2MS Corrigé - Exp & Log - Exercices de révision

Ex 1

a) $\left(3^{x-3}\right)^3 = \dfrac{3^{x+3}}{3^2} \iff 3^{3x-9} = 3^{x+3-2}$

$\iff 3x-9 = x+1$

$\iff 2x = 10$

$\iff x = 5 \implies S = \{5\}$

b) $16 \cdot 2^x - 4^{3x+5} = 0 \iff 16 \cdot 2^x = 4^{3x+5}$

$\iff 2^4 \cdot 2^x = \left(2^2\right)^{3x+5}$

$\iff 2^{4+x} = 2^{2(3x+5)}$

$\iff 4+x = 2(3x+5)$

$\iff 4+x = 6x+10$

$\iff x = -\dfrac{6}{5} \implies S = \left\{-\dfrac{6}{5}\right\}$

c) $11^{2x} = 15 \iff \log_{11}(15) = 2x$

$\iff x = \dfrac{\log_{11}(15)}{2} \implies S = \left\{\dfrac{\log_{11}(15)}{2}\right\}$

d) $11e^{3x+1} + 1 = 122 \iff 11e^{3x+1} = 121$

$\iff e^{3x+1} = 11$

$\iff 3x+1 = \ln(11)$

$\iff x = \dfrac{\ln(11)-1}{3}$

$\implies S = \left\{\dfrac{\ln(11)-1}{3}\right\}$

Ex 2

a) $6\log_7(x) + 1 = 11 \quad \Leftrightarrow \quad 6\log_7(x) = 10$

$\Leftrightarrow \quad \log_7(x) = \frac{10}{6} = \frac{5}{3} \qquad | \; 7^{()}$

$\Leftrightarrow \quad x = 7^{5/3} = \sqrt[3]{7^5}$

vérif. $6\log_7(7^{5/3}) + 1 \overset{?}{=} 11$

$6 \cdot \frac{5}{3} + 1 \overset{?}{=} 11$

$10 + 1 = 11 \quad \checkmark \qquad \Rightarrow \quad S = \{\sqrt[3]{7^5}\}$

b) $\log(x-4) = 4 \qquad | \; 10^{()}$

$x - 4 = 10^4 = 10000$

$x = 10'004$

vérif: $\log(10'000) = 4 \; \checkmark \qquad \Rightarrow \quad S = \{10'004\}$

c) $\ln(9 - x^2) = 0 \qquad | \; e^{()}$

$9 - x^2 = e^0$

$9 - x^2 = 1$

$8 - x^2 = 0$

$(\sqrt{8} + x)(\sqrt{8} - x) = 0$

$x = \begin{cases} -\sqrt{8} = -2\sqrt{2} \\ \sqrt{8} = 2\sqrt{2} \end{cases}$

vérif: $\ln(9-8) = \ln(1) = 0 \quad \checkmark$

$\ln(9-8) = \quad$ " $\quad \checkmark$

$\Rightarrow \quad S = \{\pm 2\sqrt{2}\}$

d) $2\log(-x) = \log(2-x)$

$$\log((-x)^2) = \log(2-x)$$
$$x^2 = 2-x$$
$$x^2+x-2 = 0$$
$$(x+2)(x-1) = 0$$

$x = 1$ or $x = -2$

vérif. : $2\log(-1)$ ↯ ✗

$2\log(2) \stackrel{?}{=} \log(2+2)$

$\log(4) = \log(4)$ ✓

$\Rightarrow S = \{-2\}$

e) $\log_5(3x) = 1 + 2\log_5(2) - \frac{1}{2}\log_5(400)$

$$\log_5(3x) = \log_5(5) + \log_5(2^2) - \log_5(\underbrace{400^{1/2}}_{\sqrt{400} = 20})$$
$$\log_5(3x) = \log_5\left(\frac{5\cdot 4}{20}\right)$$
$$\log_5(3x) = \log_5(1)$$
$$3x = 1$$
$$x = \frac{1}{3}$$

vérif : $\log_5(3\cdot\frac{1}{3}) = \log_5(1)$ ✓ $\Rightarrow S = \{\frac{1}{3}\}$

Variante : $\log_5(3x) = 1 + 2\log_5(2) - \frac{1}{2}\log_5(400)$

$$\log_5(3x) - 2\log_5(2) + \frac{1}{2}\log_5(400) = 1$$
$$\log_5(3x) - \log_5(4) + \log_5(20) = 1$$
$$\log_5\left(\frac{3x\cdot 20}{4}\right) = 1$$
$$\log_5(15x) = 1 \quad | \; 5^{()}$$
$$15x = 5 \iff x = \frac{1}{3} \quad \ldots$$

Ex 3

a) $P(10) = \dfrac{500}{1+50\cdot e^{-1,66}} \cong 47,59$

47 poules sont malades après 10 jours.

b) $\dfrac{500}{3} = \dfrac{500}{1+50\cdot e^{-0,166t}}$

$\dfrac{1}{3} = \dfrac{1}{1+50\cdot e^{-0,166t}}$

$1+50\cdot e^{-0,166t} = 3$

$50\cdot e^{-0,166t} = 2$

$e^{-0,166t} = \dfrac{1}{25} = 0,04$

$-0,166t = \ln(0,04)$

$t = \dfrac{\ln(0,04)}{-0,166} = 19,39$

Après 20 jours plus du tiers du poulailler est touché.

Ex 4

$A(t) = 2\cdot 2^{t/18}$ population de A en millions d'habitants après t années.

$B(t) = 6\cdot 2^{t/27}$ " B " " "

$A(t) = B(t)$

$2\cdot 2^{t/18} = 6\cdot 2^{t/27} \quad | \div 2$

$2^{t/18} = 3\cdot 2^{t/27} \quad | \div 2^{t/27}$

$2^{t/18 - t/27} = 3$

$2^{\frac{3t-2t}{54}} = 3$

$\Leftrightarrow \quad 2^{\frac{t}{54}} = 3 \quad | \log_2()$

$\dfrac{t}{54} = \log_2(3)$

$t = 54\cdot\log_2(3) = 54\cdot\dfrac{\log(3)}{\log(2)}$

$\cong 85,59$

$\Rightarrow$ Dans 86 ans la population de A aura rattrapé celle de B

Ex 5

$N(0) = 50$

$N(1) = 50 + 50 \cdot 0{,}6 = 50(1+0{,}6) = 50 \cdot 1{,}6$ $\cdot 1{,}6$

$\Rightarrow \quad N(t) = 50 \cdot 1{,}6^t$ nbre de lapins après t années.

a) $q(8) = 50 \cdot 1{,}6^8 \cong 2147{,}48$

Il y aura 2'147 lapins après 8 ans

b) $20'000 = 50 \cdot 1{,}6^t$

$400 = 1{,}6^t$

$t = \log_{1{,}6}(400) = \dfrac{\log(400)}{\log(1{,}6)} \cong 12{,}75$

Le nombre de lapins dépassera 20'000 dans 13 ans.

Ex 6

$C(n) = C(0) \cdot (1+i)^n$

a) $C(10) = 40'000 \cdot 1{,}0375^{10} \cong 57'801{,}76$ CHF

b) $10'730{,}4 = C(0) \cdot 1{,}0525^7$

$C(0) = \dfrac{10'730{,}4}{1{,}0525^7} \cong 7500$ CHF

c) $14'751{,}05 = 12'000\,(1+i)^6$

$\dfrac{14'751{,}05}{12'000} = (1+i)^6$

$\sqrt[6]{\dfrac{14'751{,}05}{12'000}} = 1+i$

$i = \sqrt[6]{\dfrac{14'751{,}05}{12'000}} - 1 \cong 0{,}035 \quad \Rightarrow$ le taux était de 3,5%

d)

$$364'248,25 = 100'000 \cdot 1,09^t$$

$$3,6424825 = 1,09^t \qquad \Big| \log_{1,09}(\;)$$

$$t = \log_{1,09}(3,6424825) = \frac{\log(3,6424825)}{\log(1,09)} \simeq \underline{15 \text{ ans}}$$