Ex 3.2.2

a) $d_1 : 5x - y - 7 = 0$ $\qquad$ $d_2 : 3x + 2y = 0$

$m_1 = 5$ et $m_2 = -\frac{3}{2}$ $\Rightarrow$ $\tan(\varphi) = \left| \frac{-\frac{3}{2} - 5}{1 - \frac{15}{2}} \right| = 1$ $\Rightarrow$ $\varphi = 45°$

ou $\vec{d_1} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ et $\vec{d_2} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\qquad$ (ou avec $\vec{n_1} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ et $\vec{n_2} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\Rightarrow$ ...)

$\Rightarrow$ $\cos(\varphi) = \frac{|-2 + 15|}{\sqrt{1+25}\sqrt{4+9}} = \frac{13}{\sqrt{26}\sqrt{13}} = \frac{13}{\sqrt{2} \cdot 13} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\Leftrightarrow$ $\varphi = 45°$

b) $d_1 : 3x - 2y + 7 = 0$ $\qquad$ $d_2 : 2x + 3y - 5 = 0$

$m_1 = \frac{3}{2}$ et $m_2 = -\frac{2}{3}$ $\Rightarrow$ $\frac{3}{2} \cdot \left(-\frac{2}{3}\right) = -1$ $\Rightarrow$ $d_1 \perp d_2$ $\Rightarrow$ $\varphi = 90°$

ou $\vec{d_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{n_2}$ $\quad$ ...

c) $d_1 : x - 2y - 4 = 0$ $\qquad$ $d_2 : 2x - 4y + 3 = 0$

$m_1 = \frac{1}{2}$ et $m_2 = \frac{1}{2}$ $\Rightarrow$ $\varphi = 0°$

d) $d_1 : 3x + 2y - 1 = 0$ $\qquad$ $d_2 : 5x - 2y + 3 = 0$

$m_1 = -\frac{3}{2}$ et $m_2 = \frac{5}{2}$ $\Rightarrow$ $\tan(\varphi) = \left| \frac{\frac{5}{2} + \frac{3}{2}}{1 - \frac{15}{4}} \right| = \left| \frac{4}{-\frac{11}{4}} \right| = \frac{16}{11}$ $\Rightarrow$ $\varphi \cong 55{,}49°$

Ex 3.2.5

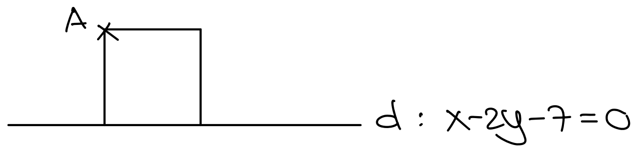
a) $\delta(P;d) = \dfrac{|8-3+10|}{\sqrt{16+9}} = \dfrac{15}{5} = \underline{3\,u}$

b) $\delta(P;d) = \dfrac{|0+36-23|}{\sqrt{25+144}} = \dfrac{13}{13} = \underline{1\,u}$

c) $\delta(P;d) = \dfrac{|-6-12-2|}{\sqrt{9+16}} = \dfrac{20}{5} = \underline{4\,u}$

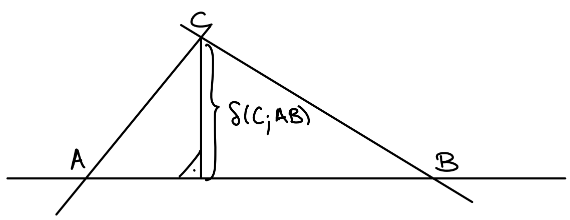
d) $\delta(P;d) = \dfrac{|1+4-5|}{\sqrt{1+4}} = \underline{0\,u}$ (le point est sur la drte)

Ex 3.2.6



$\delta(A;d) = \dfrac{|2+10-7|}{\sqrt{1+4}} = \dfrac{5}{\sqrt{5}} = \sqrt{5} \Rightarrow$ Aire $= (\sqrt{5})^2 = \underline{5\,u^2}$

Ex 3.2.7



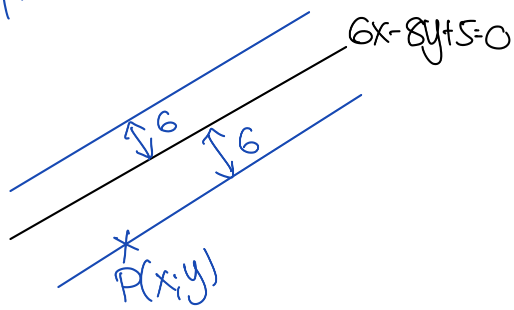
- $C = (BC) \cap (AC)$: $\begin{cases} x+3y = -3 \\ 2x+3y = 0 \end{cases} \begin{vmatrix} -1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} x+3y = -3 \\ x = 3 \end{cases} \Leftrightarrow \begin{cases} x = 3 \\ y = -2 \end{cases}$

$\Rightarrow C(3;-2)$

- longueur de la hauteur : $\delta(C;(AB)) = \dfrac{|3-2+1|}{\sqrt{1+1}} = \dfrac{2}{\sqrt{2}} = \underline{\sqrt{2}\,u}$

Ex 3.2.8

on cherche l'ensemble des points $P(x;y)$ tq.



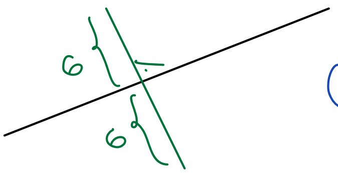
$$\delta(P;d) = 6 \iff \frac{|6x-8y+5|}{\sqrt{36+64}} = 6$$

$$\iff |6x-8y+5| = 60$$

$$\iff 6x-8y+5 = \pm 60$$

$$\iff \left\langle \begin{array}{l} 6x-8y+65 = 0 \\ 6x-8y-55 = 0 \end{array} \right.$$

Variante :



On commence par chercher un point $A \in d$:

$$y = \frac{3}{4}x + \frac{5}{8} \implies A\left(0;\frac{5}{8}\right)$$

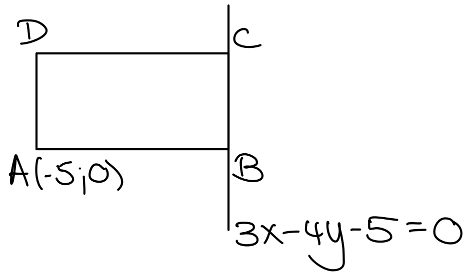
On cherche ensuite une droite $d' // d$ tq $\delta(A;d') = 6$

$d' : 6x-8y+c = 0$

$$\delta(A,d') = \frac{|0-5+c|}{\sqrt{36+64}} = \frac{|-5+c|}{10} = 6 \iff |-5+c| = 60$$

$$\iff -5+c = \pm 60 \iff c = \left\langle \begin{array}{l} 65 \\ -55 \end{array} \right. \implies \begin{array}{l} d'_1 : 6x-8y+65=0 \\ d'_2 : 6x-8y-55=0 \end{array}$$

Ex 3.2.9



- $(AD) \parallel (BC) \Rightarrow 3x-4y+c=0$
  $A \in (AD) \Rightarrow -15+0+c=0 \Leftrightarrow c=15$
  $(AD): 3x-4y+15=0$

- $(AB) \perp (BC) \Rightarrow 4x+3y+c=0$
  $A \in (AB) \Rightarrow -20+0+c=0 \Leftrightarrow c=20$
  $(AB): 4x+3y+20=0$

- $\delta(A;(BC)) = \dfrac{|-15+0-5|}{\sqrt{9+16}} = \dfrac{20}{5} = 4u$

$\Rightarrow \delta(A,(CD)) = \dfrac{20}{4} = 5u$

Comme $(CD) \parallel (AB) \Rightarrow (CD): 4x+3y+c=0$

$\Rightarrow \delta(A;(CD)) = \dfrac{|-20+0+c|}{\sqrt{16+9}} = \dfrac{|-20+c|}{5} = 5$

$\Leftrightarrow |-20+c| = 25$

$\Leftrightarrow -20+c = \pm 25 \Leftrightarrow c = \begin{cases} 45 \Rightarrow (CD_1): 4x+3y+45=0 \\ -5 \Rightarrow (CD_2): 4x+3y-5=0 \end{cases}$

Ex 3.2.10

- L'ensemble des points équidistants de $A(0;1)$ et $B(2;5)$ est la médiatrice de AB

$\vec{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \vec{n}$ vect. normal de la m_{AB} $\Rightarrow$ $m_{AB}: \; x+2y+c=0$

$M\left(\frac{0+2}{2}; \frac{1+5}{2}\right) = (1;3)$ milieu de AB $\in m_{AB}$ $\Rightarrow$ $1+6+c=0 \Leftrightarrow c=-7$

$\Rightarrow \quad m_{AB}: \quad x+2y-7=0$

- L'ensemble des points $P(x;y)$ situés à une distance 2 de $d: 3x-4y-4=0$:

$$\delta(P;d) = 2 \Leftrightarrow \frac{|3x-4y-4|}{\sqrt{9+16}} = 2$$

$$\Leftrightarrow |3x-4y-4| = 10$$

$$\Leftrightarrow 3x-4y-4 = \pm 10$$

$$\Leftrightarrow \begin{cases} 3x-4y-14 = 0 & : d_1 \\ 3x-4y+6 = 0 & : d_2 \end{cases}$$

- $m_{AB} \cap d_1: \begin{cases} x+2y=7 \\ 3x-4y=14 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \begin{vmatrix} 3 \\ -1 \end{vmatrix} \Leftrightarrow \begin{cases} 5x = 28 \\ 10y = 7 \end{cases} \Leftrightarrow \begin{cases} x = \frac{28}{5} \\ y = \frac{7}{10} \end{cases}$

$\Rightarrow \; I_1\left(\frac{28}{5}; \frac{7}{10}\right)$

$m_{AB} \cap d_2: \begin{cases} x+2y=7 \\ 3x-4y=-6 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \begin{vmatrix} 3 \\ -1 \end{vmatrix} \Leftrightarrow \begin{cases} 5x = 8 \\ 10y = 27 \end{cases} \Leftrightarrow \begin{cases} x = 8/5 \\ y = 27/10 \end{cases}$

$\Rightarrow \; I_2\left(\frac{8}{5}; \frac{27}{10}\right)$

Ex 3.2.11

$d_1 : 3x+4y-13=0 \qquad d_2 : 3x+4y-3=0$

Soit $D(1;0) \in d_2 \Rightarrow \delta(D;d_1) = \dfrac{|3+0-13|}{\sqrt{9+16}} = \dfrac{10}{5} = 2\,u$

$d_3 : 3x+4y+\dfrac{-13-3}{2} = 0 \Leftrightarrow d_3 : 3x+4y-8=0$

Ex 3.2.12

$$\frac{2x-3y-5}{\underbrace{\sqrt{4+9}}_{\sqrt{13}}} = \pm \frac{6x-4y+7}{\underbrace{\sqrt{36+16}}_{\sqrt{52}=2\sqrt{13}}} \quad \Big|\cdot 2\sqrt{13}$$

$\Leftrightarrow 2(2x-3y-5) = \pm(6x-4y+7)$

$\Leftrightarrow$
- $4x-6y-10 = 6x-4y+7 \Leftrightarrow -2x-2y-17=0 \Leftrightarrow 2x+2y+17=0 : b_1$
- $4x-6y-10 = -6x+4y-7 \Leftrightarrow 10x-10y-3=0 : b_2$

$O_x : y=0 \Rightarrow b_1 \cap O_x : 2x+17=0 \Leftrightarrow x=-\dfrac{17}{2} < 0 \Rightarrow b_1$

$\left(\Rightarrow b_2 \cap O_x : 10x-3=0 \Leftrightarrow x=\dfrac{3}{10} > 0\right)$

Ex 3.2.13

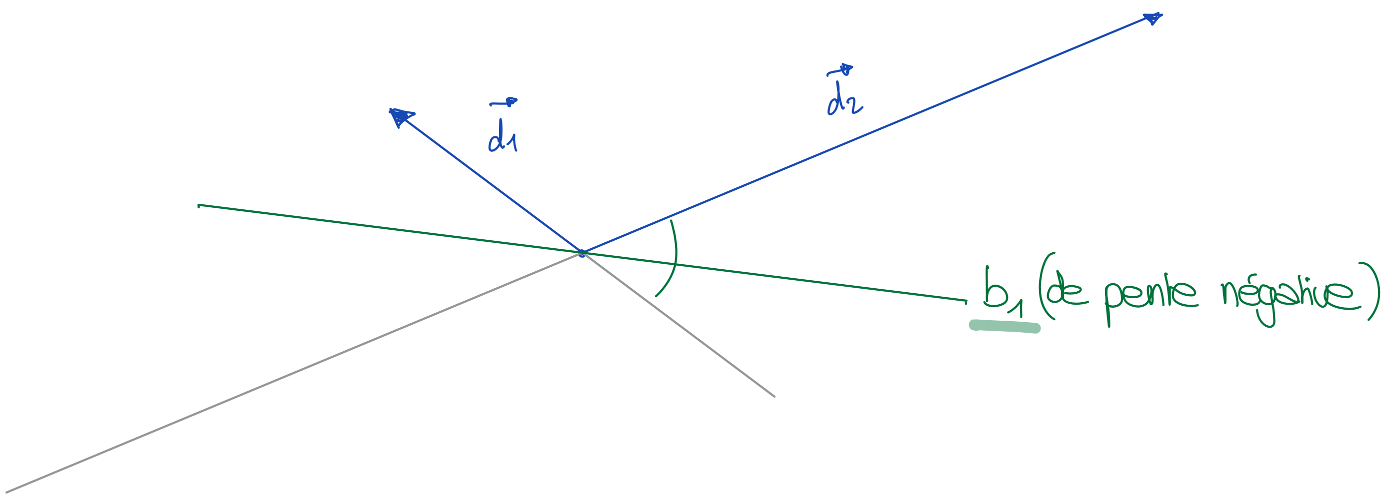
$$\frac{3x+4y-5}{\underbrace{\sqrt{9+16}}_{5}} = \pm \frac{5x-12y+3}{\underbrace{\sqrt{25+144}}_{13}} \qquad \Big| \cdot 65$$

$$\Leftrightarrow \quad 13(3x+4y-5) = \pm 5(5x-12y+3)$$

$$\Leftrightarrow \quad 39x+52y-65 = \begin{cases} 25x-60y+15 & \Leftrightarrow \quad 14x+112y-80=0 \\ -25x+60y-15 & \Leftrightarrow \quad 64x-8y-50=0 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} 7x+56y-40=0 \; : \; b_1 & m_{b_1} = -\frac{1}{8} \\ 32x-4y-25=0 \; : \; b_2 & m_{b_2} = 8 \end{cases}$$

Pour déterminer quelle est la bissectrice de l'angle aigu on représente les vecteurs directeurs $\vec{d_1} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ et $\vec{d_2} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ (suffisant ici)



Ex 3.2.14

$$\frac{x-3y+5}{\sqrt{1+9}} = \pm \frac{3x-y+15}{\sqrt{9+1}} \qquad \Big| \cdot \sqrt{10}$$

$$\Leftrightarrow \quad x-3y+5 = \pm(3x-y+15)$$

$$\Leftrightarrow \quad x-3y+5 = \begin{cases} 3x-y+15 & \Leftrightarrow \; 2x+2y+10=0 \; \Leftrightarrow \; x+y+5=0 : b_1 \\ -3x+y-15 & \Leftrightarrow \; 4x-4y+20=0 \; \Leftrightarrow \; x-y+5=0 : b_2 \end{cases}$$

$T(-1;-4) \in b_1$? : $-1-4+5=0$ ✓ oui $\Rightarrow$ b_1

$\left(T(-1;-4) \notin b_2 : \; -1+4+5 = 8 \neq 0 \right)$