

Ex 1.1.3

$$a) \int 2e^x dx = 2 \int e^x dx = \underline{2e^x + c}$$

$$b) \int e^{2x} dx = \frac{1}{2} \int e^{2x} \cdot 2 dx = \underline{\frac{1}{2} e^{2x} + c}$$

$u = 2x$
 $u' = 2$

$$c) \int (2 - e^x) dx = \int 2 dx - \int e^x dx = \underline{2x - e^x + c}$$

$$d) \int e^{2-x} dx = - \int e^{2-x} \cdot (-1) dx = \underline{-e^{2-x} + c}$$

$u = 2-x$
 $u' = -1$

Ex 1.1.4

$$a) \int_1^2 e^x dx = e^x \Big|_1^2 = e^2 - e^1 = \underline{e^2 - e}$$

$$b) \int_1^2 e^{3x-7} dx = \frac{1}{3} \int_1^2 e^{3x-7} \cdot 3 dx = \frac{1}{3} e^{3x-7} \Big|_1^2$$

$u = 3x-7$
 $u' = 3$

$$= \frac{1}{3} (e^{-1} - e^{-4}) = \frac{1}{3} \left(\frac{1}{e} - \frac{1}{e^4} \right) = \frac{1}{3} \frac{e^3 - 1}{e^4}$$
$$= \underline{\frac{e^3 - 1}{3e^4}}$$

$$c) \int_0^2 x e^{x^2} dx = \frac{1}{2} \int_0^2 2x e^{x^2} dx = \frac{1}{2} e^{x^2} \Big|_0^2$$

$u = x^2$
 $u' = 2x$

$$= \frac{1}{2} (e^4 - e^0) = \underline{\frac{1}{2} (e^4 - 1)} = \underline{\frac{e^4 - 1}{2}}$$

$$d) \int_1^2 \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = -2 \int_1^2 -\frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot e^{\sqrt{x}} dx = -2 e^{-\sqrt{x}} \Big|_1^2$$

si $u = \sqrt{x}$
 $u' = \frac{1}{2\sqrt{x}}$ | si $u = -\sqrt{x}$
 $u' = -\frac{1}{2\sqrt{x}} = -\frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$$= -2 e^{-\sqrt{2}} + 2 e^{-1}$$
$$= \frac{-2}{e^{\sqrt{2}}} + \frac{2}{e} = \underline{-2 \left(\frac{1}{e^{\sqrt{2}}} - \frac{1}{e} \right)}$$

Ex 1.1.7

$$a) \int \frac{1}{x+1} dx = \underline{\ln(|x+1|) + C}$$

$$b) \int \frac{1}{3x+2} dx = \frac{1}{3} \int \frac{3}{3x+2} dx = \underline{\frac{1}{3} \ln(|3x+2|) + C} = \underline{\ln(\sqrt[3]{|3x+2|}) + C}$$

$u = 3x+2$
 $u' = 3$

$$c) \int \frac{x-1}{x^2-2x+4} dx = \frac{1}{2} \int \frac{2(x-1)}{x^2-2x+4} dx = \underline{\frac{1}{2} \ln(|x^2-2x+4|) + C}$$

$u = x^2-2x+4$
 $u' = 2x-2 = 2(x-1)$

$$= \underline{\ln(\sqrt{|x^2-2x+4|}) + C}$$

$$d) \int \left(x^2 + x + 1 + \frac{3}{5x-1} \right) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C_1 + \frac{3}{5} \int \frac{5}{5x-1} dx$$
$$= \underline{\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{3}{5} \ln(|5x-1|) + C}$$

$$e) \int \frac{x^2+2x-2}{x-1} dx$$

$\deg(N) \geq \deg(D) \Rightarrow$ div. euclidienne

$$\begin{array}{r|l} x^2+2x-2 & x-1 \\ -x^2+x & \\ \hline 3x-2 & \\ -3x+3 & \\ \hline 1 & \end{array}$$

$$\Rightarrow f(x) = x+3 + \frac{1}{x-1}$$

$$= \int \left(x+3 + \frac{1}{x-1} \right) dx = \underline{\frac{1}{2}x^2 + 3x + \ln(|x-1|) + C}$$

$$f) \int \tan(x) dx = - \int \frac{-\sin(x)}{\cos(x)} dx = \underline{-\ln(|\cos(x)|) + C}$$

$u = \cos(x)$
 $u' = -\sin(x)$

Ex 1.1.8

$$a) \int_2^5 \frac{dx}{x} = \ln(|x|) \Big|_2^5 = \ln(5) - \ln(2) = \underline{\ln\left(\frac{5}{2}\right)}$$

$$b) \int_{-1}^{-3} \frac{dx}{x} = \ln(|x|) \Big|_{-1}^{-3} = \ln(3) - \ln(1) = \underline{\ln(3)}$$

$$c) \int_{-1}^4 \frac{dx}{x} = \lim_{\varepsilon \rightarrow 0} \left(\int_{-1}^{0-\varepsilon} \frac{dx}{x} + \int_{0+\varepsilon}^4 \frac{dx}{x} \right) = \lim_{\varepsilon \rightarrow 0} \left(\ln(|x|) \Big|_{-1}^{0-\varepsilon} + \ln(|x|) \Big|_{0+\varepsilon}^4 \right)$$

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$$= \lim_{\varepsilon \rightarrow 0} \left(\ln(0-\varepsilon) - \ln(1) + \ln(4) - \ln(0+\varepsilon) \right)$$

$$= \underline{\ln(4)}$$

$$d) \int_1^4 \frac{dx}{2x+3} = \frac{1}{2} \int_1^4 \frac{2}{2x+3} dx = \frac{1}{2} \ln(12x+3) \Big|_1^4$$

$u=2x+3$
 $u'=2$

$$= \frac{1}{2} \left(\ln(u) - \ln(5) \right) = \frac{1}{2} \ln\left(\frac{u}{5}\right) = \underline{\ln\left(\sqrt{\frac{u}{5}}\right)}$$

$$e) \int_2^6 \frac{8x^3 + 19x^2 + 15x + 4}{x^2 + 2x + 1} dx$$

$$= \int_2^6 \left(8x + 3 + \frac{x+1}{x^2+2x+1} \right) dx$$

$$= \int_2^6 (8x+3) dx + \frac{1}{2} \int_2^6 \frac{2(x+1)}{x^2+2x+1} dx$$

$\deg(N) \geq \deg(D) \Rightarrow \text{div. eucl.}$

$$\begin{array}{r|l} 8x^3 + 19x^2 + 15x + 4 & x^2 + 2x + 1 \\ -8x^3 - 16x^2 - 8x & 8x + 3 \\ \hline 3x^2 + 7x + 4 & \\ -3x^2 - 6x - 3 & \\ \hline x + 1 & \end{array}$$

$$= 4x^2 + 3x \Big|_2^6 + \frac{1}{2} \ln(|x^2+2x+1|) \Big|_2^6$$

$$= 162 - 22 + \frac{1}{2} \ln(49) - \frac{1}{2} \ln(9)$$

$$= 140 + \ln(7) - \ln(3) = \underline{\underline{140 + \ln\left(\frac{7}{3}\right)}}$$

$$1) \int_{\pi/3}^{\pi/2} \frac{\sin(x)}{1-\cos(x)} dx = \ln |1-\cos(x)| \Big|_{\pi/3}^{\pi/2} = \ln(1) - \ln\left(\frac{1}{2}\right) = -\ln\left(\frac{1}{2}\right) = \underline{\ln(2)}$$

$$u = 1 - \cos(x)$$

$$u' = -(-\sin(x)) \\ = \sin(x)$$