Exples

$\frac{u'}{u}$ ✓

1) $\int \frac{4x}{2x^2}\,dx = \ln(|2x^2|) + C$ $\qquad u = 2x^2$, $u' = 4x$

2) $\int \frac{1}{2x^2}\,dx = \int \frac{1}{2}\cdot x^{-2}\,dx = \frac{1}{2}\int x^{-2}\,dx = \dots \frac{1}{2}\cdot\left(-\frac{1}{x}\right) + C = -\frac{1}{2x} + C$

3) $\int \frac{1}{(2x)^2}\,dx = \int (2x)^{-2}\,dx = \frac{1}{2}\int 2\cdot(2x)^{-2}\,dx = \frac{1}{2}\cdot\frac{1}{-1}(2x)^{-1} + C$

$u = 2x$
$u' = 2$

$= -\frac{1}{2}\cdot\frac{1}{2x} + C = -\frac{1}{4x} + C$

4) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{1}{-2+1}x^{-2+1} + C = -x^{-1} + C$

$= -\frac{1}{x} + C$

$u^n \cdot u'$ ✓

5) $\int \frac{2x^3 + 5x^2 + 3x + 4}{x^2}\,dx = \int \left(\frac{2x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{4}{x^2}\right) dx$

$= \int \left(2x + 5 + \frac{3}{x} + \frac{4}{x^2}\right) dx$

$= x^2 + 5x + 3\ln(|x|) + \int 4x^{-2}\,dx$

$= x^2 + 5x + 3\ln(|x|) - \frac{4}{x} + C$

ex 1.3.6

+ ex suppl. II

6) $\int \frac{2x^2-x-5}{2x-3}\,dx$

$= \int \left(x+1 - \frac{2}{2x-3}\right) dx$

$= \frac{1}{2}x^2 + x - \ln(|2x-3|) + C$

$$\begin{array}{r|l} 2x^2-x-5 & 2x-3 \\ \hline -2x^2+3x & x+1 \\ 2x-5 & \\ -2x+3 & \\ \hline -2 & \end{array}$$

$2x^2-x-5 = (2x-3)(x+1) - 2$

$\frac{2x^2-x-5}{2x-3} = x+1 - \frac{2}{2x-3}$

ex 1.1.7 e)

$\int \frac{x^2+2x-2}{x-1}\,dx$

$= \int \left(x+3+\frac{1}{x-1}\right) dx$

$= \frac{1}{2}x^2 + 3x + \ln(|x-1|) + C$

$$\begin{array}{r|l} x^2+2x-2 & x-1 \\ \hline -x^2+x & x+3 \\ 3x-2 & \\ -3x+3 & \\ \hline 1 & \end{array}$$

$f(x) = x+3+\frac{1}{x-1}$

f) $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{-\sin(x)}{\cos(x)}\,dx = -\ln(|\cos(x)|) + C$

$u = \cos(x)$

$u' = -\sin(x)$