Primitives - Ex supplémentaires 2 — Corrigé

a) $\int (2x^2-3x+2)\,dx = \frac{2}{3}x^3-\frac{3}{2}x^2+2x+C$

b) $\int \frac{1}{3x^4}\,dx = \int \frac{1}{3}\cdot\frac{1}{x^4}\,dx = \frac{1}{3}\int x^{-4}\,dx = \frac{1}{3}\cdot\frac{1}{-3}x^{-3}+C = -\frac{1}{9x^3}+C$

c) $\int \frac{6x^2+8}{x^3+4x}\,dx = \int \frac{2(3x^2+4)}{x^3+4x}\,dx = 2\int \underbrace{\frac{3x^2+4}{x^3+4x}}_{\frac{u'}{u}}\,dx = 2\ln(|x^3+4x|)+C$

$u = x^3+4x$
$u' = 3x^2+4$

d) $\int \left(\frac{3x^4}{2}+\frac{x}{5}\right)dx = \int \left(\frac{3}{2}x^4+\frac{1}{5}x\right)dx = \frac{3}{2}\cdot\frac{1}{5}x^5+\frac{1}{5}\cdot\frac{1}{2}x^2+C = \frac{3}{10}x^5+\frac{1}{10}x^2+C$

$= \frac{3x^5+x^2}{10}+C$

e) $\int \underbrace{\frac{x^4-3x^2}{x^4}}_{\deg(N)=\deg(D)\Rightarrow \text{division}}\,dx = \int \left(\frac{x^4}{x^4}-\frac{3x^2}{x^4}\right)dx = \int \left(1-\frac{3}{x^2}\right)dx = \int (1-3x^{-2})\,dx$

$= x-3\cdot\frac{1}{-1}x^{-1}+C = x+\frac{3}{x}+C$

f) $\int (3x^2+x)^3(6x+1)\,dx = \frac{1}{4}(3x^2+x)^4+C$

$u = 3x^2+x$
$u' = 6x+1$

g) $\int \frac{2x+3}{2x^2+6x+5}\,dx = \frac{1}{2}\int \frac{2(2x+3)}{2x^2+6x+5}\,dx = \frac{1}{2}\ln(|2x^2+6x+5|)+C$

$u = 2x^2+6x+5$
$u' = 4x+6 = 2\cdot(2x+3)$

h) $\int \frac{1}{(3-2x)^2}\,dx = \int (3-2x)^{-2}\,dx = -\frac{1}{2}\int -2(3-2x)^{-2}\,dx = -\frac{1}{2}\cdot\frac{1}{-1}(3-2x)^{-1}+C$

$u = 3-2x$
$u' = -2$

$= \frac{1}{2(3-2x)}+C$

i) $\int \sqrt{x^2+2x-1}\cdot(x+1)\,dx = \frac{1}{2}\int (x^2+2x-1)^{1/2}\cdot 2(x+1)\,dx = \frac{1}{2}\cdot\frac{1}{3/2}(x^2+2x-1)^{3/2}+C$

$u = x^2+2x-1$
$u' = 2x+2 = 2(x+1)$

$= \frac{1}{2}\cdot\frac{2}{3}\sqrt{x^2+2x-1}+C = \frac{1}{3}\sqrt{x^2+2x-1}+C$

j) $\int (7x-2)^5\,dx = \frac{1}{7}\int 7(7x-2)^5\,dx = \frac{1}{7}\cdot\frac{1}{6}(7x-2)^6+C = \frac{1}{42}(7x-2)^7+C$

k) $\int e^{-2x+5}\,dx = -\frac{1}{2}\int -2e^{-2x+5}\,dx = -\frac{1}{2}e^{-2x+5}+C$

l) $\int \frac{x}{(x^2+1)^3}\,dx = \frac{1}{2}\int 2x\cdot(x^2+1)^{-3}\,dx = \frac{1}{2}\cdot\frac{1}{-2}(x^2+1)^{-2}+C = \frac{1}{-4(x^2+1)^2}+C$

$u = x^2+1$
$u' = 2x$

m) $\int x\cos(x^2)\,dx = \frac{1}{2}\int 2x\cos(x^2)\,dx = \frac{1}{2}\sin(x^2)+C$

$u = x^2$
$u' = 2x$

n) $\int (\cos(x)-\sin^2(x)\cos(x))\,dx = \int \cos(x)\,dx - \int \sin^2(x)\cos(x)\,dx = \sin(x)-\frac{1}{3}\sin^3(x)+C$

$u = \sin(x)$
$u' = \cos(x)$