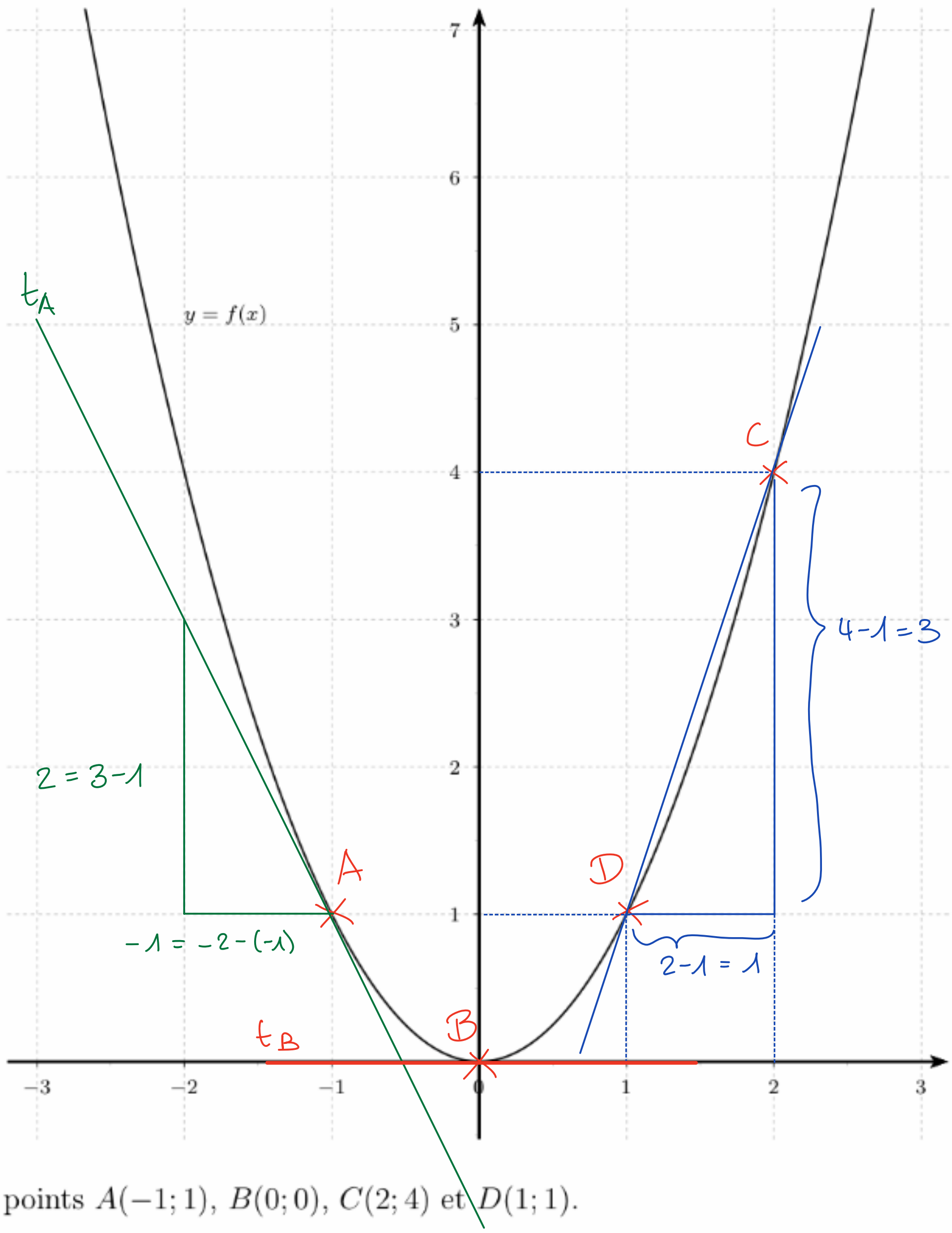
2.7.1 On donne une représentation du graphique de la fonction $f(x) = x^2$.



Soit les points $A(-1; 1)$, $B(0; 0)$, $C(2; 4)$ et $D(1; 1)$.

a) Déterminer la pente de la droite qui passe par les points D et C.

b) Déterminer graphiquement la pente de la tangente à la courbe $y = f(x)$ aux points $A(-1; 1)$, $B(0; 0)$, $C(2; 4)$ et D.

c) Donner ensuite une équation de la tangente en ces points.

a) $m_{DC} = \dfrac{4-1}{2-1} = \dfrac{3}{1} = 3$

b) $m_A = \dfrac{2}{-1} = -2$ $\quad m_B = 0$ $\quad m_C = 4$ $\quad m_D = 2$

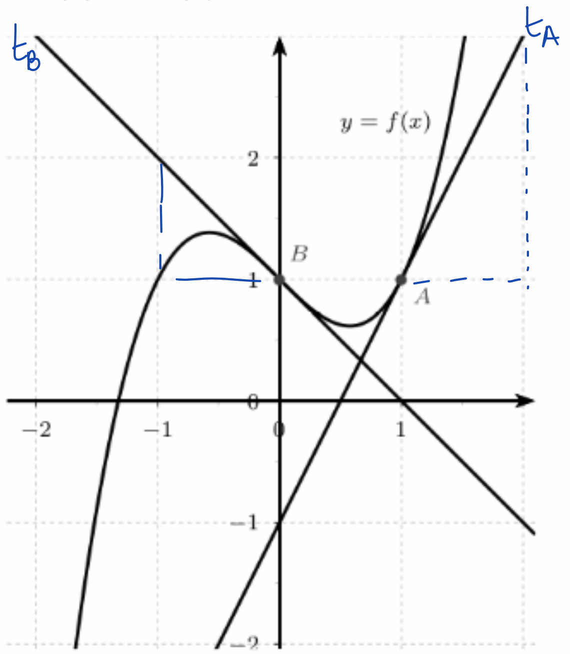
c) $t_A : \; y = \underbrace{-2}_{\text{pente}}x \underbrace{-1}_{\text{ordonnée à l'origine}}$

$t_C : \; y = 4x - 4$

$t_D : \; y = 2x - 1$

$t_B : \; y = 0$

2.7.2 Voici la représentation graphique d'une fonction $f(x)$. Les tangentes en A et B sont représentées.
Déterminer graphiquement $f'(0)$ et $f'(1)$.

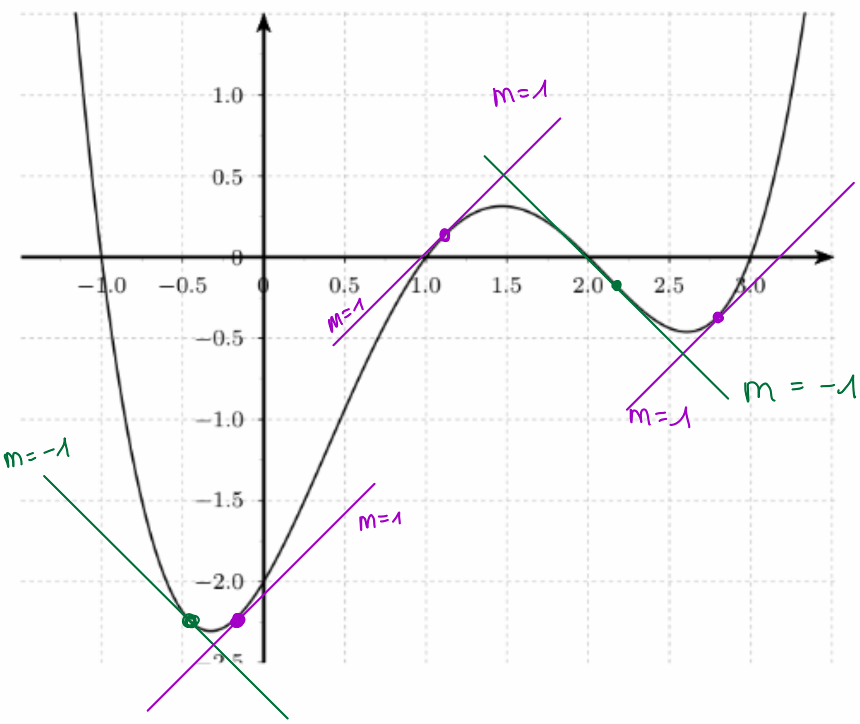


$f'(0) = m_B = -1$

$f'(1) = m_A = \frac{2}{1} = 2$

2.7.3 Sur le graphe de la fonction $f(x)$ ci-dessous, indiquer les valeurs approximatives de x pour lesquelles :

a) $f(x) = 0$ b) $f'(x) = 0$ c) $f'(x) = 1$ d) $f'(x) = -1$



a) $x = -1$
$x = 1$
$x = 2$
$x = 3$

zéros de f.

b) $x \cong -0{,}3$
$x \cong 1{,}4$
$x \cong 2{,}6$

extrema de f.

c) $x \cong -0{,}1$
$x \cong 1{,}1$
$x \cong 2{,}8$

d) $x \cong -0{,}4$
$x = 2{,}2$

Ex 2.7.12

$$f(x) = -x^2 + x + 2$$

a) $f'(x) = -2x + 1$

b) • coupe l'axe Oy : ord. à l'O. : $f(0) = 2 \Rightarrow A(0;2)$

$\Rightarrow$ pente : $m_A = f'(0) = -2\cdot 0 + 1 = \underline{1}$

• coupe l'axe Ox : zéros de f : $f(x) = 0 \Leftrightarrow -x^2 + x + 2 = 0$

$\Leftrightarrow -(x^2 - x - 2) = 0$

$\Leftrightarrow -(x-2)(x+1) = 0$

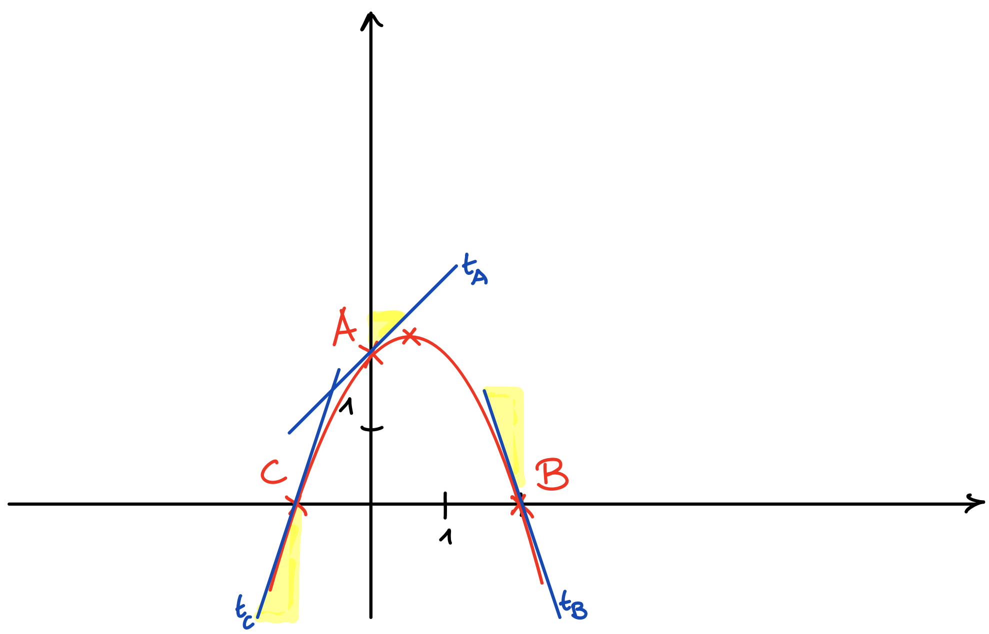
$\swarrow$ 2 $\quad$ $\downarrow$ -1

$\Rightarrow B(2;0)$ et $C(-1;0)$

$\Rightarrow$ pente : $m_B = f'(2) = -2\cdot 2 + 1 = \underline{-3}$

$m_C = f'(-1) = -2\cdot(-1) + 1 = \underline{3}$

c)



sommet : $x = \frac{-1}{-2} = \frac{1}{2}$

$y = f\left(\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + 2 = \frac{9}{4}$

Ex 2.7.17

a) $f(x) = 47$ $\quad f'(x) = 0$

b) $f(x) = 3x$ $\quad f'(x) = 3$

c) $f(x) = x^5$ $\quad f'(x) = 5x^4$

d) $f(x) = 8x^7$ $\quad f'(x) = 56x^6$

e) $f(x) = 5x^0 = 5 \implies f'(x) = 0$

f) $f(x) = \frac{1}{3}x^3$ $\quad f'(x) = \frac{1}{3} \cdot 3x^2 = x^2$

g) $f(x) = x^3 + x^2 + x + 1$ $\quad f'(x) = 3x^2 + 2x + 1$

h) $f(x) = 7x^4 - 3x + 8$ $\quad f'(x) = 28x^3 - 3$

i) $f(x) = x^2 + 5x - 6$ $\quad f'(x) = 2x + 5$

j) $f(x) = x^3 + 5x^2 - 2x + 4$ $\quad f'(x) = 3x^2 + 10x - 2$

k) $f(x) = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 2x + 4$

$f'(x) = \frac{2}{3} \cdot 3x^2 - \frac{5}{2} \cdot 2x + 2 = 2x^2 - 5x + 2$

l) $f(x) = 2x^5 - \frac{7}{6}x^3 + \frac{3}{4}x^2 - x + \sqrt{2}$

$f'(x) = 10x^4 - \frac{7}{6} \cdot 3x^2 + \frac{3}{4} \cdot 2x - 1 + 0$

$= 10x^4 - \frac{7}{2}x^2 + \frac{3}{2}x - 1$

Ex 2.7.18

a) $\left((x+1)(x-3)\right)' = 1\cdot(x-3) + 1\cdot(x+1) = x-3+x+1 = \underline{2x-2}$

$u = x+1$ $\quad v = x-3$
$u' = 1$ $\quad v' = 1$

variante: $\left(x^2-3x+x-3\right)' = \left(x^2-2x-3\right)' = 2x-2$

b) $\left(x(x^2+5)\right)' = 1\cdot(x^2+5) + x\cdot 2x = x^2+5+2x^2 = \underline{3x^2+5}$

$u = x$ $\quad v = x^2+5$
$u' = 1$ $\quad v' = 2x$

variante: $\left(x^3+5x\right)' = 3x^2+5$

c) $\left((7x^2-4x+3)(5-2x)\right)' = (14x-4)(5-2x) + (7x^2-4x+3)\cdot(-2)$

$u = 7x^2-4x+3$ $\quad v = 5-2x$
$u' = 14x-4$ $\quad v' = -2$

$= 70x - 28x^2 - 20 + 8x - 14x^2 + 8x - 6$

$= \underline{-42x^2 + 86x - 26}$

d) $\left((2x-1)(2-2x)(1+x)\right) = 2(2-2x)(1+x) + (2x-1)\cdot(-2)(1+x) + (2x-1)(2-2x)\cdot 1$

$= (4-4x)(1+x) + (2x-1)(-2-2x) + 4x-4x^2-2+2x$

$= 4+4x-4x-4x^2-4x-4x^2+2+2x+4x-4x^2-2+2x$

$= \underline{-12x^2+4x+4}$

e) $\left(\dfrac{4-3x}{2x-1}\right)' = \dfrac{-3(2x-1)-(4-3x)\cdot 2}{(2x-1)^2} = \dfrac{-6x+3-8+6x}{(2x-1)^2} = \underline{\dfrac{-5}{(2x-1)^2}}$

$u = 4-3x$ $\quad v = 2x-1$
$u' = -3$ $\quad v' = 2$

f) $\left(\dfrac{x-2}{3-x}\right)' = \dfrac{1(3-x)-(x-2)\cdot(-1)}{(3-x)^2} = \dfrac{3-x+x-2}{(3-x)^2} = \dfrac{1}{(3-x)^2}$

g) $\left(\dfrac{5}{2x^2-1}\right)' = 5\cdot\left(\dfrac{1}{2x^2-1}\right)' = 5\cdot\dfrac{-4x}{(2x^2-1)^2} = \underline{-\dfrac{20x}{(2x^2-1)^2}}$

$v = 2x^2-1$
$v' = 4x$

h) $$\left(\frac{x^3-10x^2}{1-x}\right)' = \frac{(3x^2-20x)(1-x) - (x^3-10x^2)(-1)}{(1-x)^2}$$

$u = x^3-10x^2$
$u' = 3x^2-20x$

$v = 1-x$
$v' = -1$

$$= \frac{3x^2 - 3x^3 - 20x + 20x^2 + x^3 - 10x^2}{(1-x)^2}$$

$$= \frac{-2x^3+13x^2-20x}{(1-x)^2}$$

i) $$\left(\frac{8x^2-8x+3}{4x^2-1}\right)'$$

$u = 8x^2-8x+3$
$u' = 16x-8$

$v = 4x^2-1$
$v' = 8x$

$$= \frac{(16x-8)(4x^2-1) - (8x^2-8x+3)\cdot 8x}{(4x^2-1)^2}$$

$$= \frac{64x^3 - 16x - 32x^2 + 8 - 64x^3 + 64x^2 - 24x}{(4x^2-1)^2} = \frac{32x^2-40x+8}{(4x^2-1)^2}$$

j) $$\left(\frac{x^3}{x+1}\right)' = \frac{3x^2(x+1) - x^3\cdot 1}{(x+1)^2} = \frac{3x^3+3x^2-x^3}{(x+1)^2} = \frac{2x^3+3x^2}{(x+1)^2}$$

k) $$\left(1+\frac{1}{x}-\frac{2}{x^2}\right)' = 0 - \frac{1}{x^2} - 2\cdot\left(-\frac{2x}{x^4}\right) = -\frac{1}{x^2} + \frac{4}{x^3} = \frac{-x+4}{x^3}$$

l) $$\left(\frac{x^3-4}{3x}+x\right)' = \frac{3x^2\cdot 3x - (x^3-4)\cdot 3}{9x^2} + 1 = \frac{9x^3-3x^3+12}{9x^2} + 1$$

$$= \frac{6x^3+12}{9x^2} + \frac{9x^2}{9x^2} = \frac{\overbrace{6x^3+9x^2+12}^{3(2x^3+3x^2+4)}}{9x^2} = \frac{2x^3+3x^2+4}{3x^2}$$

m) $\left(\dfrac{x(x+5)}{x^2+x}\right)' = \left(\dfrac{x^2+5x}{x^2+x}\right)' = \dfrac{(2x+5)(x^2+x)-(x^2+5x)(2x+1)}{(x^2+x)^2}$

$= \dfrac{2x^3+\underline{2x^2}+\underline{5x^2}+5x-2x^3\underline{-x^2}\underline{-10x^2}-5x}{(x^2+x)^2}$

$= \dfrac{-4x^2}{(x^2+x)^2} = \dfrac{-4x^2}{x^2(x+1)^2} = \dfrac{-4}{(x+1)^2}$

n) $\left(\left(x+\dfrac{2}{x^2}\right)x\right)'$ (erreur rép. brochure)

$u = x+\dfrac{2}{x^2}$ $\quad v = x$

$u' = 1+2.\left(-\dfrac{2x}{x^4}\right)$ $\quad v' = 1$

$= 1-\dfrac{4}{x^3}$

$= \left(1-\dfrac{4}{x^3}\right)x + \left(x+\dfrac{2}{x^2}\right).1$

$= x-\dfrac{4}{x^2} + x+\dfrac{2}{x^2} = -\dfrac{2}{x^2}$

o) $\left(\dfrac{2x+3(x^2-1)}{3}\right)' = \dfrac{1}{3}\left(2x+3x^2-3\right)' = \dfrac{1}{3}(2+6x) = \dfrac{6x+2}{3}$

p) $\left(\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}\right)' = \dfrac{\left(1+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)-\left(x-\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2}$

$u = x-\dfrac{1}{x}$ $\quad v = x+\dfrac{1}{x}$

$u' = 1+\dfrac{1}{x^2}$ $\quad v' = 1-\dfrac{1}{x^2}$

$= \dfrac{x+\frac{1}{x}+\frac{1}{x}+\frac{1}{x^3}-x+\frac{1}{x}+\frac{1}{x}-\frac{1}{x^3}}{\left(x+\frac{1}{x}\right)^2}$

$= \dfrac{\frac{4}{x}}{\left(x+\frac{1}{x}\right)^2} = \dfrac{4}{x} \div \left(x+\dfrac{1}{x}\right)^2 = \dfrac{4}{x} \div \left(\dfrac{x^2+1}{x}\right)^2$

$= \dfrac{4}{x}\cdot\dfrac{x^2}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$

Ex 2.7.20

Rappel : $\boxed{(u^n)' = n \cdot u^{n-1} \cdot \underbrace{u'}_{\text{dérivée interne}}}$

a) $f(x) = (2x+3)^4$

$f'(x) = 4(2x+3)^3 \cdot \underbrace{2}_{\text{dérivée interne}} = \underline{8(2x+3)^3}$

b) $f(x) = (3-x)^5$

$f'(x) = 5(3-x)^4 \cdot \underbrace{(-1)}_{\text{dérivée interne}} = \underline{-5(3-x)^4}$

c) $f(x) = (x^2+5x+1)^3$

$f'(x) = \underline{3(x^2+5x+1)^2 \underbrace{(2x+5)}_{\text{dérivée interne}}}$

d) $f(x) = (x^3-2x)^7$

$f'(x) = \underline{7(x^3-2x)^6 \underbrace{(3x^2-2)}_{\text{dérivée interne}}}$

Rappel : $\boxed{(u \cdot v)' = u'v + uv'}$

e) $f(x) = x^2(5x+2)^3$

$u = x^2 \qquad v = (5x+2)^3$

$u' = 2x \qquad v' = 3(5x+2)^2 \cdot \underline{5} = 15(5x+2)^2$

$f'(x) = 2x(5x+2)^3 + 15x^2(5x+2)^2$ (mise en évidence)

$= x(5x+2)^2 [\overbrace{2(5x+2)}^{10x+4} + 15x]$

$= \underline{x(5x+2)^2(25x+4)}$

f) $f(x) = (2+x)^2(1-x)^3$

$u = (2+x)^2$ $\qquad$ $v = (1-x)^3$

$u' = 2(2+x)\cdot 1$ $\qquad$ $v' = 3(1-x)^2\cdot(-1)$

$= 2(2+x)$ $\qquad$ $= -3(1-x)^2$

$$\begin{aligned} f'(x) &= 2(2+x)(1-x)^3 - 3(2+x)^2(1-x)^2 \qquad \text{(mee)} \\ &= (2+x)(1-x)^2\left[2(1-x) - 3(2+x)\right] \\ &= (2+x)(1-x)^2(2-2x-6-3x) \\ &= \underline{(2+x)(1-x)^2(-5x-4)} = \underline{-(2+x)(1-x)^2(5x+4)} \end{aligned}$$

g) $f(x) = (2x+5)^3(3x-1)^4$

$u = (2x+5)^3$ $\qquad$ $v = (3x-1)^4$

$u' = 3(2x+5)^2\cdot 2$ $\qquad$ $v' = 4(3x-1)^3\cdot 3$

$= 6(2x+5)^2$ $\qquad$ $= 12(3x-1)^3$

$$\begin{aligned} f'(x) &= 6(2x+5)^2(3x-1)^4 + 12(2x+5)^3(3x-1)^3 \qquad \text{(mee)} \\ &= 6(2x+5)^2(3x-1)^3\left[(3x-1) + \overbrace{2(2x+5)}^{4x+10}\right] \\ &= \underline{6(2x+5)^2(3x-1)^3(7x+9)} \end{aligned}$$

h) $f(x) = (1-3x)^2(2-x)(x+3)^3$

$$\begin{aligned} f'(x) &= -6(1-3x)(2-x)(x+3)^3 - (1-3x)^2(x+3)^3 + 3(1-3x)^2(2-x)(x+3)^2 \\ &= (1-3x)(x+3)^2[-6(2-x)(x+3) - (1-3x)(x+3) + 3(1-3x)(2-x)] \\ &= (1-3x)(x+3)^2(6x^2+6x-36+3x^2+8x-3+9x^2-21x+6) \\ &= (1-3x)(x+3)^2(18x^2-7x-33) \end{aligned}$$

Rappel : $\boxed{\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}}$

i) $f(x) = \frac{1}{(x^2+3)^2}$

$v = (x^2+3)^2$

$v' = 2(x^2+3)\cdot 2x = 4x(x^2+3)$

$$f'(x) = -\frac{4x(x^2+3)}{(x^2+3)^4} = -\frac{4x}{(x^2+3)^3}$$

Rappel : $\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$

j) $f(x) = \frac{x}{(3x+2)^2}$

$u = x$ $\qquad v = (3x+2)^2$

$u' = 1$ $\qquad v' = 2(3x+2)\cdot 3 = 6(3x+2)$

$$f'(x) = \frac{(3x+2)^2 - 6x(3x+2)}{(3x+2)^4}$$ (mee)

$$= \frac{\cancel{(3x+2)}\left[(3x+2) - 6x\right]}{(3x+2)^{\cancel{4}\,3}} = \frac{-3x+2}{(3x+2)^3}$$

k) $f(x) = \frac{(1-x)^3}{(1+x)^2}$

$u = (1-x)^3$ $\qquad v = (1+x)^2$

$u' = 3(1-x)^2\cdot(-1)$ $\qquad v' = 2(1+x)$

$= -3(1-x)^2$

$$f'(x) = \frac{-3(1-x)^2(1+x)^2 - 2(1-x)^3(1+x)}{(1+x)^4}$$

$$= \frac{-(1-x)^2\cancel{(1+x)}\left[\overbrace{3(1+x) + 2(1-x)}^{3+3x+2-2x}\right]}{(1+x)^{\cancel{4}\,3}} = -\frac{(1-x)^2(x+5)}{(1+x)^3}$$

l) $f(x) = \dfrac{x(x-3)^2}{(x-2)^2}$

$u = x(x-3)^2$

$u' = 1 \cdot (x-3)^2 + x \cdot 2(x-3) \cdot 1$

$= (x-3)^2 + 2x(x-3)$

$= (x-3)[(x-3)+2x]$

$= (x-3)(3x-3)$

$= 3(x-3)(x-1)$

$v = (x-2)^2$

$v' = 2(x-2)$

$$f'(x) = \frac{3(x-3)(x-1)(x-2)^2 - 2x(x-3)^2(x-2)}{(x-2)^4}$$

$$= \frac{(x-3)\cancel{(x-2)}\left[3\overbrace{(x-1)(x-2)}^{x^2-3x+2} - 2x(x-3)\right]}{(x-2)^{\cancel{4}\,3}}$$

$$= \frac{(x-3)(3x^2-9x+6-2x^2+6x)}{(x-2)^3} = \frac{(x-3)(\overbrace{x^2-3x+6}^{\Delta = 9-24<0})}{(x-2)^3}$$

Ex 2.7.21

Rappel : $\boxed{(x^n)' = nx^{n-1}}$

a) $\left(\sqrt[3]{x}\right)' = \left(x^{1/3}\right)' = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \dfrac{1}{3\sqrt[3]{x^2}}$

b) $\left(\sqrt[5]{x}\right)' = \left(x^{1/5}\right)' = \frac{1}{5}x^{\frac{1}{5}-1} = \frac{1}{5}x^{-\frac{4}{5}} = \dfrac{1}{5\sqrt[5]{x^4}}$

c) $\left(\sqrt[7]{x^4}\right)' = \left(x^{4/7}\right)' = \frac{4}{7}x^{\frac{4}{7}-1} = \frac{4}{7}x^{-\frac{3}{7}} = \dfrac{4}{7\sqrt[7]{x^3}}$

Rappel : $\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}}$

d) $f(x) = \sqrt{8x^2-5x+3}$

$$f'(x) = \frac{16x-5}{2\sqrt{8x^2-5x+3}}$$

e) $f(x) = \sqrt{x^2+4}$

$$f'(x) = \frac{2x}{2\sqrt{x^2+4}} = \frac{x}{\sqrt{x^2+4}}$$

i) $f(x) = \frac{1}{\sqrt{x}}$

avec $\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

$$f'(x) = -\frac{\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = -\frac{\frac{1}{2\sqrt{x}}}{\frac{x}{1}} = -\frac{1}{2\sqrt{x}} \cdot \frac{1}{x} = -\frac{1}{2x\sqrt{x}}$$

ou avec $(x^n)' = nx^{n-1}$

$$f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2} \cdot \frac{1}{\sqrt{x^3}} = -\frac{1}{2\sqrt{x^3}}$$

j) $\left(\frac{1}{\sqrt[3]{x^2}}\right)' = \left(x^{-\frac{2}{3}}\right)' = -\frac{2}{3}x^{-\frac{2}{3}-1} = -\frac{2}{3}x^{-\frac{5}{3}} = -\frac{2}{3\sqrt[3]{x^5}}$

k) $f(x) = (1+x)\sqrt{1-x}$

$u = 1+x \qquad v = \sqrt{1-x}$

$u' = 1 \qquad v' = \frac{-1}{2\sqrt{1-x}}$

$$f'(x) = \sqrt{1-x} + (1+x)\cdot\frac{-1}{2\sqrt{1-x}} = \frac{\sqrt{1-x}}{1} - \frac{1+x}{2\sqrt{1-x}}$$

$$= \frac{\sqrt{1-x}\cdot 2\sqrt{1-x}}{2\sqrt{1-x}} - \frac{1+x}{2\sqrt{1-x}}$$

$$= \frac{2(1-x)-(1+x)}{2\sqrt{1-x}} = \frac{2-2x-1-x}{2\sqrt{1-x}} = \frac{1-3x}{2\sqrt{1-x}}$$

l) $f(x) = \sqrt{\frac{3x-2}{x+1}}$

$u = \frac{3x-2}{x+1} \qquad u' = \frac{3(x+1)-(3x-2)\cdot 1}{(x+1)^2} = \frac{3x+3-3x+2}{(x+1)^2} = \frac{5}{(x+1)^2}$

$$f'(x) = \frac{\frac{5}{(x+1)^2}}{2\sqrt{\frac{3x-2}{x+1}}} = \frac{5}{(x+1)^2}\cdot\frac{1}{2\sqrt{\frac{3x-2}{x+1}}} = \frac{5}{2(x+1)^2}\sqrt{\frac{x+1}{3x-2}}$$

Ex 2.7.22

a) $f(x) = \sin(x) + \cos(x)$

$f'(x) = \underline{\cos(x) - \sin(x)}$

b) $f(x) = \tan(x) - x$

$f'(x) = 1 + \tan^2(x) - 1 = \underline{\tan^2(x)}$

ou $f'(x) = \dfrac{1}{\cos^2(x)} - 1 = \dfrac{1-\cos^2(x)}{\cos^2(x)} = \dfrac{\sin^2(x)}{\cos^2(x)} = \left(\dfrac{\sin(x)}{\cos(x)}\right)^2 = \tan^2(x)$

c) $f(x) = \dfrac{1}{\sin(x)}$

$v = \sin(x)$
$v' = \cos(x)$

$f'(x) = \underline{-\dfrac{\cos(x)}{\sin^2(x)}}$

d) $f(x) = \dfrac{\sin(x)}{1+\cos(x)}$

$u = \sin(x)$ $\quad$ $v = 1+\cos(x)$
$u' = \cos(x)$ $\quad$ $v' = -\sin(x)$

$f'(x) = \dfrac{\cos(x)(1+\cos(x)) - (-\sin^2(x))}{(1+\cos(x))^2}$

$= \dfrac{\cos(x) + \cos^2(x) + \sin^2(x)}{(1+\cos(x))^2} = \dfrac{1+\cos(x)}{(1+\cos(x))^2} = \underline{\dfrac{1}{1+\cos(x)}}$

e) $f(x) = \dfrac{\cos(x)+2}{\cos(x)+3}$

$u = \cos(x)+2$ $\quad$ $v = \cos(x)+3$
$u' = -\sin(x)$ $\quad$ $v' = -\sin(x)$

$f'(x) = \dfrac{-\sin(x)(\cos(x)+3) + \sin(x)(\cos(x)+2)}{(\cos(x)+3)^2}$

$= \dfrac{\sin(x)\left[-\cos(x) - 3 + \cos(x) + 2\right]}{(\cos(x)+3)^2} = \underline{\dfrac{-\sin(x)}{(\cos(x)+3)^2}}$

f) $f(x) = \frac{\sin(x)+1}{1-\sin(x)}$

$u = \sin(x)+1$ $\quad$ $v = 1-\sin(x)$

$u' = \cos(x)$ $\quad$ $v' = -\cos(x)$

$$f'(x) = \frac{\cos(x)(1-\sin(x)) + \cos(x)(\sin(x)+1)}{(1-\sin(x))^2}$$

$$= \frac{\cos(x)(1-\sin(x)+\sin(x)+1)}{(1-\sin(x))^2} = \underline{\frac{2\cos(x)}{(1-\sin(x))^2}}$$

g) $f(x) = \sin(2x)$ $\quad$ $(\sin(u))' = \cos(u)\cdot u'$

$f'(x) = \cos(2x)\cdot 2 = \underline{2\cos(2x)}$

h) $f(x) = \sin^2(x)$ $\quad$ $u^k = k u^{k-1}\cdot u'$ $\quad$ avec $u = \sin(x)$

$u' = \cos(x)$

$f'(x) = \underline{2\sin(x)\cos(x)}$

et $k=2$

$k-1 = 1$

Ex 2.7.25

a) $f(x) = 1 + 2x - x^3$ $\quad a = 1$ $\quad t: y = mx + h$

1) dérivée : $f'(x) = -3x^2 + 2$

pente de la tgte : $m = f'(1) = -3 + 2 = -1 \Rightarrow t: y = -x + h$

2) $f(1) = 1 + 2 - 1 = 2 \Rightarrow T(1;2)$

$T(1;2) \in t \Rightarrow 2 = -1 + h \Leftrightarrow h = 3$

$\Rightarrow t: y = -x + 3$

b) $f(x) = \frac{x+3}{x}$ $\quad a = 3$

1) dérivée : $f'(x) = \frac{1 \cdot x - (x+3) \cdot 1}{x^2} = \frac{x - x - 3}{x^2} = \frac{-3}{x^2}$

pente de la tgte : $m = f'(3) = \frac{-3}{9} = -\frac{1}{3} \Rightarrow t: y = -\frac{1}{3}x + h$

2) $f(3) = \frac{3+3}{3} = \frac{6}{3} = 2 \Rightarrow T(3;2)$

$T(3;2) \in t \Rightarrow 2 = -\frac{1}{3} \cdot 3 + h \Leftrightarrow 2 = -1 + h \Leftrightarrow h = 3$

$\Rightarrow t: y = -\frac{1}{3}x + 3$

c) $f(x) = \sqrt{2x+1}$, $\quad a = 4$

1) dérivée : $f'(x) = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$

pente de la tgte : $m = f'(4) = \frac{1}{\sqrt{8+1}} = \frac{1}{3} \Rightarrow t: y = \frac{1}{3}x + h$

2) $f(4) = \sqrt{8+1} = 3 \Rightarrow T(4;3)$

$T(4;3) \in t \Rightarrow 3 = \frac{1}{3} \cdot 4 + h \Leftrightarrow h = 3 - \frac{4}{3} = \frac{5}{3}$

$\Rightarrow t: y = \frac{1}{3}x + \frac{5}{3}$

d) $f(x) = \dfrac{\sin(x)}{\sin(x)+\cos(x)}$, $a=0$

1) $f'(x) = \dfrac{\cos(x)(\sin(x)+\cos(x)) - \sin(x)(\cos(x)-\sin(x))}{(\sin(x)+\cos(x))^2}$

$u = \sin(x)$ $\quad v = \sin(x)+\cos(x)$
$u' = \cos(x)$ $\quad v' = \cos(x)-\sin(x)$

$= \dfrac{\cos(x)\sin(x) + \cos^2(x) - \cos(x)\sin(x) + \sin^2(x)}{(\sin(x)+\cos(x))^2}$

$= \dfrac{\cos^2(x)+\sin^2(x)}{(\sin(x)+\cos(x))^2} = \dfrac{1}{(\sin(x)+\cos(x))^2}$

pente de la tgte : $m = f'(0) = \dfrac{1}{(0+1)^2} = 1 \Rightarrow t : y = x + h$

2) $f(0) = \dfrac{0}{0+1} = 0 \Rightarrow T(0;0)$

$T(0;0) \in t \Rightarrow 0 = 0 + h \Leftrightarrow h = 0$

$\Rightarrow t : y = x$

Ex 2.7.26

$y = x^2$ et $m = -3$

$f'(x) = 2x \Rightarrow 2x = -3 \Leftrightarrow x = -\frac{3}{2}$ (au point d'abscisse $-\frac{3}{2}$)

$f\left(-\frac{3}{2}\right) = \frac{9}{4} \Rightarrow P\left(-\frac{3}{2} ; \frac{9}{4}\right)$

Ex 2.7.27

$f(x) = x^3 - x^2 - 5x + 2$ et $A(-3;2)$ et $B(1;14)$

les tangentes sont parallèles à la droite AB $\Rightarrow$ la pente est la même

$m_{AB} = \frac{14-2}{1-(-3)} = 3 \Rightarrow f'(x) = 3x^2 - 2x - 5 = 3 \Leftrightarrow 3x^2 - 2x - 8 = 0$

$\Delta = 100 \Rightarrow x_{1,2} = \frac{2 \pm 10}{6} = \begin{cases} -4/3 \\ 2 \end{cases}$

Ex 2.7.28

$y = \frac{x}{x^2+9}$ tgte horizontale $\Rightarrow m = 0$

$m = 0 \Rightarrow f'(x) = \frac{1(x^2+9) - x \cdot 2x}{(x^2+9)^2} = \frac{-x^2+9}{(x^2+9)^2} = 0$

$\Rightarrow -x^2 + 9 = 0 \Leftrightarrow x^2 - 9 = 0 \Leftrightarrow (x+3)(x-3) = 0$

$\Rightarrow x = \begin{cases} -3 \Rightarrow f(-3) = -1/6 \Rightarrow P_1(-3 ; -1/6) \\ 3 \Rightarrow f(3) = 1/6 \Rightarrow P_2(3 ; 1/6) \end{cases}$

Ex 2.7.31

$$f(x) = \frac{x^2+ax+b}{cx^2+dx-2}$$

- Pas d'AH $\Rightarrow$ $c = 0$
- AV $x=2$ $\Rightarrow$ $dx-2 = x-2 \Leftrightarrow d=1 \Rightarrow f(x) = \dfrac{x^2+ax+b}{x-2}$
- $P(1;-2) \in y=f(x) \Rightarrow -2 = \dfrac{1+a+b}{-1} \Leftrightarrow 2 = 1+a+b \Leftrightarrow a+b=1$ (1)

$$f'(x) = \frac{(2x+a)(x-2) - 1(x^2+ax+b)}{(x-2)^2}$$

$$\Rightarrow m = f'(1) = -5 \Rightarrow \frac{(2+a)\cdot(-1) - (1+a+b)}{1} = -2a-b-3 = -5$$

$$\Leftrightarrow -2a-b = -2 \quad (2)$$

(1) et (2) $\Rightarrow \begin{cases} a+b = 1 \\ -2a-b = -2 \end{cases} \left|\begin{matrix} 1 \\ 1 \end{matrix}\right. \Leftrightarrow \begin{cases} a+b = 1 \\ -a = -1 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 0 \end{cases}$

Ex 2.7.32

$f(x) = x^3 + ax^2 + bx \qquad T(-1; \ldots) \qquad t: y = x+4$

$\Rightarrow m = f'(-1) = 1$ (1)

$T(-1;\ldots) \in t \Rightarrow y = -1+4 = 3 \Rightarrow T(-1;3) \Rightarrow f(-1) = 3$ (2)

$f'(x) = 3x^2 + 2ax + b$

(1) $\Rightarrow 3-2a+b = 1 \Leftrightarrow$

(2) $\Rightarrow -1+a-b = 3 \Leftrightarrow$

$\begin{cases} -2a+b = -2 \\ a-b = 4 \end{cases} \left|\begin{matrix} 1 \\ 1 \end{matrix}\right. \Leftrightarrow \begin{cases} -a = 2 \\ a-b = 4 \end{cases}$

$\Leftrightarrow \begin{cases} a = -2 \\ b = -6 \end{cases}$