

2.2.1 Déterminer l'ensemble de définition des fonctions suivantes :

↓

a) $f(x) = 4 - 5x$

b) $f(x) = x^2 - x - 2$

c) $f(x) = (x + 4)^2(2 + x)$

d) $f(x) = -6x^3 + 11x^2 - 3x$

e) $f(x) = x^3 + 2x^2 - 4x - 8$

f) $f(x) = x^4 + 5x^2 - 36$

Comme toutes les fcts sont polynomiales $ED(f) = \mathbb{R}$

+ Etude de signe :

a) zéro : $\frac{4}{5}$ signe :

x	$\frac{4}{5}$
$\text{sgn}(f)$	$+ \quad 0 \quad -$

 $(m = -5 < 0 \searrow)$

b) zéros : $x^2 - x - 2 = 0 \Leftrightarrow (x-2)(x+1) = 0$ ou $\Delta = \dots$
 $\downarrow \quad \downarrow$
 $2 \quad -1$

signe :

x	-1	2
$\text{sgn}(f)$	$+ \quad 0 \quad - \quad 0 \quad +$	

 $(a = 1 > 0 \cup)$

c) zéros : $(x+4)^2(2+x) = 0 \Leftrightarrow x = -4$ et $x = -2$
 (2)

signe :

x	-4	-2
$\text{sgn}(f)$	$- \quad 0 \quad - \quad 0 \quad +$	

 $f(1000) : + \cdot + = +$
 (2)

d) zéros : $-6x^3 + 11x^2 - 3x = 0$

$\Leftrightarrow x(-6x^2 + 11x - 3) = 0$

$\Delta = 49$

$\Leftrightarrow x = 0$ ou $x_{1,2} = \frac{-11 \pm 7}{-12} = \begin{cases} \frac{1}{3} \\ \frac{3}{2} \end{cases}$

signe :

x	0	$\frac{1}{3}$	$\frac{3}{2}$
$\text{sgn}(f)$	$+ \quad 0 \quad - \quad 0 \quad + \quad 0 \quad -$		

 $f(1000) : -$

e) zéros : $x^3 + 2x^2 - 4x - 8 = 0$

$$\Leftrightarrow x^2(x+2) - 4(x+2) = 0$$

$$\Leftrightarrow (x+2)(x^2-4) = 0$$

$$\Leftrightarrow (x+2)(x+2)(x-2) = 0$$

$$\Leftrightarrow (x+2)^2(x-2) \Leftrightarrow x = -2 \text{ ou } x = 2$$

(2)

signe :

x	-2	2	
$\text{sgn}(f)$	- 0	- 0	+
	(2)		

$f(1000) : +$

f) zéros : $x^4 + 5x^2 - 36 = 0$

chgmt de variable, on pose $y = x^2$ ⊗

$$\Rightarrow y^2 + 5y - 36 = 0$$

$$\Leftrightarrow (y+9)(y-4) = 0$$

$$\Leftrightarrow (x^2+9)(x^2-4) = 0 \Leftrightarrow \underbrace{(x^2+9)}_{>0}(x+2)(x-2) = 0$$

$$\Leftrightarrow x = -2 \text{ ou } x = 2$$

signe :

x	-2	2	
$\text{sgn}(f)$	+ 0	- 0	+

$f(1000) : +$

Ex 2.2.2 ED(f) + signe

a) $f(x) = \frac{x(x+4)}{3-2x}$ cond: $3-2x \neq 0 \Leftrightarrow x \neq \frac{3}{2}$ (v.i.)

$$ED(f) = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

zéro : $x(x+4) = 0 \Leftrightarrow x = 0$ ou $x = -4$

signe :

x	-4	0	3/2	
sgn(f)	+	0	-	+

$\text{à } f(1000) : \frac{+}{-} = -$

b) $f(x) = \frac{2x}{16-x^2}$

cond: $16-x^2 \neq 0 \Leftrightarrow (4+x)(4-x) \neq 0$
 $\Leftrightarrow x \neq -4$ et $x \neq 4$ (v.i.)

$$ED(f) = \mathbb{R} - \{ \pm 4 \}$$

zéro : $2x = 0 \Leftrightarrow x = 0$

signe :

x	-4	0	4	
sgn(f)	+	0	-	+

$\text{à } f(1000) : \frac{+}{-} = -$

c) $f(x) = \frac{(x+2)^2(x+1)}{x^2+x}$

cond: $x^2+x \neq 0 \Leftrightarrow x(x+1) \neq 0$
 $\Leftrightarrow x \neq 0$ et $x \neq -1$ (v.i.)

$$ED(f) = \mathbb{R}^* - \{-1\}$$

zéros : $(x+2)^2(x+1) = 0 \Leftrightarrow x = -2$ (ou $x = -1$) (2) (2)

signe :

x	-2	-1	0	
sgn(f)	-	0	-	+

$\text{à } f(1000) : \frac{+ \cdot +}{+} = +$

$$d) f(x) = x - \frac{1}{x} = \frac{x^2 - 1}{x} = \frac{(x+1)(x-1)}{x} \quad \text{zéros: } -1 \text{ et } 1$$

$$\text{v.i.: } 0$$

$$\text{ED}(f) = \mathbb{R}^*$$

$$\text{signe: } \begin{array}{c|cccc} x & -1 & 0 & 1 & \\ \hline \text{sgn}(f) & - & 0 & + & - & 0 & + \end{array} \quad \text{↖ } f(\infty): \frac{+}{+}$$

$$e) f(x) = \frac{1}{x-5} + \frac{3}{x+1} = \frac{x+1 + 3(x-5)}{(x-5)(x+1)} = \frac{4x-14}{(x-5)(x+1)} = \frac{2(2x-7)}{(x-5)(x+1)}$$

$$\text{ED}(f) = \mathbb{R} - \{-1; 5\}$$

$$\text{zéros: } 2x-7=0 \Leftrightarrow x = \frac{7}{2}$$

$$\text{signe: } \begin{array}{c|ccc} x & -1 & \frac{7}{2} & 5 \\ \hline \text{sgn}(f) & - & 0 & - & 0 & + \end{array} \quad \text{↖ } f(\infty): \frac{+}{+}$$

$$f) f(x) = \frac{-5(4-x)^2}{(1-x^2)(2-x)} \quad \text{cond: } (1-x^2)(2-x) \neq 0 \Leftrightarrow \begin{array}{ccc} (1+x)(1-x)(2-x) \neq 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ -1 \quad 1 \quad 2 \quad (\text{v.i.}) \end{array}$$

$$\text{ED}(f) = \mathbb{R} - \{-1; 1; 2\}$$

$$\text{zéro: } -5(4-x)^2 = 0 \Leftrightarrow x = 4 \quad (2)$$

$$\text{signe: } \begin{array}{c|cccc} x & -1 & 1 & 2 & 4 \\ \hline \text{sgn}(f) & + & 0 & - & 0 & + & 0 & - \end{array} \quad \text{↖ } f(\infty): \frac{- \cdot +}{- \cdot -} = \frac{-}{+} = -$$

Ex 2.2.3

a) $f(x) = \sqrt{x^2 + x + 1}$

cond: $x^2 + x + 1 \geq 0$ $\Delta = -3 < 0$ pas de zéro

x	
$x^2 + x + 1$	+

car $a = 1 > 0$

$ED(f) = \mathbb{R}$

pas de zéro

signe :

x	
$\text{sgn}(f)$	+

b) $f(x) = \sqrt{x-1} \sqrt{x-5}$

cond: $x-1 \geq 0$ et $x-5 \geq 0$

$x \geq 1$ et $x \geq 5$
 $\rightarrow x \geq 5$

$ED(f) = [5; +\infty[$

zéro : $x-1=0 \Leftrightarrow x=1 \notin ED(f)$

$x-5=0 \Leftrightarrow x=5 \in ED(f)$

signe :

x	
$\text{sgn}(f)$	0
	+

c) $f(x) = \sqrt{(x-1)(x-5)}$

cond: $(x-1)(x-5) \geq 0$

$ED(f) =]-\infty; 1] \cup [5; +\infty[$

x	
$(x-1)(x-5)$	+
	0
	-
	0
	+

zéros : $\sqrt{(x-1)(x-5)} = 0 \Leftrightarrow (x-1)(x-5) = 0 \Leftrightarrow x=1$ ou $x=5$

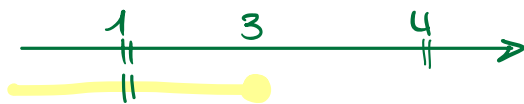
signe :

x	
$\text{sgn}(f)$	+
	0
	-
	0
	+

$$d) f(x) = \frac{\sqrt{6-2x}}{x^2-5x+4} = \frac{\sqrt{2(3-x)}}{(x-1)(x-4)}$$

cond: $2(3-x) \geq 0$ et $x \neq 1$ et $x \neq 4$
 $3-x \geq 0$
 $3 \geq x$

$$\Rightarrow \text{ED}(f) =]-\infty; 3] - \{1\}$$



zéro de f : $\sqrt{6-2x} = 0 \Leftrightarrow 6-2x=0 \Leftrightarrow x=3$

signe :

x		1		3	
$\text{sgn}(f)$		+		-	0 // //

$f(0) = \frac{\sqrt{6}}{4} : +$

$$e) f(x) = \sqrt{\frac{x+1}{x-4}}$$

cond: $\frac{x+1}{x-4} \geq 0$

x		-1		4	
$\frac{x+1}{x-4}$		+	0	-	+

$$\Rightarrow \text{ED}(f) =]-\infty; -1] \cup]4; +\infty[$$

zéro de f : $\sqrt{\frac{x+1}{x-4}} = 0 \Leftrightarrow x+1=0 \Leftrightarrow x=-1$

signe de f :

x		-1		4	
		+	0 // //	-	+

$$f) f(x) = \frac{x^2+7x}{\sqrt{1-x^2}} = \frac{x(x+7)}{\sqrt{(1+x)(1-x)}}$$

cond: $1-x^2 \geq 0$ et $\sqrt{1-x^2} \neq 0$
 $1-x^2 > 0$

$$\Rightarrow \text{ED}(f) =]-1; 1[$$

x		-1		1	
$1-x^2$		-	0	+	0

zéro de f : $x(x+7) = 0 \Leftrightarrow x=0$ ou $x=-7$
 $\in \text{ED}(f)$ $\notin \text{ED}(f)$

signe de f :

x		-1		0		1	
f		// //		-	0	+	// //

$f(-0,5) : \frac{0,25-3,5}{+} < 0$

Ex 2.2.5

a) $f(x) = \ln(7-2x)$

cond: $7-2x > 0$
 $\downarrow$
 $\frac{7}{2}$

x	$\frac{7}{2}$
$7-2x$	+ 0

$\Rightarrow \underline{ED(f) =]-\infty; \frac{7}{2}[}$

b) $f(x) = e^{x-1}$

$\underline{ED(f) = \mathbb{R}}$

car aucune opération interdite

c) $f(x) = \frac{3-x}{1-\log(x)}$

cond: $1-\log(x) \neq 0$ or $x > 0$
 $x \neq \log(x)$
 $10 \neq x$

$\Rightarrow \underline{ED(f) = \mathbb{R}_+^* - \{10\}}$

d) $f(x) = 3^{\frac{1}{x+2}}$

cond: $x+2 \neq 0 \Leftrightarrow x \neq -2$

$\underline{ED(f) = \mathbb{R} - \{-2\}}$

Ex 2.2.6

$$a) \quad f(x) = 3 \quad g(x) = x^2 \quad \text{ED}(f) = \mathbb{R} \quad \text{ED}(g) = \mathbb{R}$$

$$(f+g)(x) = 3+x^2 \quad \text{ED}(f+g) = \mathbb{R}$$

$$(f-g)(x) = 3-x^2 \quad \text{ED}(f-g) = \mathbb{R}$$

$$(f \cdot g)(x) = 3x^2 \quad \text{ED}(f \cdot g) = \mathbb{R}$$

$$\left(\frac{f}{g}\right)(x) = \frac{3}{x^2} \quad \text{ED}\left(\frac{f}{g}\right) = \mathbb{R}^*$$

$$b) \quad f(x) = \frac{2x}{x-4} \quad \text{ED}(f) = \mathbb{R} - \{4\}$$

$$g(x) = \frac{x}{x+5} \quad \text{ED}(g) = \mathbb{R} - \{-5\}$$

$$(f+g)(x) = \frac{2x}{x-4} + \frac{x}{x+5} = \dots = \frac{3x^2+6x}{(x-4)(x+5)}$$

$$\text{ED}(f+g) = \mathbb{R} - \{-5, 4\}$$

$$(f-g)(x) = \frac{2x}{x-4} - \frac{x}{x+5} = \dots = \frac{x^2+14x}{(x-4)(x+5)}$$

$$\text{ED}(f-g) = "$$

$$(f \cdot g)(x) = \frac{2x}{x-4} \cdot \frac{x}{x+5} = \frac{2x^2}{(x-4)(x+5)}$$

$$\text{ED}(f \cdot g) = "$$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{2x}{x-4}}{\frac{x}{x+5}} = \frac{2x}{x-4} \cdot \frac{x+5}{x} = \frac{2x(x+5)}{x(x-4)}$$

$$\text{ED}\left(\frac{f}{g}\right) = \mathbb{R}^* - \{-5, 4\}$$

$$c) \quad f(x) = \sqrt{x} \quad \text{ED}(f) = \mathbb{R}_+$$

$$g(x) = \sqrt{4x} \quad \text{ED}(g) = \mathbb{R}_+$$

$$(f+g)(x) = \sqrt{x} + \sqrt{4x} = \sqrt{x} + 2\sqrt{x} = 3\sqrt{x}$$

$$\text{ED}(f+g) = \mathbb{R}_+$$

$$(f-g)(x) = \sqrt{x} - \sqrt{4x} = \sqrt{x} - 2\sqrt{x} = -\sqrt{x}$$

$$\text{ED}(f-g) = "$$

$$(f \cdot g)(x) = \sqrt{x} \cdot \sqrt{4x} = \sqrt{4x^2} = 2x$$

$$\text{ED}(f \cdot g) = "$$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{\sqrt{4x}} = \sqrt{\frac{x}{4x}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{ED}\left(\frac{f}{g}\right) = \mathbb{R}_+^*$$

Ex 2.2.7

$$f(x) = 2x$$

$$g(x) = 2x - 1$$

$$h(x) = x^2$$

$$\text{ED} = \mathbb{R}$$

$$a) (f \circ g)(x) = f(g(x)) = f(2x - 1) = 2(2x - 1) = \underline{4x - 2}$$

$$b) (h \circ f)(x) = h(f(x)) = h(2x) = (2x)^2 = \underline{4x^2}$$

$$c) (g \circ h \circ f)(x) = g(h(f(x))) = g(h(2x)) = g(4x^2) = 2(4x^2) - 1 = \underline{8x^2 - 1}$$

Ex 2.2.8

$$a) f(x) = x^2 - 3x$$

$$g(x) = \sqrt{x+2}$$

$$\text{ED}(f) = \mathbb{R}$$

$$\text{ED}(g) = [-2; +\infty[$$

$$\cdot (f \circ g)(x) = f(g(x)) = f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 3\sqrt{x+2} = \underline{x+2 - 3\sqrt{x+2}}$$

$$\underline{\text{ED}(f \circ g) = [-2; +\infty[}$$

$$\cdot (g \circ f)(x) = g(f(x)) = g(x^2 - 3x) = \underline{\sqrt{x^2 - 3x + 2}}$$

$$\text{ED} : \text{cond: } x^2 - 3x + 2 \geq 0$$

$$(x-2)(x-1) \geq 0$$

x	1	2
$\text{sgn}(x^2 - 3x + 2)$	+	+

$$\Rightarrow \underline{\text{ED}(g \circ f) =]-\infty; 1] \cup [2; +\infty[}$$