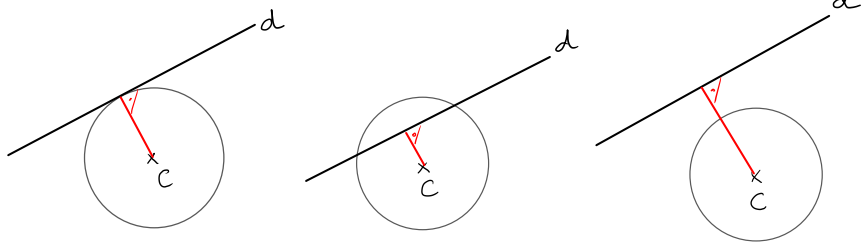
# Position relative

## 1. Droite - cercle




tangents
$\delta(C;d) = r$

sécants
$\delta(C;d) < r$

disjoints
$\delta(C;d) > r$

Exple : $\mathcal{C}: x^2+y^2-4x-6y-12=0$ et $d: 3x-4y-19=0$

Quel est la pos. rel. de la drte et du cercle ?

$\mathcal{C}: x^2-4x+4+y^2-6y+9 = 12+4+9$

$(x-2)^2 + (y-3)^2 = 25 \Rightarrow C(2;3)$ et $r=5$ u

$\delta(C;d) = \dfrac{|3\cdot 2-4\cdot 3-19|}{\sqrt{9+16}} = \dfrac{|-25|}{5} = 5 = r \Rightarrow$ tangents

Quelles sont les coords du point d'∩ ?

$$\begin{cases} x^2+y^2-4x-6y=12 \\ 3x-4y=19 \end{cases} \Leftrightarrow 4y = 3x-19 \text{ (substitution)} \Leftrightarrow y = \frac{3x-19}{4}$$

$$x^2 + \left(\frac{3x-19}{4}\right)^2 - 4x - 6\,\frac{3x-19}{4} - 12 = 0$$

$$x^2 + \frac{(3x-19)^2}{16} - 4x - 6\,\frac{3x-19}{4} - 12 = 0 \quad |\cdot 16$$

$$16x^2 + (3x-19)^2 - 64x - 24(3x-19) - 192 = 0$$

$$16x^2 + 9x^2 - 114x + 361 - 64x - 72x + 456 - 192 = 0$$

$$25x^2 - 250x + 625 = 0 \quad |\div 25$$

$$x^2 - 10x + 25 = 0$$

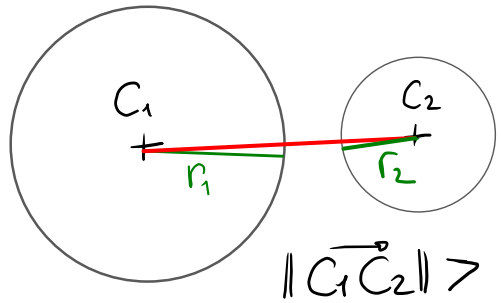
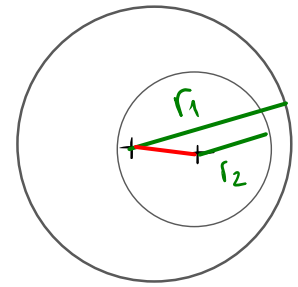
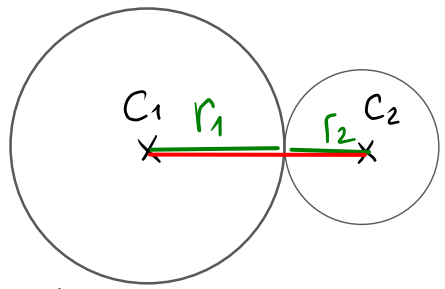
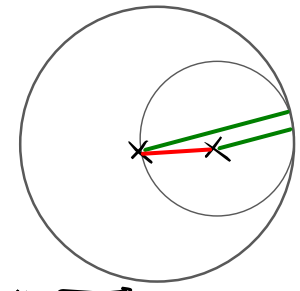
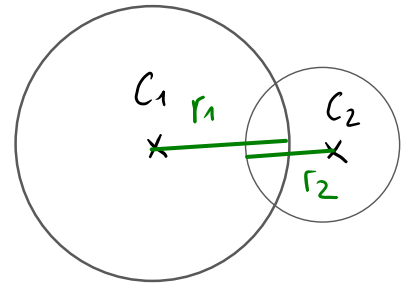
$$(x-5)^2 = 0$$

$x = 5 \Rightarrow y = \dfrac{3\cdot 5-19}{4} = \dfrac{-4}{4} = -1$

$\Rightarrow T(5;-1)$

ex 2.1.3

## 2. Cercle - cercle

| | extérieurement | intérieurement |
|---|---|---|
| disjoints |   $\|\|\overrightarrow{C_1C_2}\|\| > r_1 + r_2$ |   $\|\|\overrightarrow{C_1C_2}\|\| < r_1 - r_2$ |
| tangents |   $\|\|\overrightarrow{C_1C_2}\|\| = r_1 + r_2$ |  $\|\|\overrightarrow{C_1C_2}\|\| = r_1 - r_2$ |
| sécants |   $r_1 - r_2 < \|\|\overrightarrow{C_1C_2}\|\| < r_1 + r_2$ | |

$r_1 > r_2$

ex 2.1.4

Exple : $\mathcal{C}_1 : x^2 + y^2 = 25$ $\quad C_1(0;0)$ et $r = 5$ u

$\mathcal{C}_2 : (x-14)^2 + (y-2)^2 = 125$ $\quad C_2(14;2)$ et $r = 5\sqrt{5}$ u $\cong 11{,}18$

$$\| \overrightarrow{C_1C_2} \| = \left\| \begin{pmatrix} 14 \\ 2 \end{pmatrix} \right\| = \sqrt{196+4} = \sqrt{200} = 10\sqrt{2} \cong 14{,}14$$

$$11{,}18 - 5 < 14{,}14 < 5 + 11{,}18$$

$\Rightarrow$ $\mathcal{C}_1$ et $\mathcal{C}_2$ sont sécants

Pts d'$\cap$ : (1) $\begin{cases} x^2 + y^2 = 25 \\ (x-14)^2 + (y-2)^2 = 125 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 25 \\ x^2 + y^2 - 28x - 4y = -75 \end{cases} \begin{matrix} \\ \cdot(-1) \end{matrix}$

$$\begin{array}{rl} x^2 + y^2 & = 25 \\ -x^2 - y^2 + 28x + 4y & = 75 \\ \hline 28x + 4y & = 100 \quad | \div 4 \\ 7x + y & = 25 \end{array} \Rightarrow y = -7x + 25$$

subst. $\Rightarrow$ dans (1)

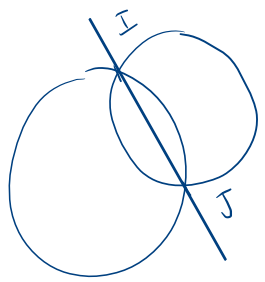
$$x^2 + (-7x+25)^2 = 25$$

$$x^2 + 49x^2 - 350x + 625 = 25$$

$$50x^2 - 350x + 600 = 0 \quad | \div 50$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$




$$x_{1,2} = \begin{cases} 3 \Rightarrow y_1 = -7 \cdot 3 + 25 = 4 \Rightarrow I(3;4) \\ 4 \Rightarrow y_2 = -7 \cdot 4 + 25 = -3 \Rightarrow J(4;-3) \end{cases}$$

ex 2.1.8

## Ex 2.1.3

a) $x^2 - 3x + \frac{9}{4} + y^2 + 2y + 1 = 3 + \frac{9}{4} + 1$

$(x - \frac{3}{2})^2 + (y+1)^2 = \frac{25}{4} \Rightarrow C\left(\frac{3}{2}; -1\right)$ et $r = \frac{5}{2}$

$$S(C;d) = \frac{|2\cdot\frac{3}{2} - (-1) - 3|}{\sqrt{4+1}} \quad \text{avec } d : 2x - y - 3 = 0$$

$$= \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \simeq 0{,}45 < \frac{5}{2} = 2{,}5 \Rightarrow \text{sécants}$$

$I(0;-3)$ $\quad$ $J\left(\frac{11}{5}; \frac{7}{5}\right)$

b) $x^2 - 8x + 16 + y^2 + 2y + 1 = -12 + 16 + 1$ $\quad d: x - 2y - 1 = 0$

$(x-4)^2 + (y+1)^2 = 5 \Rightarrow C(4;-1)$ et $r = \sqrt{5}$

$$S(C;d) = \frac{|4 - 2\cdot(-1) - 1|}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5} = r \Rightarrow \text{tangente}$$

$T(3;1)$

c) $x^2+y^2=1 \Rightarrow C(0;0)$ et $r=1$ $\quad d: x-y+10=0$

$$\delta(C;d) = \frac{|10|}{\sqrt{1+1}} = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \simeq 7,07 > 1 \Rightarrow \text{disjoints.}$$