+} \frac{\frac{1}{x}}{-\frac{2\cancel{x}}{x^{\cancel{4}\,3}}} = \lim_{x\to 0_+} -\frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{3}\,2}}{2} = 0$

$\Rightarrow$ trou en $(0;0)$

AH : à gauche : $\lim_{x\to-\infty} f(x)$ n'a pas de sens vu $ED(f)$

à droite : $\lim_{x\to+\infty} x^2\cdot\ln(x) = +\infty \Rightarrow$ pas d'AH

4. croissance : $f'(x) = 2x\ln(x) + x^2\cdot\frac{1}{x} = 2x\ln(x) + x = x(2\ln(x)+1)$

zéros de f' : ~~$x=0$~~ $\notin ED(f)$ ou $2\ln(x)+1 = 0 \Leftrightarrow \ln(x) = -\frac{1}{2} \Leftrightarrow x = e^{-1/2} = \frac{1}{\sqrt{e}}$ ✓ $\sim 0{,}6$

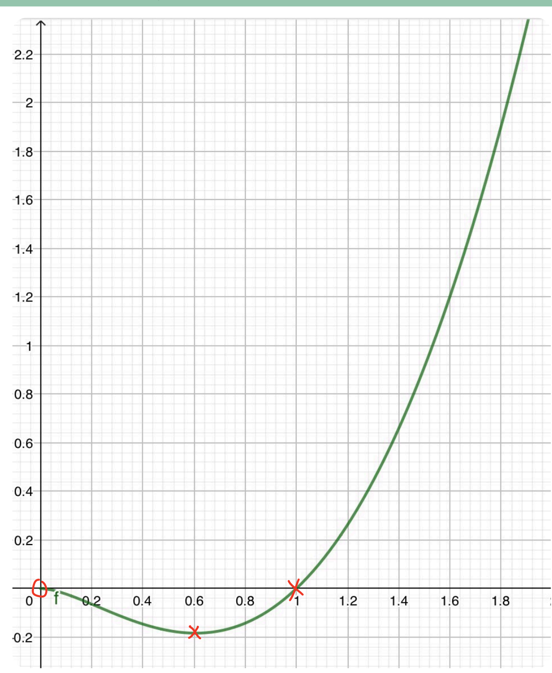
x	0		$1/\sqrt{e}$	
sgn(f')	$\|\|$	$-$	0	$+$
croiss. de f	$\|\|$	↘	min	↗

$f'(0{,}5) \cong -0{,}4$

$f'(1) = 1$

min $\left(\frac{1}{\sqrt{e}}; f\left(\frac{1}{\sqrt{e}}\right)\right) = \left(\frac{1}{\sqrt{e}}; -\frac{1}{2e}\right) \cong (0{,}6; -0{,}2)$

5. graphe :



c) $f(x) = \dfrac{\ln(x)}{x}$

1. $ED(f) = \mathbb{R}^*_+$

2. zéros et signe : zéro : $x = 1$

x	0		1	
$f(x)$	‖	$-$	ϕ	$+$

3. asymptotes : AV - trou : $\lim\limits_{x \to 0_+} \dfrac{\ln(x)}{x} = \dfrac{-\infty}{0_+} = -\infty \Rightarrow x = 0_+$ est une AV

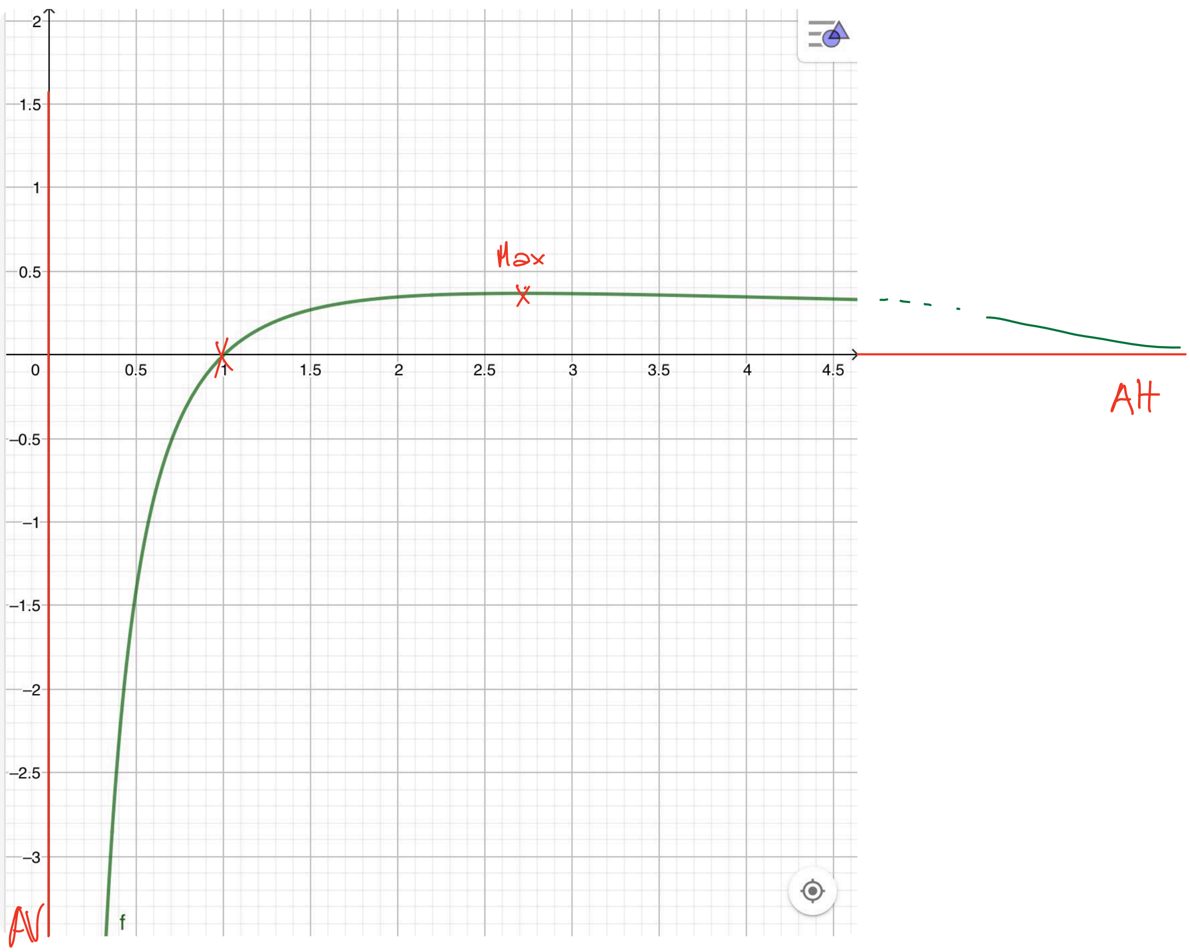
AH : à droite (pas de sens à gauche) : $\lim\limits_{x \to +\infty} \dfrac{\ln(x)}{x} \overset{\frac{+\infty}{+\infty}}{\underset{B.H.}{=}} \lim\limits_{x \to +\infty} \dfrac{\frac{1}{x}}{1} = \dfrac{1}{+\infty} = 0 \Rightarrow$ AHG en $y = 0$

4. croissance : $f'(x) = \dfrac{\frac{1}{x} \cdot x - \ln(x)}{x^2} = \dfrac{1 - \ln(x)}{x^2}$ zéro : $1 = \ln(x) \Leftrightarrow x = e$

x	0		e	
sgn (f')	‖	$+$	ϕ	$-$
croiss. de f	‖	↗	Max	↘

$\text{Max}(e ; f(e)) = (e ; \frac{1}{e}) \simeq (2{,}7 ; 0{,}4)$

5. graphe :



Ex 1.1.27

$$s(t) = \frac{20\ln(t+1)+t}{t+1} \qquad ED(s) =]-1;+\infty[\quad \text{et} \quad EV(s) = [1;+\infty[$$

a) $s(5) = \dfrac{20\ln(6)+5}{6} \cong 6{,}81$

b) $s'(t) = \dfrac{\left(\frac{20}{t+1}+1\right)(t+1) - (20\ln(t+1)+t)}{(t+1)^2} = \dfrac{20+(t+1)-20\ln(t+1)-t}{(t+1)^2} = \dfrac{21-20\ln(t+1)}{(t+1)^2}$

$s'(t) = 0 \Leftrightarrow 21 - 20\ln(t+1) = 0 \Leftrightarrow \ln(t+1) = \dfrac{21}{20} \Leftrightarrow t+1 = e^{\frac{21}{20}} \Leftrightarrow t = e^{\frac{21}{20}} - 1 \cong 1{,}86$

t	1		$\sim 1{,}86$	
sgn(s')	//	$+$	ϕ	$-$
s	// min	$\nearrow$	Max	$\searrow$

L'indice de satisfaction est maximal au cours du mois de février.

c) $\lim\limits_{t\to+\infty} s(t) \underset{B.H.}{\overset{\text{"}\frac{+\infty}{+\infty}\text{"}}{=}} \lim\limits_{t\to+\infty} \dfrac{\frac{20}{t+1}+1}{1} = \dfrac{20}{+\infty} + 1 = 0+1 = 1$

Ex 1.1.28

$$d(t) = \frac{t\cdot e^{-t/30}+2}{2} \qquad ED(d) = \mathbb{R} \quad \text{et} \quad EV(d) = [0;45]$$

a) $d(0) = \dfrac{0\cdot e^{0}+2}{2} = 1$

b) $d(20) = \dfrac{20e^{-2/3}+2}{2} \cong 6{,}13$

c) $d'(t) = \left[\frac{1}{2}\left(te^{-t/30}+2\right)\right]' = \frac{1}{2}\left(1\cdot e^{-t/30} + te^{-t/30}\cdot\left(-\frac{1}{30}\right)\right) = \frac{1}{2}\underbrace{e^{-t/30}}_{>0}\left(1-\frac{t}{30}\right)$

$d'(t) = 0 \Leftrightarrow 1 - \dfrac{t}{30} = 0 \Leftrightarrow t = 30$

t	0		30		45
sgn(d')	//	$+$	ϕ	$-$	//
d	// min	$\nearrow$	Max	$\searrow$	min //

$\text{Max}(30;\, d(30)) \sim (30;6{,}52)$

Le degré maximal est atteint après 30 min et vaut environ 6,52

Ex 1.1.29

$$h(t) = \frac{40}{1+200e^{-0{,}2t}}$$ hauteur en fct du temps en année.

a) $h(30) = \frac{40}{1+200e^{-0{,}2\cdot 30}} \cong 26{,}74\,m$

b) $16 = \frac{40}{1+200e^{-0{,}2t}} \Leftrightarrow 16(1+200e^{-0{,}2t}) = 40$

$\Leftrightarrow 1+200e^{-0{,}2t} = \frac{10}{4} = 2{,}5$

$\Leftrightarrow 200e^{-0{,}2t} = 1{,}5$

$\Leftrightarrow e^{-0{,}2t} = 0{,}0075$

$\Leftrightarrow -0{,}2t = \ln(0{,}0075)$

$\Leftrightarrow t = \frac{\ln(0{,}0075)}{-0{,}2} \cong 24{,}46$ ans

c) $h'(t) = -\frac{40\cdot(-40e^{-0{,}2t})}{(1+200e^{-0{,}2t})^2}$

$= \frac{1600e^{-0{,}2t}}{(1+200e^{-0{,}2t})^2}$

$v = 1+200e^{-0{,}2t}$

$v' = 200e^{-0{,}2t}\cdot(-0{,}2)$

$= -40e^{-0{,}2t}$

zéro de h' : aucun.

croissance :

t	
h'	+
h	↗

le maximum est atteint lorsque $t \to +\infty$:

$$\lim_{t\to+\infty} h(t) = \frac{40}{1+200\cdot \underbrace{e^{-\infty}}_{\to 0}} = 40$$

$\Rightarrow$ la hauteur maximal de l'arbre est de 40m.