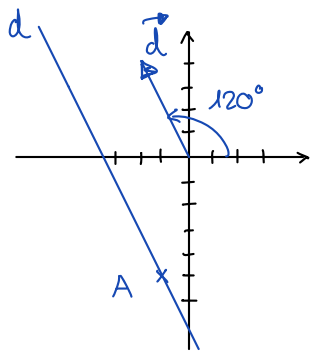
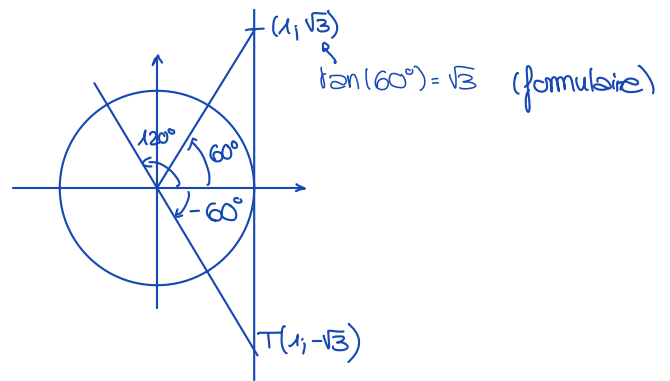


Ex 3.2.1



$$m = \tan(120^\circ) = -\sqrt{3}$$

$$\Rightarrow d: \sqrt{3}x + y + c = 0$$



$$A(-1, -5) \in d \Rightarrow \sqrt{3}(-1) - 5 + c = 0 \Leftrightarrow c = \sqrt{3} + 5$$

$$\Rightarrow d: \underline{\sqrt{3}x + y + \sqrt{3} + 5 = 0}$$

Ex 3.2.2

a) $d_1: 5x - y - 7 = 0$ $d_2: 3x + 2y = 0$

$$m_1 = 5 \text{ et } m_2 = -\frac{3}{2} \Rightarrow \tan(\varphi) = \left| \frac{-\frac{3}{2} - 5}{1 - \frac{15}{2}} \right| = 1 \Rightarrow \varphi = 45^\circ$$

ou $\vec{d}_1 = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ et $\vec{d}_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ (ou avec $\vec{n}_1 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ et $\vec{n}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow \dots$)

$$\Rightarrow \cos(\varphi) = \frac{|-2 + 15|}{\sqrt{1+25}\sqrt{4+9}} = \frac{13}{\sqrt{26}\sqrt{13}} = \frac{13}{\sqrt{2} \cdot 13} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \underline{\varphi = 45^\circ}$$

b) $d_1: 3x - 2y + 7 = 0$ $d_2: 2x + 3y - 5 = 0$

$$m_1 = \frac{3}{2} \text{ et } m_2 = -\frac{2}{3} \Rightarrow \frac{3}{2} \cdot \left(-\frac{2}{3}\right) = -1 \Rightarrow d_1 \perp d_2 \Rightarrow \underline{\varphi = 90^\circ}$$

ou $\vec{d}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{n}_2 \dots$

c) $d_1: x - 2y - 4 = 0$ $d_2: 2x - 4y + 3 = 0$

$$m_1 = \frac{1}{2} \text{ et } m_2 = \frac{1}{2} \Rightarrow \underline{\varphi = 0^\circ}$$

d) $d_1: 3x + 2y - 1 = 0$ $d_2: 5x - 2y + 3 = 0$

$$m_1 = -\frac{3}{2} \text{ et } m_2 = \frac{5}{2} \Rightarrow \tan(\varphi) = \left| \frac{\frac{5}{2} + \frac{3}{2}}{1 - \frac{15}{4}} \right| = \left| \frac{4}{-\frac{11}{4}} \right| = \frac{16}{11} \Rightarrow \underline{\varphi \approx 55,49^\circ}$$

Ex 3.2.5

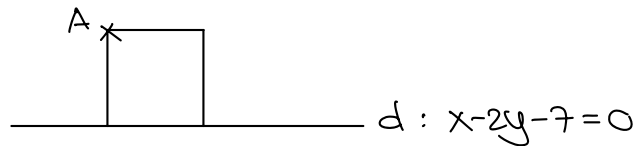
$$a) \delta(P;d) = \frac{|8-3+10|}{\sqrt{16+9}} = \frac{15}{5} = \underline{3u}$$

$$b) \delta(P;d) = \frac{|0+36-23|}{\sqrt{25+144}} = \frac{13}{13} = \underline{1u}$$

$$c) \delta(P;d) = \frac{|-6-12-2|}{\sqrt{9+16}} = \frac{20}{5} = \underline{4u}$$

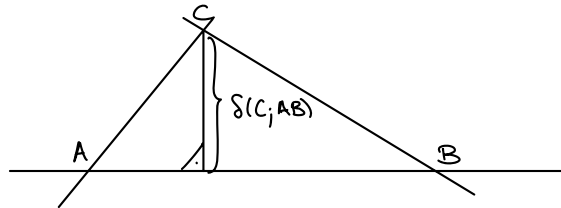
$$d) \delta(P;d) = \frac{|1+4-5|}{\sqrt{1+4}} = \underline{0u} \quad (\text{le point est sur la droite})$$

Ex 3.2.6



$$\delta(A;d) = \frac{|2+10-7|}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \sqrt{5} \Rightarrow \text{Aire} = (\sqrt{5})^2 = \underline{5u^2}$$

Ex 3.2.7



$$\bullet C = (B) \cap (AC) : \begin{cases} x+3y = -3 & | -1 \\ 2x+3y = 0 & | 1 \end{cases} \Leftrightarrow \begin{cases} x+3y = -3 \\ x = 3 \end{cases} \Leftrightarrow \begin{cases} x=3 \\ y=-2 \end{cases}$$

$$\Rightarrow C(3; -2)$$

$$\bullet \text{longueur de la hauteur: } \delta(C; (AB)) = \frac{|3-2+1|}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \underline{\sqrt{2}u}$$

Ex 3.2.8

on cherche l'ensemble des points $P(x;y)$ tq.

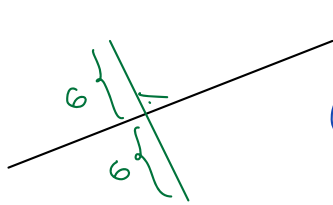
$$\delta(P; d) = 6 \Leftrightarrow \frac{|6x - 8y + 5|}{\sqrt{36 + 64}} = 6$$

$$\Leftrightarrow |6x - 8y + 5| = 60$$

$$\Leftrightarrow 6x - 8y + 5 = \pm 60$$

$$\Leftrightarrow \begin{cases} \underline{6x - 8y + 65 = 0} \\ \underline{6x - 8y - 55 = 0} \end{cases}$$

Variante :



$d: 6x - 8y + 5 = 0$

On commence par chercher un point $A \in d$:

$$y = \frac{3}{4}x + \frac{5}{8} \Rightarrow A\left(0; \frac{5}{8}\right)$$

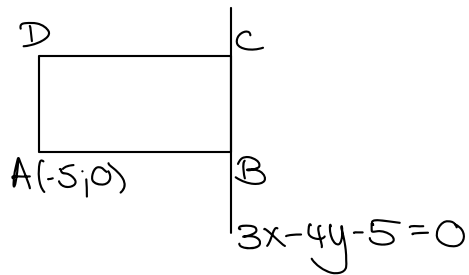
On cherche ensuite une droite $d' \parallel d$ tq $\delta(A; d') = 6$

$$d': 6x - 8y + c = 0$$

$$\delta(A, d') = \frac{|0 - 5 + c|}{\sqrt{36 + 64}} = \frac{|-5 + c|}{10} = 6 \Leftrightarrow |-5 + c| = 60$$

$$\Leftrightarrow -5 + c = \pm 60 \Leftrightarrow c = \begin{cases} 65 \\ -55 \end{cases} \Rightarrow \begin{cases} d'_1: 6x - 8y + 65 = 0 \\ d'_2: 6x - 8y - 55 = 0 \end{cases}$$

Ex 3.2.9



$$\begin{aligned} \bullet (AD) \parallel (BC) &\Rightarrow 3x - 4y + c = 0 \\ A \in (AD) &\Rightarrow -15 + 0 + c = 0 \Leftrightarrow c = 15 \end{aligned} \left. \vphantom{\begin{aligned} \bullet (AD) \parallel (BC) &\Rightarrow 3x - 4y + c = 0 \\ A \in (AD) &\Rightarrow -15 + 0 + c = 0 \end{aligned}} \right\} \underline{(AD): 3x - 4y + 15 = 0}$$

$$\begin{aligned} \bullet (AB) \perp (BC) &\Rightarrow 4x + 3y + c = 0 \\ A \in (AB) &\Rightarrow -20 + 0 + c = 0 \Leftrightarrow c = 20 \end{aligned} \left. \vphantom{\begin{aligned} \bullet (AB) \perp (BC) &\Rightarrow 4x + 3y + c = 0 \\ A \in (AB) &\Rightarrow -20 + 0 + c = 0 \end{aligned}} \right\} \underline{(AB): 4x + 3y + 20 = 0}$$

$$\bullet \delta(A; (BC)) = \frac{|-15 + 0 - 5|}{\sqrt{9 + 16}} = \frac{20}{5} = 4u$$

$$\Rightarrow \delta(A; (CD)) = \frac{20}{4} = 5u$$

Comme $(CD) \parallel (AB) \Rightarrow (CD): 4x + 3y + c = 0$

$$\Rightarrow \delta(A; (CD)) = \frac{|-20 + 0 + c|}{\sqrt{16 + 9}} = \frac{|-20 + c|}{5} = 5$$

$$\Leftrightarrow |-20 + c| = 25$$

$$\Leftrightarrow -20 + c = \pm 25 \Leftrightarrow c = \begin{cases} 45 &\Rightarrow \underline{(CD_1): 4x + 3y + 45 = 0} \\ -5 &\Rightarrow \underline{(CD_2): 4x + 3y - 5 = 0} \end{cases}$$

Ex 3.2.10

- l'ensemble des points équidistants de $A(0;1)$ et $B(2;5)$ est la médiatrice de AB

$$\vec{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \vec{n} \text{ vect. normal de la } m_{AB} \Rightarrow m_{AB}: x+2y+c=0$$

$$M\left(\frac{0+2}{2}; \frac{1+5}{2}\right) = (1; 3) \text{ milieu de } AB \in m_{AB} \Rightarrow 1+6+c=0 \Leftrightarrow c=-7$$

$$\Rightarrow m_{AB}: x+2y-7=0$$

- l'ensemble des points $P(x;y)$ situés à une distance 2 de $d: 3x-4y-4=0$:

$$S(P; d) = 2 \Leftrightarrow \frac{|3x-4y-4|}{\sqrt{9+16}} = 2$$

$$\Leftrightarrow |3x-4y-4| = 10$$

$$\Leftrightarrow 3x-4y-4 = \pm 10$$

$$\Leftrightarrow \begin{cases} 3x-4y-14 = 0 & : d_1 \\ 3x-4y+6 = 0 & : d_2 \end{cases}$$

$$\begin{aligned} m_{AB} \cap d_1 &: \begin{cases} x+2y=7 & | & 2 & | & 3 \\ 3x-4y=14 & | & 1 & | & -1 \end{cases} \Leftrightarrow \begin{cases} 5x = 28 \\ 10y = 7 \end{cases} \Leftrightarrow \begin{cases} x = \frac{28}{5} \\ y = \frac{7}{10} \end{cases} \\ &\Rightarrow \underline{I_1\left(\frac{28}{5}; \frac{7}{10}\right)} \end{aligned}$$

$$\begin{aligned} m_{AB} \cap d_2 &: \begin{cases} x+2y=7 & | & 2 & | & 3 \\ 3x-4y=-6 & | & 1 & | & -1 \end{cases} \Leftrightarrow \begin{cases} 5x = 8 \\ 10y = 27 \end{cases} \Leftrightarrow \begin{cases} x = \frac{8}{5} \\ y = \frac{27}{10} \end{cases} \\ &\Rightarrow \underline{I_2\left(\frac{8}{5}; \frac{27}{10}\right)} \end{aligned}$$

Ex 3.2.11

$$d_1: 3x+4y-13=0$$

$$d_2: 3x+4y-3=0$$

$$\text{Seit } D(1;0) \in d_2 \Rightarrow S(D, d_2) = \frac{|3+0-13|}{\sqrt{9+16}} = \frac{10}{5} = \underline{2 \text{ u}}$$

$$d_3: 3x+4y + \frac{-13-3}{2} = 0 \Leftrightarrow \underline{d_3: 3x+4y-8=0}$$

Ex 3.2.12

$$\frac{2x-3y-5}{\underbrace{\sqrt{4+9}}_{\sqrt{13}}} = \pm \frac{6x-4y+7}{\underbrace{\sqrt{36+16}}_{\sqrt{52} = 2\sqrt{13}}} \quad | \cdot 2\sqrt{13}$$

$$\Leftrightarrow 2(2x-3y-5) = \pm (6x-4y+7)$$

$$\Leftrightarrow \begin{cases} 4x-6y-10 = 6x-4y+7 & \Leftrightarrow -2x-2y-17=0 \Leftrightarrow \underline{2x+2y+17=0 : b_1} \\ 4x-6y-10 = -6x+4y-7 & \Leftrightarrow 10x-10y-3=0 : b_2 \end{cases}$$

$$O_x: y=0 \Rightarrow b_1 \cap O_x: 2x+17=0 \Leftrightarrow x = -\frac{17}{2} < 0 \Rightarrow \underline{b_1}$$
$$\left(\Rightarrow b_2 \cap O_x: 10x-3=0 \Leftrightarrow x = \frac{3}{10} > 0 \right)$$

Ex 3.2.13

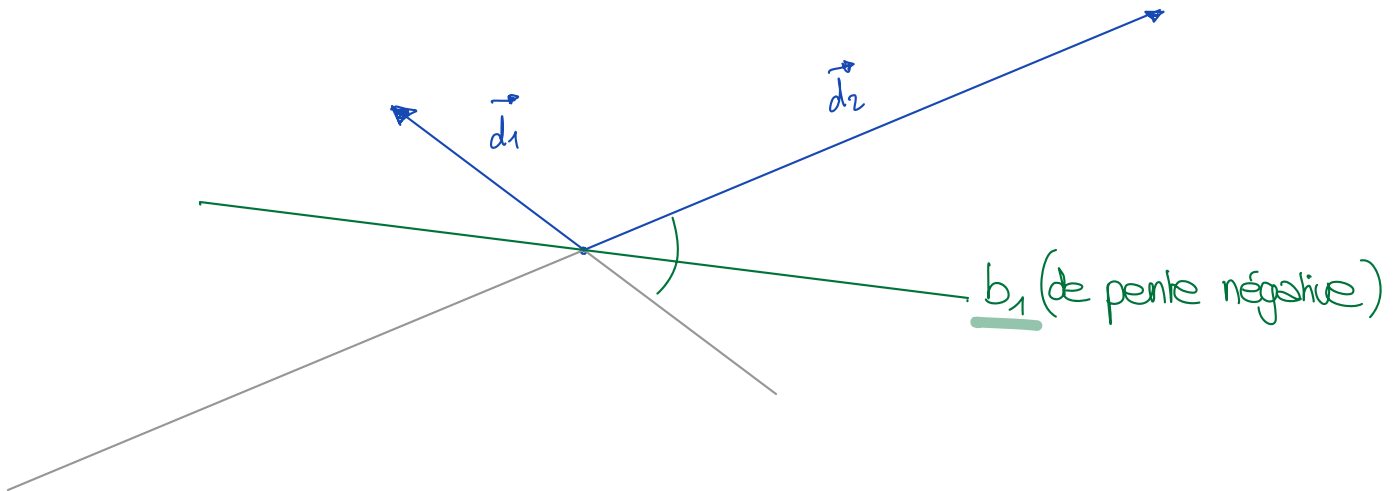
$$\frac{3x+4y-5}{\sqrt{9+16}} = \pm \frac{5x-12y+3}{\sqrt{25+144}} \quad | \cdot 65$$

$$\Leftrightarrow 13(3x+4y-5) = \pm 5(5x-12y+3)$$

$$\Leftrightarrow 39x+52y-65 = \begin{cases} 25x-60y+15 & \Leftrightarrow 14x+112y-80=0 \\ -25x+60y-15 & \Leftrightarrow 64x-8y-50=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \underline{7x+56y-40=0} : b_1 & m_{b_1} = -\frac{1}{8} \\ 32x-4y-25=0 : b_2 & m_{b_2} = 8 \end{cases}$$

Pour déterminer quelle est la bissectrice de l'angle aigu on représente les vecteurs directeurs $\vec{d}_1 = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ et $\vec{d}_2 = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ (suffisant ici)



Ex 3.2.14

$$\frac{x-3y+5}{\sqrt{1+9}} = \pm \frac{3x-y+15}{\sqrt{9+1}} \quad | \cdot \sqrt{10}$$

$$\Leftrightarrow x-3y+5 = \pm (3x-y+15)$$

$$\Leftrightarrow x-3y+5 = \begin{cases} 3x-y+15 & \Leftrightarrow 2x+2y+10=0 \Leftrightarrow \underline{x+y+5=0 : b_1} \\ -3x+y-15 & \Leftrightarrow 4x-4y+20=0 \Leftrightarrow x-y+5=0 : b_2 \end{cases}$$

$$\pi(-1; -4) \in b_1? : -1-4+5=0 \quad \checkmark \quad \text{oui} \quad \Rightarrow \underline{b_1}$$

$$(\pi(-1; -4) \notin b_2 : -1+4+5=8 \neq 0)$$