

Etude de fonction

1. ED(f)
2. signe
3. asymptotes (étude de position pas très demandée)
4. croissance
5. graphe

Exemple $f(x) = \frac{-x^2 + 2x - 1}{x^2 - 5x + 6} = \frac{-(x-1)^2}{(x-3)(x-2)}$

← zéro : 1 (2)
← v.i : 2 et 3

1. ED(f) = $\mathbb{R} - \{2; 3\}$

2.

x	1	2	3
sgn(f)	-		+

 $f(1000) : \frac{-}{+}$

3. AV/mou : $\lim_{x \rightarrow 2} f(x) = \frac{-1}{0} = \infty \Rightarrow$ AV : x=2 du type $\sqrt{\quad}$

$\lim_{x \rightarrow 3} f(x) = \frac{-4}{0} = \infty \Rightarrow$ AV : x=3 du type $\frac{1}{\sqrt{\quad}}$

AH : car $\deg(N) = \deg(D)$

$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{-x^2}{x^2} = -1 \Rightarrow$ AH : y = -1

4. croissance

$$f'(x) = \frac{-2(x-1)(x-2)(x-3) + (x-1)^2(2x-5)}{(x-2)^2(x-3)^2}$$

$$= \frac{(x-1)[-2(x-2)(x-3) + (x-1)(2x-5)]}{(x-2)^2(x-3)^2}$$

$$= \frac{(x-1)[-2x^2+10x-12 + 2x^2-7x+5]}{(x-2)^2(x-3)^2} = \frac{(x-1)(3x-7)}{(x-2)^2(x-3)^2}$$

$$u = \underbrace{-x^2+2x-1}_{-2(x-1)}$$

$$v = \underbrace{x^2-5x+6}_{(x-2)(x-3)}$$

zéros de f' : 1 et $\frac{7}{3}$

v.i. 2 et 3
(2) (2)

x	1	2	$\frac{7}{3}$	3	
sign(f')	+	0	-	0	+
f					
		(2)		(2)	

Diagram showing a local maximum at x=1 and a local minimum at x=7/3. The function is increasing for x < 1, decreasing for 1 < x < 7/3, increasing for 7/3 < x < 3, and increasing for x > 3.

$$f'(1000) : \frac{+}{+}$$

Max : $f(1) = 0 \Rightarrow \text{Max}(1, 0)$

min : $f(\frac{7}{3}) = 8 \Rightarrow \text{min}(\frac{7}{3}; 8) = (2, \bar{3}; 8)$