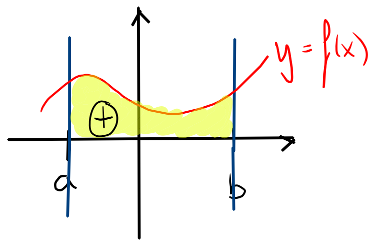
Aire géométrique

1) Aire "sous" une courbe

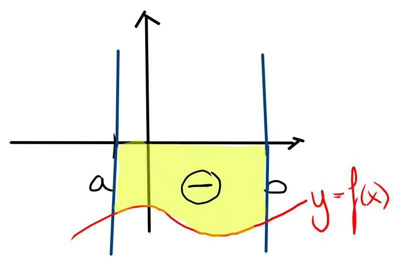
1.1 Soit $\mathcal{D}$ le domaine fermé, délimité par $y = f(x)$, l'axe Ox et $x = a$ et $x = b$

si $f(x) > 0$ sur $[a ; b]$



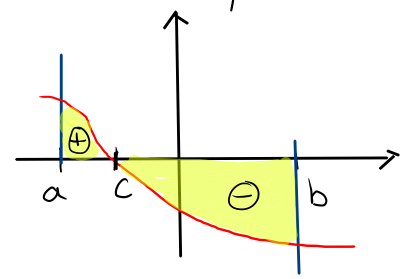
$$\mathcal{A} = \int_a^b f(x)\,dx$$

si $f(x) < 0$ sur $[a ; b]$



$$\mathcal{A} = \left| \int_a^b f(x)\,dx \right|$$

si $f(x) > 0$ sur $[a ; c[$ et $f(x) < 0$ sur $]c ; b]$ et c un zéro de f.



$$\mathcal{A} = \int_a^c f(x)\,dx + \left| \int_c^b f(x)\,dx \right|$$

Exple : $y = x^3+1$ $\quad$ $x=-2$ et $x=2$

$ED(f) = \mathbb{R}$

Signe : zéro : $\underbrace{x^3+1}_{(x+1)(x^2-x+1)} = 0 \iff x = -1$

$(x+1) \to -1$, $\underbrace{x^2-x+1}_{\Delta<0}$

x	-2		-1		2
$sgn(f)$		$-$	0	$+$	

$$\mathcal{A} = \underbrace{\left| \int_{-2}^{-1} f(x)\,dx \right|}_{I_1} + \underbrace{\int_{-1}^{2} f(x)\,dx}_{I_2}$$

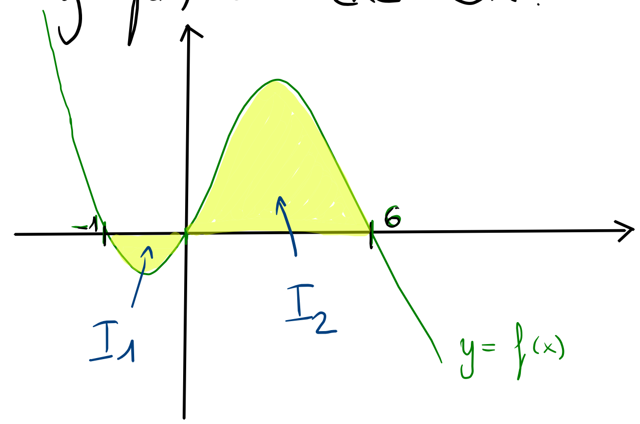
$$I_1 = \int_{-2}^{-1} (x^3+1)\,dx = \frac{1}{4}x^4 + x \Big|_{-2}^{-1} = \left(\frac{1}{4}-1\right) - (4-2) = \frac{1}{4} - 3 = -\frac{11}{4}$$

$$I_2 = \frac{1}{4}x^4 + x \Big|_{-1}^{2} = (4+2) - \left(\frac{1}{4}-1\right) = 7 - \frac{1}{4} = \frac{27}{4}$$

$$\Rightarrow \mathcal{A} = \left|-\frac{11}{4}\right| + \frac{27}{4} = \frac{11}{4} + \frac{27}{4} = \frac{38}{4} = \frac{19}{2}\ u^2$$

ex 1.3.16

1.2 Soit D un domaine fermé délimité par $y = f(x)$ et l'axe Ox.



$$f(x) = -x^3 + 5x^2 + 6x$$

zéros de f : $f(x) = 0$

$$\Leftrightarrow \quad -x\left(x^2 - 5x - 6\right) = 0$$

$$\Leftrightarrow \quad -x\,(x-6)(x+1) = 0$$

$$0 \qquad 6 \qquad -1$$

signe :

x		-1		0		6	
f	$+$	0	$-$	0	$+$	0	$-$

$\leftarrow f(1000) : -$

$$\underbrace{- \; 0}_{I_1} \qquad \underbrace{+ \; 0}_{I_2}$$

$$I_1 = \int_{-1}^{0} \left(-x^3 + 5x^2 + 6x\right) dx$$

$$= -\frac{1}{4}x^4 + \frac{5}{3}x^3 + 3x^2 \Bigg|_{-1}^{0} = 0 - \left(-\frac{1}{4} - \frac{5}{3} + 3\right) = -\frac{13}{12}$$

$$I_2 = -\frac{1}{4}x^4 + \frac{5}{3}x^3 + 3x^2 \Bigg|_{0}^{6} = \left(-324 + 360 + 108\right) - 0 = 144$$

$$\Rightarrow \quad A = \left|-\frac{13}{12}\right| + 144 = \frac{1741}{12}\,u^2$$

ex 1.3.17 a) b)