Règles de calcul / Dérivée de fonctions usuelles

1. $f(x) = k \Rightarrow f'(x) = \boxed{k' = 0}$ avec $k \in \mathbb{R}$

2. $f(x) = kx \Rightarrow f'(x) = \boxed{(kx)' = k}$ " "

3. $f(x) = x^n \Rightarrow f'(x) = \boxed{(x^n)' = nx^{n-1}}$ $n \in \mathbb{R}$

Exples

- $\left(\frac{1}{x^3}\right)' = (x^{-3})' = -3x^{-3-1} = -3x^{-4} = -\frac{3}{x^4}$
- $\left(\sqrt[4]{x}\right)' = \left(x^{\frac{1}{4}}\right)' = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4} \cdot \frac{1}{x^{3/4}} = \frac{1}{4\sqrt[4]{x^3}}$
- $\boxed{\left(\sqrt{x}\right)'} = (x^{1/2})' = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$
- $\boxed{\left(\frac{1}{x}\right)'} = (x^{-1})' = -1x^{-2} = \boxed{-\frac{1}{x^2}}$

Règles de dérivation

Soit $u(x)$ et $v(x)$ deux fonctions de x, notée u et v

et $k \in \mathbb{R}$

Alors

- $\boxed{(u+v)' = u' + v'}$

exple : $(3x+5)' = (3x)' + (5)'$
$= 3 + 0$
$= 3$

- $\boxed{(ku)' = k \cdot u'}$

exple : $(4x^3)'$
$= 4 \cdot 3x^2 = 12x^2$

exple : $(3x^2 - 7x + 18)' = (3x^2)' - (7x)' + (18)'$
$= 3 \cdot 2x - 7 + 0$
$= 6x - 7$

ex 2.7.17
2.7.21 a) b) c) i) j)

Règles de dérivation (suite)

- $f(x) = u \cdot v$

  $f'(x) = \boxed{(u \cdot v)' = u' \cdot v + u \cdot v'}$ ⚠

- $f(x) = \dfrac{u}{v}$

  $f'(x) = \boxed{\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}}$ ⚠

Exemples :

a) $f(x) = (x+5)(2x-1)$

$u = x+5 \qquad v = 2x-1$

$u' = 1 \qquad v' = 2$

$f'(x) = 1 \cdot (2x-1) + 2(x+5)$

$= 2x-1 + 2x+10 = 4x+9$

variante : $f(x) = 2x^2 - x + 10x - 5 = 2x^2 + 9x - 5$

$f'(x) = 4x + 9$

b) $f(x) = (x^2-1)\sqrt{x}$

$u = x^2-1 \qquad v = \sqrt{x}$

$u' = 2x \qquad v' = \frac{1}{2\sqrt{x}}$

$$f'(x) = 2x\sqrt{x} + (x^2-1)\cdot\frac{1}{2\sqrt{x}}$$

$$= 2x\sqrt{x} + \frac{x^2-1}{2\sqrt{x}}$$

c) $f(x) = 2(x^2+1)(2x^3+3x-1)$

$u = 2(x^2+1) \qquad v = 2x^3+3x-1$

$u' = 2\cdot 2x \qquad v' = 2\cdot 3x^2+3$

$= 4x \qquad = 6x^2+3$

$$f'(x) = 4x(2x^3+3x-1) + 2(x^2+1)(6x^2+3)$$

$$= 8x^4+12x^2-4x + 12x^4+6x^2+12x^2+6$$

$$= 20x^4+30x^2-4x+6$$

4) $f(x) = \dfrac{x+5}{2x-1}$

$u = x+5$ $\quad v = 2x-1$

$u' = 1$ $\quad v' = 2$

$$f'(x) = \frac{1(2x-1) - 2(x+5)}{(2x-1)^2} = \frac{2x-1-2x-10}{(2x-1)^2} = \frac{-11}{(2x-1)^2}$$

5) $f(x) = \dfrac{2x}{x^2-1}$

$u = 2x$ $\quad v = x^2-1$

$u' = 2$ $\quad v' = 2x$

$$f'(x) = \frac{2(x^2-1) - 2x \cdot 2x}{(x^2-1)^2} = \frac{2x^2-2-4x^2}{(x^2-1)^2} = \frac{-2x^2-2}{(x^2-1)^2} = \frac{-2(x^2+1)}{(x^2-1)^2}$$