Etude de fonction

1. ED(f)
2. signe
3. asymptotes (étude de position pas tjrs demandée)
4. croissance
5. graphe

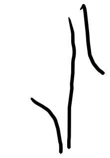
Exemple $f(x) = \dfrac{-x^2+2x-1}{x^2-5x+6} = \dfrac{-(x-1)^2}{(x-3)(x-2)}$ ← zéro : 1 (2)
← v.i : 2 et 3

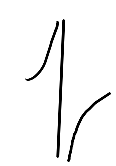
1. $ED(f) = \mathbb{R} - \{2;3\}$

2.

x		1		2		3	
sgn(f)	−	0 (2)	−	‖	+	‖	−

$f(1000) : \dfrac{-}{+}$

3. AV/trou : $\lim\limits_{x \to 2} f(x) = "\dfrac{-1}{0}" = \infty \Rightarrow$ AV : $x=2$ du type 

$\lim\limits_{x \to 3} f(x) = "\dfrac{-4}{0}" = \infty \Rightarrow$ AV : $x=3$ du type 

AH : car deg(N) = deg(D)

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{-\cancel{x^2}^{\,-1}}{\cancel{x^2}_{\,1}} = -1 \Rightarrow$ AH : $y = -1$

4. croissance

$$u = \overbrace{-x^2+2x-1}^{-(x-1)^2} \qquad v = \overbrace{x^2-5x+6}^{(x-2)(x-3)}$$

$$u' = \underbrace{-2x+2}_{-2(x-1)} \qquad v' = 2x-5$$

$$f'(x) = \frac{-2(x-1)(x-2)(x-3) + (x-1)^2(2x-5)}{(x-2)^2(x-3)^2}$$

$$= \frac{(x-1)\left[-2(x-2)(x-3) + (x-1)(2x-5)\right]}{(x-2)^2(x-3)^2}$$

$$= \frac{(x-1)\left[\cancel{-2x^2}+10x-12+\cancel{2x^2}-7x+5\right]}{(x-2)^2(x-3)^2} = \frac{(x-1)(3x-7)}{(x-2)^2(x-3)^2}$$

zéros de f' : 1 et $\frac{7}{3}$

v.i. $\underset{(2)}{2}$ et $\underset{(2)}{3}$

x		1		2		$7/3$		3	
sgn(f')	$+$	0	$-$	$\Vert$ (2)	$-$	0	$+$	$\Vert$ (2)	$+$
f	$\nearrow$	Max	$\searrow$	$\Vert$	$\searrow$	min	$\nearrow$	$\Vert$	$\nearrow$

$f'(1000)$: $\frac{+}{+}$

Max : $f(1) = 0 \Rightarrow$ Max$(1;0)$

min : $f\left(\frac{7}{3}\right) = 8 \Rightarrow$ min$\left(\frac{7}{3}\,;\,8\right) = (2,\overline{3}\,;\,8)$