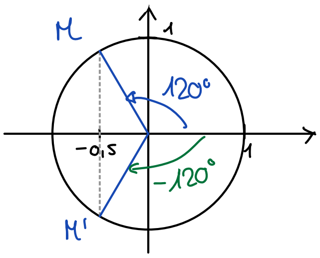
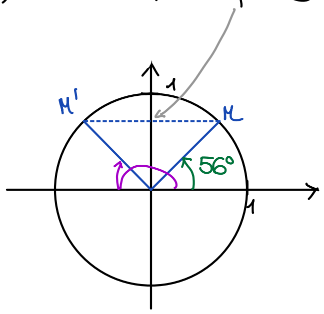
Ex 6.1

a) $\cos(t) = -\frac{1}{2}$ à la màc. : $\cos^{-1}\left(-\frac{1}{2}\right) = 120°$



$$\Leftrightarrow \quad t = \begin{cases} 120° + k\cdot 360° \\ \underbrace{-120° + k\cdot 360°}_{\text{ou } 210° + k\cdot 360°} \end{cases}, \ k \in \mathbb{Z}$$

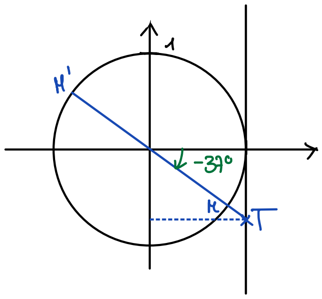
b) $\sin(t) = 0{,}829$ à la màc: $\sin^{-1}(0{,}829) \cong 55{,}996 \cong 56°$



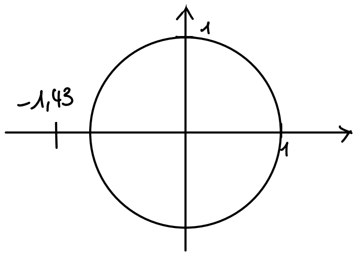
$$\Leftrightarrow \quad t \cong \begin{cases} 56° + k\cdot 360° \\ 180° - 56° + k\cdot 360° = 124° + k\cdot 360° \end{cases}, \ k \in \mathbb{Z}$$

c) $\tan(t) = -0{,}754$ à la màc : $\tan^{-1}(-0{,}754) \cong -37°$

$\Leftrightarrow \quad t \cong -37° + k\cdot 180° \ , \ k \in \mathbb{Z}$

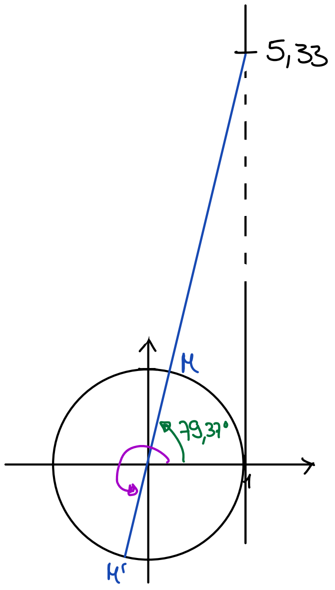


d) $\cos(t) = -1{,}43$ ↯ impossible car $-1{,}43 < -1$

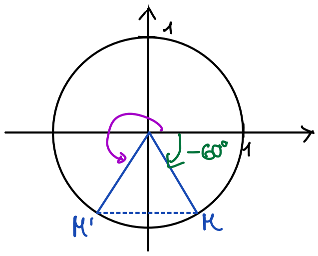


e) $\tan(t) = 5{,}33$ $\tan^{-1}(5{,}33) \cong 79{,}37°$

$\Leftrightarrow \quad t \cong 79{,}37° + k\cdot 180° \quad k \in \mathbb{Z}$



f) $\sin(3t) = -\frac{\sqrt{3}}{2}$ $\qquad \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60°$



$$\Leftrightarrow \quad 3t = \begin{cases} -60° + k\cdot 360° & \mid \div 3 \\ 180°+60°+k\cdot 360° = 240°+k\cdot 360° & \mid \div 3 \end{cases}$$

$$\Leftrightarrow \quad t = \begin{cases} -20° + k\cdot 120° \\ 80° + k\cdot 120° \end{cases}, \quad k \in \mathbb{Z}$$

g) $\tan(5t) = 3{,}273$ $\qquad \tan^{-1}(3{,}273) \hat{=} 73{,}01°$

$$5t \hat{=} 73{,}01° + k\cdot 180° \qquad \mid \div 5$$

$$t \cong 14{,}6° + k\cdot 36°, \quad k \in \mathbb{Z}$$

h) $\cos\left(\frac{t}{2}\right) = -\frac{1}{2}$

$$\Leftrightarrow \quad \frac{t}{2} = \begin{cases} 120° + k\cdot 360° & \mid \cdot 2 \\ -120° + k\cdot 360° & \mid \cdot 2 \end{cases}$$

$$\Leftrightarrow \quad t = \begin{cases} 240° + k\cdot 720° \\ -240° + k\cdot 720° \end{cases}, \quad k \in \mathbb{Z}$$

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f) solutions entre 0 et 360° :

$$t = \begin{cases} \overset{k=1}{100°} \text{ ou } \overset{k=2}{220°} \\ \underset{k=0}{80°} \text{ ou } \underset{k=1}{200°} \text{ ou } \underset{k=2}{320°} \end{cases}$$

g) solutions entre 0° et 180° :

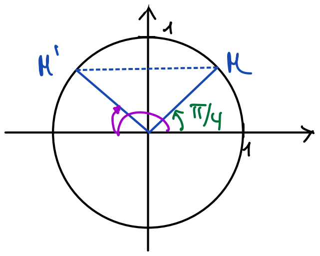
$$t \cong \underset{k=0}{14{,}6°} \text{ ou } \underset{k=1}{50{,}6°} \text{ ou } \underset{k=2}{86{,}6°} \text{ ou } 122{,}6° \text{ ou } 158{,}6° \quad \ldots$$

h) solution entre 0° et 360° : $t = 240°$

k=0 ds 1e solution

Ex 6.2

a) $\sin\left(\frac{2t}{3}+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$ $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)=45^\circ=\frac{\pi}{4}$



$$\frac{2t}{3}+\frac{\pi}{4}=\begin{cases}\frac{\pi}{4}+k\cdot 2\pi & ①\\ \pi-\frac{\pi}{4}+k\cdot 2\pi = \frac{3\pi}{4}+k\cdot 2\pi & ②\end{cases}$$

① $\frac{2t}{3}+\frac{\pi}{4}=\frac{\pi}{4}+k\cdot 2\pi \quad \left|-\frac{\pi}{4}\right.$

$\frac{2t}{3}=k\cdot 2\pi \quad \left|\cdot\frac{3}{2}\right.$

$t=k\cdot\cancel{2}\pi\cdot\frac{3}{\cancel{2}}=k\cdot 3\pi$

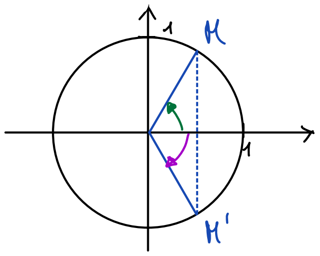
② $\frac{2t}{3}+\frac{\pi}{4}=\frac{3\pi}{4}+k\cdot 2\pi \quad \left|-\frac{\pi}{4}\right.$

$\frac{2t}{3}=\frac{2\pi}{4}+k\cdot 2\pi=\frac{\pi}{2}+k\cdot 2\pi$

$t=\left(\frac{\pi}{2}+k\cdot 2\pi\right)\cdot\frac{3}{2}=\frac{3\pi}{4}+k\cdot 3\pi$

$$\Rightarrow \quad t=\begin{cases}k\cdot 3\pi\\ \frac{3\pi}{4}+k\cdot 3\pi\end{cases} \quad k\in\mathbb{Z}$$

b) $\cos\left(\frac{t}{2}-\frac{\pi}{6}\right)=\frac{1}{2}$ $\cos^{-1}\left(\frac{1}{2}\right)=60^\circ=\frac{\pi}{3}$



$$\frac{t}{2}-\frac{\pi}{6}=\begin{cases}\frac{\pi}{3}+k\cdot 2\pi & ①\\ -\frac{\pi}{3}+k\cdot 2\pi & ②\end{cases}$$

① $\frac{t}{2}=\frac{\pi}{3}+\frac{\pi}{6}+k\cdot 2\pi=\frac{\overset{1}{\cancel{3}}\pi}{\underset{2}{\cancel{6}}}+k\cdot 2\pi$

$\frac{t}{2}=\frac{\pi}{2}+k\cdot 2\pi$

$t=\pi+k\cdot 4\pi$

② $\frac{t}{2}=-\frac{\pi}{3}+\frac{\pi}{6}+k\cdot 2\pi$

$\frac{t}{2}=-\frac{\pi}{6}+k\cdot 2\pi$

$t=-\frac{\pi}{3}+k\cdot 4\pi$

$$\Rightarrow \quad t=\begin{cases}\pi+k\cdot 4\pi\\ -\frac{\pi}{3}+k\cdot 4\pi\end{cases}, \quad k\in\mathbb{Z}$$

c) $2\cos(t)+1 = 0$

$$2\cos(t) = -1$$

$$\cos(t) = -\frac{1}{2}$$ (idem ex 6.1 a) mais en radians)

$$t = \begin{cases} \frac{2\pi}{3}+k\cdot 2\pi \\ -\frac{2\pi}{3}+k\cdot 2\pi \end{cases}, \quad k\in\mathbb{Z}$$

(solutions entre 0 et 2π : $t = \begin{cases} \frac{2\pi}{3} \\ \underset{k=1}{\frac{4\pi}{3}} \end{cases}$)

d) $4\sin^2(x)-3 = 0$

$$\Leftrightarrow \quad 4\sin^2(x) = 3$$

$$\Leftrightarrow \quad \sin^2(x) = \frac{3}{4} \quad \Big| \sqrt{\ }$$ ⚠ 2 solutions.

$$\Leftrightarrow \quad \sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

⊕ : $\sin(x) = \frac{\sqrt{3}}{2}$

$$\Leftrightarrow \quad x = \begin{cases} \frac{\pi}{3}+k\cdot 2\pi \\ \pi-\frac{\pi}{3}+k\cdot 2\pi = \frac{2\pi}{3}+k\cdot 2\pi \end{cases}, \quad k\in\mathbb{Z}$$

⊖ $\sin(x) = -\frac{\sqrt{3}}{2}$

$$\Leftrightarrow \quad x = \begin{cases} -\frac{\pi}{3}+k\cdot 2\pi \\ \pi+\frac{\pi}{3}+k\cdot 2\pi = \frac{4\pi}{3}+k\cdot 2\pi \end{cases}, \quad k\in\mathbb{Z}$$

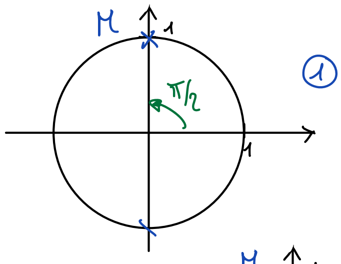
e) $(\sin(x)-1)\cos(x) = 0$

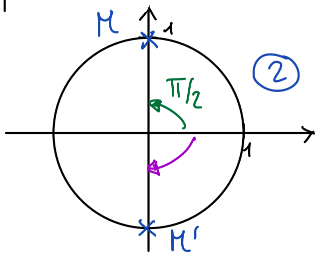
① $\sin(x)-1 = 0$ ou $\cos(x) = 0$ ②

$$\Leftrightarrow \quad \sin(x) = 1$$

$$\Leftrightarrow \quad x = \frac{\pi}{2}+k\cdot 2\pi$$

$$\Leftrightarrow \quad x = \begin{cases} \frac{\pi}{2}+k\cdot 2\pi \\ -\frac{\pi}{2}+k\cdot 2\pi \end{cases}$$





$$\Rightarrow \quad x = \frac{\pi}{2}+k\cdot 2\pi \quad \text{ou} \quad -\frac{\pi}{2}+k\cdot 2\pi, \quad k\in\mathbb{Z}$$

f) $\sqrt{3} + 2\sin(3x) = 0$

$\Leftrightarrow \quad 2\sin(3x) = -\sqrt{3}$

$\Leftrightarrow \quad \sin(3x) = -\frac{\sqrt{3}}{2}$ (idem ex 6.1 f) mais en radians)

$$\Leftrightarrow \quad x = \begin{cases} -\frac{\pi}{9} + k \cdot \frac{2\pi}{3} \\ \frac{4\pi}{9} + k \cdot \frac{2\pi}{3} \end{cases}, \ k \in \mathbb{Z}$$

Ex 6.3

a) $4\cos^2(t) - 4\cos(t) - 3 = 0$

Chgmt de variable : on pose $y = \cos(t)$ ⊛

$$\Rightarrow \quad 4y^2 - 4y - 3 = 0 \qquad \Delta = 64$$

$$y_{1,2} = \frac{4 \pm 8}{8} = \begin{cases} \frac{3}{2} & \overset{⊛}{\Longrightarrow} \cos(t) = \frac{3}{2} \quad ↯ \text{ car } \frac{3}{2} > 1 \\ -\frac{1}{2} & \overset{⊛}{\Longrightarrow} \cos(t) = -\frac{1}{2} \Rightarrow t = \begin{cases} \frac{2\pi}{3} + k \cdot 2\pi \\ -\frac{2\pi}{3} + k \cdot 2\pi \end{cases}, \ k \in \mathbb{Z} \end{cases}$$

(ou 2^e sol : $-\frac{2\pi}{3} + 2\pi = -\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{4\pi}{3}$)

b) $2\sin^2(x) - 3\sin(x) + 1 = 0$

Chgmt de variable : on pose $y = \sin(x)$

$$\Rightarrow \quad 2y^2 - 3y + 1 = 0 \qquad \Delta = 1$$

$$\Rightarrow \quad y_{1,2} = \frac{3 \pm 1}{4} = \begin{cases} 1 & \Rightarrow \sin(x) = 1 \\ \frac{1}{2} & \Rightarrow \sin(x) = \frac{1}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{2} + k \cdot 2\pi, \quad k \in \mathbb{Z} \\ x = \begin{cases} \frac{\pi}{6} + k \cdot 2\pi \\ \pi - \frac{\pi}{6} + k \cdot 2\pi = \frac{5\pi}{6} + k \cdot 2\pi \end{cases}, \quad k \in \mathbb{Z} \end{cases}$$

c) $3\sin^2(z) + 8\cos(z) + 1 = 0$

Comme $\sin^2(z) + \cos^2(z) = 1 \Leftrightarrow \sin^2(z) = 1 - \cos^2(z)$

L'équation devient :

$$3(1-\cos^2(z)) + 8\cos(z) + 1 = 0$$

$$3 - 3\cos^2(z) + 8\cos(z) + 1 = 0$$

$$-3\cos^2(z) + 8\cos(z) + 4 = 0$$

$$3\cos^2(z) - 8\cos(z) - 4 = 0$$

Chgmt de variable : on pose $y = \cos(z)$

$$3y^2 - 8y - 4 \qquad \Delta = 112$$

$$y = \frac{8 \pm \sqrt{112}}{6} \cong \begin{cases} 3{,}097 \Rightarrow \cos(z) \cong 3{,}097 \text{ ↯ car } 3{,}097 > 1 \\ -0{,}431 \Rightarrow \cos(z) \cong -0{,}431 \end{cases}$$

$$\Rightarrow z \cong \begin{cases} 2{,}016 + k\cdot 2\pi \\ -2{,}016 + k\cdot 2\pi \end{cases}, k \in \mathbb{Z}$$

d) $3\sin^2(t) + \cos^2(t) - 2 = 0$

Comme $\sin^2(t) + \cos^2(t) = 1 \Leftrightarrow \cos^2(t) = 1 - \sin^2(t)$

L'équation devient :

$$3\sin^2(t) + 1 - \sin^2(t) - 2 = 0$$

$$2\sin^2(t) - 1 = 0$$

$$\sin^2(t) = \frac{1}{2}$$

$$\sin(t) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} + \quad t = \begin{cases} \frac{\pi}{4} + k\cdot 2\pi \\ \pi - \frac{\pi}{4} + k\cdot 2\pi = \frac{3\pi}{4} + k\cdot 2\pi \end{cases}, k \in \mathbb{Z} \\ - \quad t = \begin{cases} -\frac{\pi}{4} + k\cdot 2\pi \\ \pi - (-\frac{\pi}{4}) + k\cdot 2\pi = \frac{5\pi}{4} + k\cdot 2\pi \end{cases}, k \in \mathbb{Z} \end{cases}$$

$$\Rightarrow t = \frac{\pi}{4} + k\cdot\frac{\pi}{2} \quad , k \in \mathbb{Z}$$

e) $5\sin(x) = 6\cos^2(x)$

Comme $\sin^2(x) + \cos^2(x) = 1 \Leftrightarrow \cos^2(x) = 1 - \sin^2(x)$

L'équation devient :

$$5\sin(x) = 6(1 - \sin^2(x))$$

$$6\sin^2(x) + 5\sin(x) - 6 = 0$$

Chgmt de variable : on pose $y = \sin(x)$

$6y^2 + 5y - 6 = 0 \qquad \Delta = 169$

$$y_{1,2} = \frac{-5 \pm 13}{12} = \begin{cases} \frac{2}{3} \Rightarrow \sin(x) = \frac{2}{3} \Leftrightarrow x \cong \begin{cases} 0{,}73 + k \cdot 2\pi \\ \pi - 0{,}73 + k \cdot 2\pi = \\ \qquad 2{,}412 + k \cdot 2\pi \end{cases} , k \in \mathbb{Z} \\ -\frac{3}{2} \end{cases}$$

f) $\tan^4(t) - 4\tan^2(t) + 3 = 0$

Chgmt de variable : on pose $y = \tan^2(t)$

$$y^2 - 4y + 3 = 0$$

$$(y-3)(y-1) = 0$$

$$y = \begin{cases} 1 \Rightarrow \tan^2(t) = 1 \Leftrightarrow \tan(t) = \pm 1 \begin{cases} + & t = \frac{\pi}{4} + k \cdot \pi \quad k \in \mathbb{Z} \\ - & t = -\frac{\pi}{4} + k \cdot \pi \quad " \end{cases} \\ 3 \Rightarrow \tan^2(t) = 3 \Leftrightarrow \tan(t) = \pm\sqrt{3} \begin{cases} + & t = \frac{\pi}{3} + k \cdot \pi \quad " \\ - & t = -\frac{\pi}{3} + k \cdot \pi \quad " \end{cases} \end{cases}$$