Ex 1.3.3

a) $\int 3\,dx = 3x + c$

b) $\int 5x\,dx = \frac{5}{2}x^2 + c$

c) $\int (2x+1)\,dx = x^2 + x + c$

d) $\int (5x-4)\,dx = \frac{5}{2}x^2 - 4x + c$

e) $\int (2x^2 - 3x + 2)\,dx = \frac{2}{3}x^3 - \frac{3}{2}x^2 + 2x + c$

f) $\int 5x^3\,dx = \frac{5}{4}x^4 + c$

g) $\int (-3x^4)\,dx = -\frac{3}{5}x^5 + c$

h) $\int (3x^5 + 2x^4 - 1)\,dx = \frac{3}{6}x^6 + \frac{2}{5}x^5 - x + c = \frac{1}{2}x^6 + \frac{2}{5}x^5 - x + c$

i) $\int (\cos(x) + \sin(x))\,dx = \sin(x) - \cos(x) + c$

j) $\int (1 + \tan^2(x))\,dx = \tan(x) + c$

Ex 1.3.4

a) $\int \frac{dx}{x^2} = \int x^{-2}dx = \frac{1}{-1}x^{-1} + C = -\frac{1}{x} + C$

b) $\int \frac{2dx}{x^3} = \int 2x^{-3}dx = 2 \cdot \frac{1}{-2}x^{-2} + C = -\frac{1}{x^2} + C$

c) $\int \frac{-7dx}{x^5} = \int -7x^{-5}dx = -7 \cdot \frac{1}{-4}x^{-4} + C = \frac{7}{4x^4} + C$

d) $\int \sqrt{x}\,dx = \int x^{1/2}dx = \frac{1}{\frac{3}{2}}x^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C$

e) $\int \sqrt[3]{x}\,dx = \int x^{1/3}dx = \frac{1}{\frac{4}{3}}x^{\frac{4}{3}} + C = \frac{3}{4}\sqrt[3]{x^4} + C$

f) $\int \frac{dx}{\sqrt{x}} = \int x^{-1/2}dx = \frac{1}{\frac{1}{2}}x^{1/2} + C = 2\sqrt{x} + C$

g) $\int \frac{dx}{\sqrt[3]{x^2}} = \int x^{-2/3}dx = \frac{1}{\frac{1}{3}}x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$

h) $\int\left(\frac{3}{x^4} - \sqrt[4]{x^3}\right)dx = \int\left(3x^{-4} - x^{3/4}\right)dx = 3 \cdot \frac{1}{-3}x^{-3} - \frac{1}{7/4}x^{7/4} + C$

$$= -\frac{1}{x^3} - \frac{4\sqrt[4]{x^7}}{7} + C$$

Ex 1.3.5

a) $\frac{1}{3}\int 3\cdot\cos(3x)dx = \underline{\frac{1}{3}\sin(3x)+C}$

$u=3x$
$u'=3$

b) $\frac{1}{2}\int 2\sin\left(2x-\frac{\pi}{3}\right)dx = \underline{-\frac{1}{2}\cos\left(2x-\frac{\pi}{3}\right)+C}$

$u=2x-\frac{\pi}{3}$
$u'=2$

c) $\int (x+3)^3 dx = \underline{\frac{1}{4}(x+3)^4+C}$

$u=x+3$
$u'=1$

d) $\frac{1}{2}\int 2(2x-1)^2dx = \frac{1}{2}\cdot\frac{1}{3}(2x-1)^3+C = \underline{\frac{1}{6}(2x-1)^3+C}$

$u=2x-1$
$u'=2$

e) $\frac{1}{7}\int 7(7x-2)^5dx = \frac{1}{7}\cdot\frac{1}{6}(7x-2)^6+C = \underline{\frac{1}{42}(7x-2)^6+C}$

$u=7x-2$
$u'=7$

f) $\int (3x^2+x)^3(6x+1)\,dx = \underline{\frac{1}{4}(3x^2+x)^4+C}$

$u=3x^2+x$
$u'=6x+1$

g) $\frac{1}{8}\int (4x^2+3)^4\,x\cdot 8\,dx = \frac{1}{8}\cdot\frac{1}{5}(4x^2+3)^5+C = \underline{\frac{1}{40}(4x^2+3)^5+C}$

$u=4x^2+3$
$u'=8x$

h) $\int \sin^2(x)\cos(x)dx = \underline{\frac{1}{3}\sin^3(x)+C}$

$u=\sin(x)$
$u'=\cos(x)$

i) $\int \frac{\tan^2(x)}{\cos^2(x)}dx = \int \tan^2(x)\cdot\frac{1}{\cos^2(x)}dx = \underline{\frac{1}{3}\tan^3(x)+C}$

$u=\tan(x)$
$u'=\frac{1}{\cos^2(x)}$

j) $\int \sqrt{x+3}\, dx = \int (x+3)^{1/2} dx = \frac{1}{3/2}(x+3)^{3/2} + C = \frac{2}{3}\sqrt{(x+3)^3} + C$

$u = x+3$
$u' = 1$

k) $\frac{1}{3}\int \frac{3 \cdot 1}{\sqrt{3x+1}} dx = \frac{1}{3}\int 3(3x+1)^{-1/2} dx = \frac{1}{3} \cdot \frac{1}{1/2}(3x+1)^{1/2} + C = \frac{2}{3}\sqrt{3x+1} + C$

$u = 3x+1$
$u' = 3$

l) $\frac{1}{2}\int \frac{(x+1) \cdot 2}{\sqrt{x^2+2x}} dx = \frac{1}{2}\int (x^2+2x)^{-1/2} \cdot 2(x+1)\, dx = \frac{1}{2} \cdot \frac{1}{1/2}(x^2+2x)^{1/2} + C = \sqrt{x^2+2x} + C$

$u = x^2+2x$
$u' = 2x+2 = 2(x+1)$

Ex 1.3.6

a) $\int (3x^2-2x+3)\,dx = \underline{x^3 - x^2 + 3x + C}$

b) $\int \frac{3x^4-3x^2-7}{4x^2}\,dx$

(deg(N) $\geq$ deg(D)   on divise chaque terme du numérateur par le monôme du dénominateur.)

$$= \int \left(\frac{3x^{\cancel{4}\,2}}{4\cancel{x^2}} - \frac{3\cancel{x^2}}{4\cancel{x^2}} - \frac{7}{4x^2}\right)dx$$

$\downarrow$ ⚠ $\frac{7}{4}\cdot\frac{1}{x^2} = \frac{7}{4}\cdot x^{-2}$ $(\neq 7\cdot 4x^{-2})$

$$= \int \left(\frac{3x^2}{4} - \frac{3}{4} - \frac{7}{4}\cdot x^{-2}\right)dx$$

$$= \frac{3}{4}\cdot\frac{1}{3}x^3 - \frac{3}{4}x - \frac{7}{4}\cdot\frac{1}{-1}\,x^{-1} + C \qquad = \underline{\frac{1}{4}x^3 - \frac{3}{4}x + \frac{7}{4x} + C}$$

c) $\int 7\sqrt[4]{x^3}\,dx = \int 7x^{3/4}\,dx = 7\cdot\frac{1}{\frac{7}{4}}\,x^{7/4} + C = \underline{4\sqrt[4]{x^7} + C}$

d) $\int (\sqrt{x} - \sqrt[3]{x})\,dx = \int (x^{1/2} - x^{1/3})\,dx = \frac{1}{\frac{3}{2}}x^{3/2} - \frac{1}{\frac{4}{3}}x^{4/3} + C$

$$= \underline{\frac{2}{3}\sqrt{x^3} - \frac{3}{4}\sqrt[3]{x^4} + C}$$

e) $\int (2\sin(x) - 3\cos(x))\,dx = \underline{-2\cos(x) - 3\sin(x) + C}$

f) $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)\cdot 2\,dx = \underline{\frac{1}{2}\sin(2x) + C}$

$u = 2x$
$u' = 2$

g) $\int \left(\frac{5}{\cos^2(x)} + 5\cos(x)\right)x = \underline{5\tan(x) + 5\sin(x) + C}$

h) $\int \left(8\sin(x) + \frac{4}{\sqrt{2x}}\right)dx = -8\cos(x) + C' + \int 2\cdot 2(2x)^{-1/2}\,dx$

$u = 2x$
$u' = 2$

$$= -8\cos(x) + 2\cdot\frac{1}{-\frac{1}{2}+1}\,(2x)^{-\frac{1}{2}+1} + C$$

$$= \underline{-8\cos(x) + 4\sqrt{2x} + C}$$

i) $\int (3x^2-7)^2\,dx = \int (9x^4 - 42x^2 + 49)\,dx$

$$= \frac{9}{5}x^5 - \frac{42}{3}x^3 + 49x + C = \underline{\frac{9}{5}x^5 - 14x^3 + 49x + C}$$

⚠ ne pas confondre avec $\int \underbrace{(3x^2-7)^2}_{u^2} \cdot \underbrace{6x}_{u'}\,dx = \frac{1}{3}(3x^2-7)^3 + C$

j) $\int \sqrt{x}\cdot(x^2-5)\,dx = \int x^{\frac{1}{2}}(x^2-5)\,dx = \int \left(x^{\frac{1}{2}+2} - 5x^{\frac{1}{2}}\right)dx = \int \left(x^{\frac{5}{2}} - 5x^{\frac{1}{2}}\right)dx$

Un des facteurs n'est pas la dérivée interne de l'autre ⇒ on multiplie après transformation.

$$= \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1} - 5\,\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} - \frac{10}{3}x^{\frac{3}{2}} + C$$

$$= \underline{\frac{2}{7}\sqrt{x^7} - \frac{10}{3}\sqrt{x^3} + C}$$

k) $\frac{1}{3}\int 3(3x-5)^6\,dx = \frac{1}{3}\cdot\frac{1}{7}(3x-5)^7 + C = \underline{\frac{1}{21}(3x-5)^7 + C}$

l) $\int \frac{12}{(4-3x)^4}\,dx = -4\int (-3)\cdot(4-3x)^{-4}\,dx = -4\cdot\frac{1}{-3}(4-3x)^{-3} + C = \underline{\frac{4}{3(4-3x)^3} + C}$

$u = 4-3x$
$u' = -3$

m) $\int \sqrt[3]{(3x-8)^2}\,dx = \frac{1}{3}\int 3(3x-8)^{2/3}\,dx = \frac{1}{3}\cdot\frac{1}{\frac{5}{3}}(3x-8)^{\frac{5}{3}} + C$

$u = 3x-8$
$u' = 3$

$$= \underline{\frac{1}{5}\sqrt[3]{(3x-8)^5} + C}$$

n) $\int \frac{6}{\cos^2(3x)}\,dx = 2\int \frac{1}{\cos^2(3x)}\cdot 3\,dx = \underline{2\tan(3x) + C}$

$u = 3x$
$u' = 3$

o) $\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int 2x\,(x^2+1)^{1/2}\,dx = \frac{1}{2}\cdot\frac{1}{\frac{3}{2}}(x^2+1)^{\frac{3}{2}} + C$

$u = x^2+1$

$u' = 2x$

$$= \frac{1}{3}\sqrt{(x^2+1)^3} + C$$

p) $\int \frac{2x-1}{\sqrt{x^2-x-1}}\,dx = \int (x^2-x-1)^{-\frac{1}{2}}(2x-1)\,dx = \frac{1}{\frac{1}{2}}(x^2-x-1)^{\frac{1}{2}} + C$

$u = x^2-x-1$

$u' = 2x-1$

$$= 2\sqrt{x^2-x-1} + C$$

Ex 1.3.10

a) $\int_1^4 (x^2-2x+3)\,dx = \frac{1}{3}x^3 - x^2 + 3x \Big|_1^4$

$$= \left(\frac{1}{3}\cdot 64 - 16 + 12\right) - \left(\frac{1}{3} - 1 + 3\right)$$

$$= \frac{63}{3} - 6 = 21 - 6 = 15$$

b) $\int_{-1}^{1} (2x^3+3x^2+2x-1)\,dx = \frac{1}{2}x^4 + x^3 + x^2 - x \Big|_{-1}^{1}$

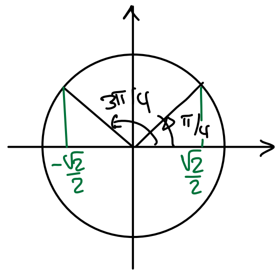
$$= \left(\frac{1}{2}+1+1-1\right) - \left(\frac{1}{2}-1+1+1\right) = 0$$

c) $\int_{-1}^{1} \sqrt[3]{x+1}\,dx = \int_{-1}^{1} (x+1)^{1/3}\,dx = \frac{1}{\frac{4}{3}}(x+1)^{4/3}\Big|_{-1}^{1}$

$$= \frac{3}{4}\left(2^{4/3} - 0^{4/3}\right) = \frac{3}{4}\sqrt[3]{\underbrace{16}_{8\cdot 2}} = \frac{3}{4}\cdot 2\sqrt[3]{2}$$

$$= \frac{3}{2}\sqrt[3]{2}$$

d) $$\int_{\pi/4}^{3\pi/4} \sin(x)\,dx = -\cos(x)\Big|_{\pi/4}^{3\pi/4} = \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) = \underline{\sqrt{2}}$$



e) $$\int_{0}^{\pi/4} (1+\tan^2(x))\,dx = \tan(x)\Big|_{0}^{\pi/4} = 1-0 = \underline{1}$$

f) $$\int_{0}^{2} (1+2x)^3\,dx = \frac{1}{2}\int_{0}^{2} (1+2x)^3\cdot 2\,dx = \frac{1}{2}\cdot\frac{1}{4}(1+2x)^4\Big|_{0}^{2}$$

$u = 1+2x$

$u' = 2$

$$= \frac{1}{8}5^4 - \frac{1}{8}\cdot 1^4 = \frac{1}{8}(625-1) = \frac{624}{8} = \underline{78}$$

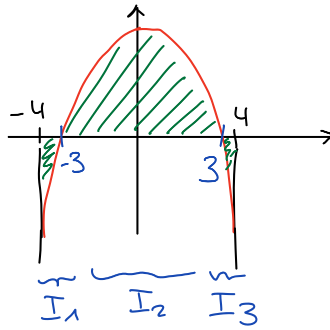
g) $$\int_{1}^{4} \frac{dx}{\sqrt{x}} = \int_{1}^{4} x^{-1/2}\,dx = \frac{1}{\frac{1}{2}}x^{1/2}\Big|_{1}^{4} = 2\sqrt{x}\Big|_{1}^{4} = 2\cdot 2 - 2\cdot 1 = \underline{2}$$

h) $$\int_{0}^{\pi/2} \underbrace{\sin^2(x)}_{u^2}\cdot\underbrace{\cos(x)}_{u'}\,dx = \frac{1}{3}\sin^3(x)\Big|_{0}^{\pi/2} = \frac{1}{3} - 0 = \underline{\frac{1}{3}}$$

Ex 1.3.16

a) $f(x) = 9 - x^2$ (parabole $\cap$) $\quad a = -4$ et $b = 4$

$ED(f) = \mathbb{R}$



zéros : $9 - x^2 = 0 \Leftrightarrow (3+x)(3-x) = 0 \Leftrightarrow x = -3$ ou $x = 3$

signe :

x	-4		-3		3		4
$sgn(f)$		$-$	0	$+$	0	$-$	
		I_1		I_2		I_3	

$$I_1 = \int_{-4}^{-3} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{-4}^{-3} = (-27 + 9) - \left(-36 + \frac{64}{3}\right) = -\frac{10}{3}$$

$$I_2 = \int_{-3}^{3} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{-3}^{3} = (27 - 9) - (-27 + 9) = 36$$

$$I_3 = \int_{3}^{4} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{3}^{4} = \left(36 - \frac{64}{3}\right) - (27 - 9) = -\frac{10}{3}$$

$$\Rightarrow \quad A = \left|-\frac{10}{3}\right| + 36 + \left|-\frac{10}{3}\right| = 36 + 2 \cdot \frac{10}{3} = \underline{\frac{128}{3}\,u^2}$$

b) $f(x) = \dfrac{4}{x^2} - 1 \qquad a = 1, \; b = 4$

$$f(x) = \frac{4 - x^2}{x^2} = \frac{(2-x)(2+x)}{x^2}$$

$\leftarrow$ zéros : ± 2

$\leftarrow$ v.i. : 0 (2)

$ED(f) = \mathbb{R}^*$

signe :

x		-2		0	1		2	4	
$sgn(f)$	$-$	0	$+$	$\Vert$ (2)	$+$		0		$-$
					I_1		I_2		

$\nwarrow f(1000)$

$$I_1 = \int_1^2 \left(\frac{4}{x^2} - 1\right) dx = \int_1^2 (4x^{-2} - 1)\,dx = 4 \cdot \frac{1}{-1} x^{-1} - x \Big|_1^2 = -\frac{4}{x} - x \Big|_1^2$$

$$= (-2 - 2) - (-4 - 1) = 1$$

$$I_2 = \int_2^4 \left(\frac{4}{x^2} - 1\right) dx = -\frac{4}{x} - x \Big|_2^4 = (-1 - 4) - (-2 - 2) = -1$$

$$\Rightarrow \quad \mathcal{A} = 1 + |-1| = 1 + 1 = \underline{2\,u^2}$$

c) $f(x) = \cos(3x) \qquad a = 0 \qquad b = \frac{\pi}{2}$

zéro(s) : $\cos(3x) = 0 \iff 3x = \begin{cases} \frac{\pi}{2} + k \cdot 2\pi \\ -\frac{\pi}{2} + k \cdot 2\pi \end{cases} \quad k \in \mathbb{Z}$

$\iff x = \begin{cases} \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \\ -\frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{cases} \Rightarrow x = \frac{\pi}{6}$ entre 0 et $\frac{\pi}{2}$

signe :

x	0		$\pi/6$		$\pi/2$
$\mathrm{sgn}(f)$		$+$	0	$-$	
		I_1		I_2	

$$I_1 = \int_0^{\pi/6} \cos(3x)\,dx = \frac{1}{3}\int_0^{\pi/6} 3\cos(3x)\,dx = \frac{1}{3}\sin(3x)\Big|_0^{\pi/6} = \frac{1}{3}(1-0) = \frac{1}{3}$$

$u = 3x$
$u' = 3$

$$I_2 = \int_{\pi/6}^{\pi/2} \cos(3x)\,dx = \frac{1}{3}\sin(3x)\Big|_{\pi/6}^{\pi/2} = \frac{1}{3}(-1-1) = -\frac{2}{3}$$

$$\Rightarrow \mathcal{A} = \frac{1}{3} + \left|-\frac{2}{3}\right| = \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1\,u^2$$

d) $f(x) = \sqrt{2x-4} \qquad a = 2,\ b = 10$

$ED(f) = [2\,;+\infty[$ car $2x-4 \geq 0$

x		2		10
	$-$	0	$+$	

$$\mathcal{A} = \int_2^{10} \sqrt{2x-4}\,dx = \int_2^{10} (2x-4)^{1/2}\,dx = \frac{1}{2}\int_2^{10} (2x-4)^{1/2} \cdot 2\,dx$$

$u = 2x-4$
$u' = 2$

$$= \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (2x-4)^{3/2}\Big|_2^{10} = \frac{1}{3}\sqrt{(2x-4)^3}\Big|_2^{10} = \frac{1}{3}(4^3-0) = \frac{64}{3}\,u^2$$

Ex 1.3.17

a) $f(x) = x^3 - 2x^2 - x + 2 = x^2(x-2) - 1(x-2) = (x-2)(x^2-1) = (x-2)(x+1)(x-1)$

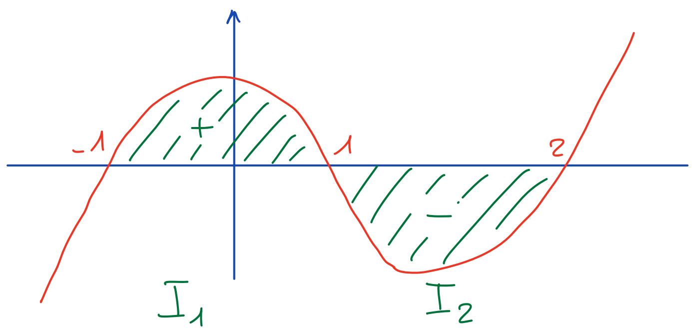
$ED(f) = \mathbb{R}$ zéros : -1, 1 et 2

signe :

x		-1		1		2	
$\operatorname{sgn}(f)$	$-$	0	$+$	0	$-$	0	$+$

$f(1000)$: $+$

croquis :



$$I_1 = \int_{-1}^{1} (x^3 - 2x^2 - x + 2)\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_{-1}^{1}$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right) = \frac{8}{3}$$

$$I_2 = \int_{1}^{2} f(x)\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_{1}^{2}$$

$$= \left(4 - \frac{16}{3} - 2 + 4\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) = -\frac{5}{12}$$

$$\Rightarrow \mathcal{A} = \frac{8}{3} + \frac{5}{12} = \frac{37}{12}\ u^2$$

b) $f(x) = x\sqrt{4-x^2}$ cond : $4-x^2 \geqslant 0 \iff (2+x)(2-x) \geqslant 0$

x		-2		2	
$4-x^2$	$-$	0	$+$	0	$-$

$ED(f) = [-2\,;2]$

zéros : $-2\,;0\,;2$

signe :

x		-2		0		2	
$\text{sgn}(f)$	////	0	$-$	0	$+$	0	////
			I_1		I_2		

$$I_1 = \int_{-2}^{0} x\sqrt{4-x^2}\,dx = \int_{-2}^{0} x\,(4-x^2)^{1/2}\,dx = -\frac{1}{2}\int_{-2}^{0} -2x\,(4-x^2)^{1/2}\,dx$$

$u = 4-x^2$

$u' = -2x$

$$= -\frac{1}{2}\cdot\frac{1}{\frac{3}{2}}\,(4-x^2)^{3/2}\Big|_{-2}^{0} = -\frac{1}{3}\sqrt{(4-x^2)^3}\,\Big|_{-2}^{0}$$

$$= -\frac{1}{3}8 - \left(-\frac{1}{3}\cdot 0\right) = -\frac{8}{3}$$

$$I_2 = \int_{0}^{2} x\sqrt{4-x^2}\,dx = -\frac{1}{3}\sqrt{(4-x^2)^3}\,\Big|_{-2}^{0} = 0-\left(-\frac{8}{3}\right) = \frac{8}{3}$$

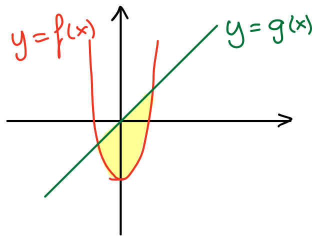
$$\Rightarrow \quad \mathcal{A} = \frac{8}{3}+\frac{8}{3} = \frac{16}{3}\,u^2$$

Ex 1.3.19

a) $f(x) = x^2 - 3$ et $g(x) = 2x$

pts d'∩ : $x^2 - 3 = 2x \Leftrightarrow x^2 - 2x - 3 = 0 \Leftrightarrow (x-3)(x+1) = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 3 \qquad -1$



$$\int_{-1}^{3} (x^2 - 3 - 2x)\,dx = \int_{-1}^{3} (x^2 - 2x - 3)\,dx = \frac{1}{3}x^3 - x^2 - 3x \Big|_{-1}^{3}$$

$$= (9 - 9 - 9) - \left(-\frac{1}{3} - 1 + 3\right) = -9 - \frac{5}{3} = -\frac{32}{3}$$

$$\Rightarrow A = \left|-\frac{32}{3}\right| = \frac{32}{3}\ u^2$$

b) $f(x) = x^2$ et $g(x) = 8 - x^2$

pts d'∩ : $x^2 = 8 - x^2 \Leftrightarrow 2x^2 = 8 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$

$$\int_{-2}^{2} (x^2 - (8 - x^2))\,dx = \int_{-2}^{2} (2x^2 - 8)\,dx = \frac{2}{3}x^3 - 8x \Big|_{-2}^{2}$$

$$= \left(\frac{16}{3} - 16\right) - \left(-\frac{16}{3} + 16\right) = \frac{32}{3} - 32 = \frac{-64}{3}$$

$$\Rightarrow A = \left|-\frac{64}{3}\right| = \frac{64}{3}\ u^2$$

c) $f(x) = x^3 - 5x^2 + 6x$ et $g(x) = x^3 - 7x^2 + 12x$

pts d'∩ : $x^3 - 5x^2 + 6x = x^3 - 7x^2 + 12x \Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow 2x(x-3) = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0 \qquad 3$

$$\Rightarrow \int_{0}^{3} (x^3 - 5x^2 + 6x - (x^3 - 7x^2 + 12x))\,dx = \int_{0}^{3} (2x^2 - 6x)\,dx$$

$$= \frac{2}{3}x^3 - 3x^2 \Big|_{0}^{3} = \left(\frac{2}{3} \cdot 27 - 3 \cdot 9\right) - 0 = 18 - 27 = -9$$

$$\Rightarrow \mathcal{A} = |-9| = 9\ u^2$$

d) $f(x) = x(6-2x^2) = -2x^3+6x \qquad g(x) = x(2-x^2) = -x^3+2x$

pts d'∩ : $-2x^3+6x = -x^3+2x \iff -x^3+4x = 0 \iff -x(x^2-4)=0$

$\iff \underset{\downarrow\atop 0}{-x}\,\underset{\downarrow\atop -2}{(x+2)}\,\underset{\downarrow\atop 2}{(x-2)} = 0$

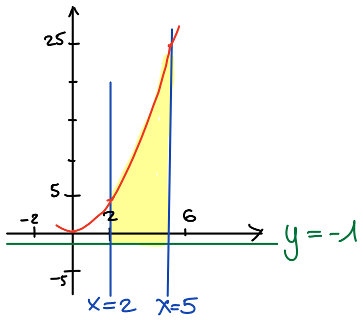
$$I_1 = \int_{-2}^{0} (f(x)-g(x))\,dx = \int_{-2}^{0} (-x^3+4x)\,dx = -\frac{1}{4}x^4+2x^2 \Big|_{-2}^{0}$$

$$= 0-\left(-\frac{1}{4}\cdot 16+2\cdot 4\right) = +4-8 = -4$$

$$I_2 = -\frac{1}{4}x^4+2x^2 \Big|_{0}^{2} = -\frac{1}{4}\cdot 16+2\cdot 4 = -4+8 = 4$$

$$\Rightarrow \mathcal{A} = |-4|+4 = \underline{8\,u^2}$$

Ex 1.3.21



$y = x^2$, $y=-1$, $\underbrace{x=2 \quad \text{et} \quad x=5}_{\text{bornes}}$

$$\mathcal{A} = \int_2^5 (x^2-(-1))\,dx = \int_2^5 (x^2+1)\,dx = \frac{1}{3}x^3+x \Big|_2^5$$

$$= \left(\frac{1}{3}\cdot 125+5\right) - \left(\frac{1}{3}\cdot 8+2\right) = \frac{140}{3} - \frac{14}{3} = \underline{42\,u^2}$$

Variante : Aire sous la courbe $y=x^2$ + Aire "sous" la courbe $y=-1$

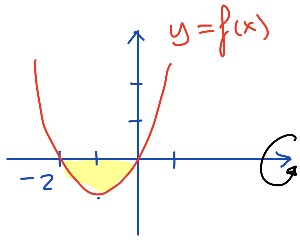
$$A_1 = \int_2^5 x^2\,dx = \frac{1}{3}x^3 \Big|_2^5 = \frac{125}{3} - \frac{8}{3} = 39$$

$A_2 = 1\cdot 3 = 3$ (aire d'un rectangle)

$\Rightarrow \mathcal{A} = 39+3 = 42\,u^2$

Ex 1.3.24

a) $f(x) = x^2+2x = x(x+2) \Rightarrow$ zéros : -2 et 0



$$V = \pi\int_{-2}^{0}(x^2+2x)^2\,dx = \pi\int_{-2}^{0}(x^4+4x^3+4x^2)\,dx$$

$$= \pi\left(\frac{1}{5}x^5+x^4+\frac{4}{3}x^3\right)\Big|_{-2}^{0} = \pi\left(0-\left(-\frac{32}{5}+16-\frac{32}{3}\right)\right) = \underline{\frac{16\pi}{15}\,u^3}$$

b) $f(x) = \sqrt{1-x^2}$ zéros : $1-x^2=0 \Leftrightarrow (1+x)(1-x)=0 \Rightarrow -1$ et 1

$$V = \pi\int_{-1}^{1}\left(\sqrt{1-x^2}\right)^2\,dx = \pi\int_{-1}^{1}(1-x^2)\,dx = \pi\left(x-\frac{1}{3}x^3\right)\Big|_{-1}^{1}$$

$$= \pi\left(1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)\right) = \pi\left(\frac{2}{3}-\left(-\frac{2}{3}\right)\right) = \underline{\frac{4\pi}{3}\,u^3}$$

↑ Volume d'une sphère de rayon $=1$ $\left(V=\frac{4}{3}\pi r^3\right)$

$y=f(x) \Leftrightarrow y=\sqrt{1-x^2}$

$\Leftrightarrow y^2 = 1-x^2$

$\Leftrightarrow \underbrace{x^2+y^2=1}_{\text{équation d'un cercle de rayon}=1}$

Ex 1.3.26

a) $f(x) = x+1 \qquad a=1 \qquad b=3$

$$V = \pi\int_1^3 (x+1)^2\,dx = \pi\int_1^3 (x^2+2x+1)\,dx = \pi\left(\frac{1}{3}x^3 + x^2 + x\right)\Big|_1^3$$

$$= \pi\left(9+9+3-\left(\frac{1}{3}+1+1\right)\right) = \pi\left(21-\frac{7}{3}\right) = \frac{56\pi}{3}\ u^3$$

Variante: $\pi\int_1^3 (x+1)^2\,dx = \pi\cdot\frac{1}{3}(x+1)^3\Big|_1^3 = \pi\cdot\frac{1}{3}(64-8) = \frac{56\pi}{3}u^3$

$u = x+1$
$u' = 1$

b) $f(x) = x^2 \qquad a=0 \qquad b=4$

$$V = \pi\int_0^4 (x^2)^2\,dx = \pi\int_0^4 x^4\,dx = \pi\cdot\frac{1}{5}x^5\Big|_0^4 = \pi\left(\frac{1024}{5}-0\right) = \frac{1024\pi}{5}\ u^3$$

c) $f(x) = \frac{1}{x+1} \qquad a=1 \qquad b=2$

$$V = \pi\int_1^2 \left(\frac{1}{x+1}\right)^2 dx = \pi\int_1^2 (x+1)^{-2}\,dx = \pi\,\frac{1}{-1}(x+1)^{-1}\Big|_1^2 = -\pi\cdot\frac{1}{x+1}\Big|_1^2$$

$u = x+1$
$u' = 1$

$$= -\pi\left(\frac{1}{3}-\frac{1}{2}\right) = -\pi\cdot\left(-\frac{1}{6}\right) = \frac{\pi}{6}\ u^3$$

Ex 1.3.28

a) $f(x) = \sqrt{x}$ $\qquad g(x) = x^2$

pts d'∩ : $\sqrt{x} = x^2 \iff x = x^4 \iff x^4 - x = 0 \iff x(x^3-1) = 0$

$\iff \underset{\downarrow\,0}{x}\,\underset{\downarrow\,1}{(x-1)}\underbrace{(x^2+x+1)}_{\Delta<0} = 0$

$$V_1 = \pi \int_0^1 (\sqrt{x})^2\,dx = \pi \int_0^1 x\,dx = \pi \cdot \frac{1}{2}x^2 \Big|_0^1 = \pi\left(\frac{1}{2} - 0\right) = \frac{\pi}{2}$$

$$V_2 = \pi \int_0^1 (x^2)^2\,dx = \pi \int_0^1 x^4\,dx = \pi \cdot \frac{1}{5}x^5 \Big|_0^1 = \pi\left(\frac{1}{5} - 0\right) = \frac{\pi}{5}$$

$$V = |V_1 - V_2| = \left|\frac{\pi}{2} - \frac{\pi}{5}\right| = \pi\left|\frac{1}{2} - \frac{1}{5}\right| = \frac{3\pi}{10}\,u^3$$

b) $f(x) = x^2 - 2x + 6$ $\qquad g(x) = -x^2 + 10$

sommet $(1,5)$

pts d'∩ : $x^2 - 2x + 6 = -x^2 + 10 \iff 2x^2 - 2x - 4 = 0$

$\iff x^2 - x - 2 = 0 \iff \underset{\downarrow\,2}{(x-2)}\underset{\downarrow\,-1}{(x+1)} = 0$

$$V_1 = \pi \int_{-1}^{2} (x^2-2x+6)^2\,dx = \pi \int_{-1}^{2} \left(x^4 + 4x^2 + 36 - 4x^3 + 12x^2 - 24x\right)dx$$

$$= \pi \int_{-1}^{2} \left(x^4 - 4x^3 + 16x^2 - 24x + 36\right)dx = \pi\left(\frac{1}{5}x^5 - x^4 + \frac{16}{3}x^3 - 12x^2 + 36x\right)\Big|_{-1}^{2}$$

$$= \pi\left(\left(\frac{32}{5} - 16 + \frac{128}{3} - 48 + 72\right) - \left(-\frac{1}{5} - 1 - \frac{16}{3} - 12 - 36\right)\right) = \pi\left(\frac{856}{15} + \frac{818}{15}\right) = \frac{558\pi}{5}$$

$$V_2 = \pi \int_{-1}^{2} (-x^2+10)^2\,dx = \pi \int_{-1}^{2} \left(x^4 - 20x^2 + 100\right)dx = \pi\left(\frac{1}{5}x^5 - \frac{20}{3}x^3 + 100x\right)\Big|_{-1}^{2}$$

$$= \pi\left(\left(\frac{32}{5} - \frac{160}{3} + 200\right) - \left(-\frac{1}{5} + \frac{20}{3} - 100\right)\right) = \pi\left(\frac{2296}{15} + \frac{1403}{15}\right) = \frac{1233\pi}{5}$$

$$\Rightarrow V = |V_1 - V_2| = \left|\frac{558\pi}{5} - \frac{1233\pi}{5}\right| = \pi\left|\frac{558}{5} - \frac{1233}{5}\right| = 135\pi$$