Ex1

a) $f(x) = x^3 + 3x^2 - 4x - 12$

$ED(f) = \mathbb{R}$

zéros : $\underbrace{x^3 + 3x^2}\underbrace{- 4x - 12} = 0$ ou Horner :

$x^2(x+3) - 4(x+3) = 0$

$(x+3)(x^2-4) = 0$

$(x+3)(x+2)(x-2) = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$-3 \quad -2 \quad 2$

Horner : $x = 2 : \; 8 + 12 - 8 - 12 = 0$ ✓

	1	3	−4	−12
2		2	10	12
	1	5	6	0

$\Rightarrow (x-2)(x^2+5x+6) = (x-2)(x+2)(x+3)$

signe :

x		−3		−2		2	
sgn(f)	−	0	+	0	−	0	+

$f(1000) : +$

b) $f(x) = \dfrac{5(x-2)^2(3-x)}{(x+2)(x-4)}$ ← zéros : 2 (2) et 3 ← v.i. : −2 et 4

$ED(f) = \mathbb{R} - \{-2 ; 4\}$

signe :

x		−2		2		3		4	
sgn(f)	+	‖	−	0 (2)	−	0	+	‖	−

$f(1000) : \dfrac{-}{+}$

c) $f(x) = \dfrac{8x}{(x-2)^2} + 1 = \dfrac{8x}{(x-2)^2} + \dfrac{(x-2)^2}{(x-2)^2} = \dfrac{8x + x^2 - 4x + 4}{(x-2)^2} = \dfrac{x^2+4x+4}{(x-2)^2} = \dfrac{(x+2)^2}{(x-2)^2}$

$ED(f) = \mathbb{R} - \{2\}$ v.i : 2 (2)

zéro : −2 (2)

signe :

x		−2		2	
sgn(f)	+	0 (2)	+	‖ (2)	+

$f(1000) : \dfrac{+}{+}$

d) $f(x) = x\sqrt{x^2-81}$

cond : $x^2 - 81 \geqslant 0 \iff (x+9)(x-9) \geqslant 0$

x		-9		9	
sgn(x^2-81)	$+$	0	$-$	0	$+$

$ED(f) =]-\infty ; -9] \cup [9 ; +\infty[$

zéros : $0 \notin ED(f)$, $x = \pm 9$

signe :

x		-9		9	
sgn(f)	$-$	0	//////	0	$+$

($\sqrt{x^2-81}$ est tjrs positif donc sgn(f) ne dépend que du sgn de x)

e) $f(x) = \dfrac{\sqrt{1-x}}{x^3+2x^2} = \dfrac{\sqrt{1-x}}{x^2(x+2)}$

$x^2 \to 0_{(2)}$, $(x+2) \to -2$

cond : $1-x \geqslant 0$ et $x \neq 0$ et $x \neq -2$

$1 \geqslant x$

$\Rightarrow ED(f) =]-\infty ; 1] - \{-2 ; 0\}$

zéro : $\sqrt{1-x} = 0 \iff 1-x = 0 \iff x = 1$

signe :

x		-2		0		1	
sgn(f)	$-$	$\|\|$	$+$	$\|\|_{(2)}$	$+$	0	//////

($\sqrt{1-x}$ tjrs pos.
x^2 " "
donc sgn(f) ne dépend que de $x+2$)

Ex 2

$f(x) = \cos(x)$ et $g(x) = 1 - x^2$

$(f \circ g)(x) = f(g(x)) = f(1-x^2) = \cos(1-x^2)$

$(g \circ f)(x) = g(f(x)) = g(\cos(x)) = 1 - \cos^2(x)$ $(= \sin^2(x))$

Ex3

a) $f(x) = \dfrac{x+1}{x^2-9} = \dfrac{x+1}{(x+3)(x-3)}$

1. $ED(f) = \mathbb{R} - \{\pm 3\}$

2. zéro : -1 signe

x		-3		-1		3	
sgn(f)	$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$

$f(1000) : \dfrac{+}{+}$

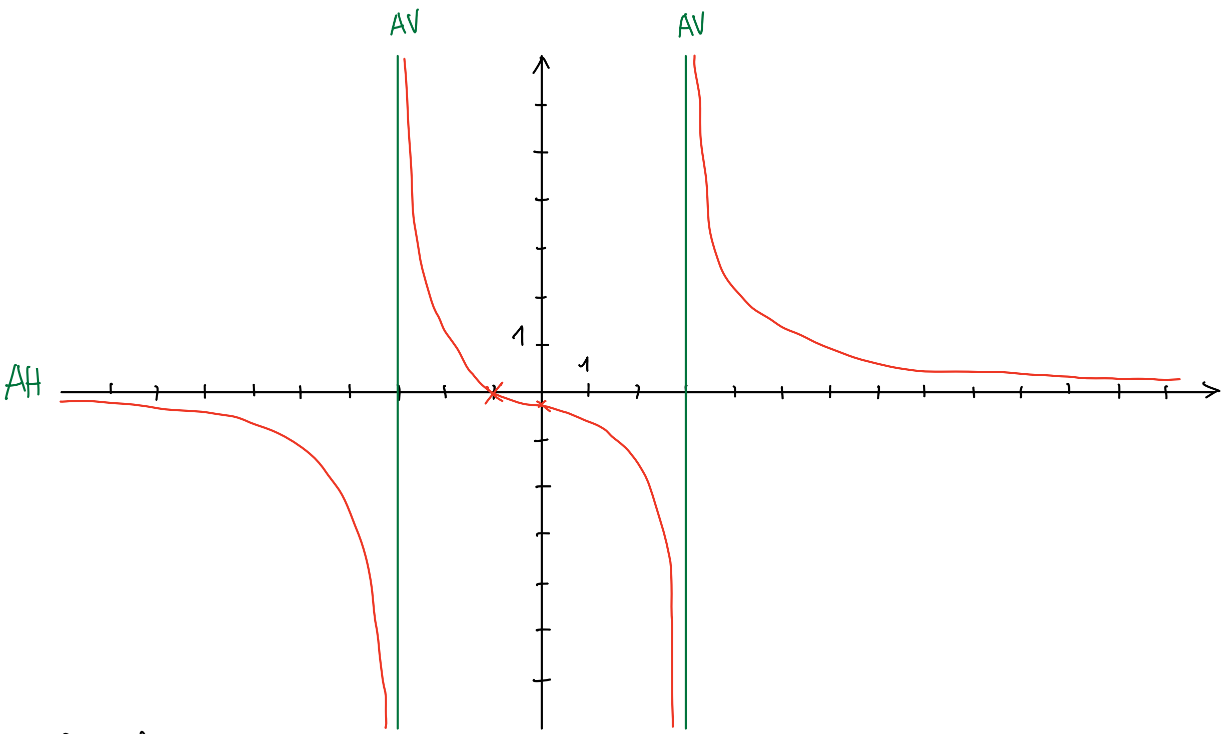
3. AV/trous :

$\lim\limits_{x \to -3} f(x) = \text{"}\dfrac{-2}{0}\text{"} = \infty \Rightarrow$ AV : $x = -3$ du type

$\lim\limits_{x \to 3} f(x) = \text{"}\dfrac{4}{0}\text{"} = \infty \Rightarrow$ AV : $x = 3$ du type

AH : $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{\cancel{x}}{x^{\cancel{2}}} = \lim\limits_{x \to \infty} \dfrac{1}{x} = \dfrac{1}{\infty} = 0 \Rightarrow$ AH : $y = 0$

4.



OàO : $f(0) = -\dfrac{1}{9}$

(ordonnée à l'origine)

b) $f(x)=\dfrac{2x^2+5x-3}{x^2-9}$

1. $ED(f)=\mathbb{R}-\{\pm 3\}$ $\qquad$ v.i : -3 et 3 $\qquad$ valeur double

2. zéros : $2x^2+5x-3=0$ $\qquad \Delta = 25+24=49 \Rightarrow x_{1,2}=\dfrac{-5\pm 7}{4}=$ $-3 \notin ED(f)$; $\dfrac{1}{2}$

signe :

x		-3		$1/2$		3	
f	$+$	$\Vert$ (2)	$+$	0	$-$	$\Vert$	$+$

$f(1000):\dfrac{+}{+}$

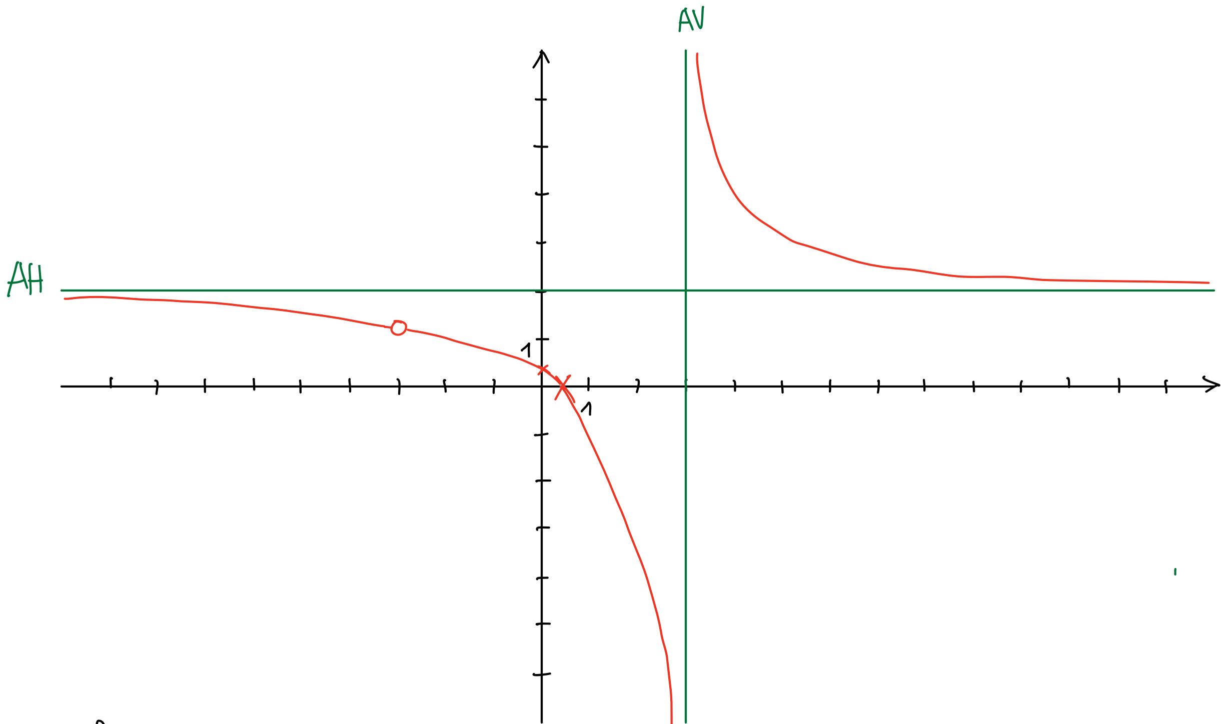
3. AV/hou :

$$\lim_{x\to -3} f(x) = \text{"}\frac{0}{0}\text{"} = \lim_{x\to -3}\frac{\cancel{(x+3)}(2x-1)}{\cancel{(x+3)}(x-3)} = \lim_{x\to -3}\frac{2x-1}{x-3} = \frac{-7}{-6}=\frac{7}{6} \Rightarrow \text{"hou"}\left(-3;\frac{7}{6}\right)$$

$$\lim_{x\to 3} f(x) = \text{"}\frac{30}{0}\text{"} = \infty \Rightarrow \text{AV : } x=3 \text{ du type}$$

AH : $\displaystyle\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{2x^2}{x^2} = 2 \Rightarrow$ AH : $y=2$

4.



OàO : $f(0)=\dfrac{-3}{-9}=\dfrac{1}{3}$

Ex 4

a) $\tan\left(2x+\frac{2\pi}{3}\right)=1 \iff 2x+\frac{2\pi}{3}=\frac{\pi}{4}+k\cdot\pi \quad , k\in\mathbb{Z}$

$\iff 2x = \frac{\pi}{4}-\frac{2\pi}{3}+k\cdot\pi$

$\iff 2x = -\frac{5\pi}{12}+k\cdot\pi$

$\iff x = -\frac{5\pi}{24}+k\cdot\frac{\pi}{2} \quad , k\in\mathbb{Z}$

b) $1+2\cos(3x)=0$

$2\cos(3x)=-1$

$\cos(3x)=-\frac{1}{2} \iff 3x=\begin{cases}\frac{2\pi}{3}+k\cdot 2\pi\\ -\frac{2\pi}{3}+k\cdot 2\pi\end{cases} \quad , k\in\mathbb{Z}$

$\iff x=\begin{cases}\frac{2\pi}{9}+k\cdot\frac{2\pi}{3}\\ -\frac{2\pi}{9}+k\cdot\frac{2\pi}{3}\end{cases} \quad , k\in\mathbb{Z}$

c) $8\cos^2(x)+10\sin(x)-1=0$

$\cos^2(x)+\sin^2(x)=1$
$\cos^2(x)=1-\sin^2(x)$

$8(1-\sin^2(x))+10\sin(x)-1=0$

$-8\sin^2(x)+10\sin(x)+7=0$

Chgmt de variable : $y=\sin(x)$

$-8y^2+10y+7=0 \qquad \Delta=324$

$y_{1,2}=\frac{-10\pm 18}{-16}=\begin{cases}-\frac{1}{2} \Rightarrow \sin(x)=-\frac{1}{2}\\ \frac{7}{4} \Rightarrow \sin(x)=\frac{7}{4}>1 \ \lightning\end{cases}$

$$\Rightarrow \quad \sin(x) = -\frac{1}{2} \quad \Longleftrightarrow \quad x = \begin{cases} -\frac{\pi}{6} + k \cdot 2\pi \\ \pi - \left(-\frac{\pi}{6}\right) + k \cdot 2\pi \end{cases}, \quad k \in \mathbb{Z}$$

$$x = \begin{cases} -\frac{\pi}{6} + k \cdot 2\pi \\ \frac{7\pi}{6} + k \cdot 2\pi \end{cases}, \quad k \in \mathbb{Z}$$