

Ex 4.2.1

$$U = \{1; 2; 3; 4; 5; 6\} \quad \#U = 6$$

$$a) P = \frac{1}{6} \quad \text{car } E = \{2\} \quad \#E = 1$$

$$b) P = \frac{3}{6} = \frac{1}{2} \quad \text{car } E = \{2; 4; 6\} \quad \#E = 3$$

$$c) P = \frac{2}{6} = \frac{1}{3} \quad \text{car } E = \{5; 6\} \quad \#E = 2$$

Ex 4.2.2

$$\# \text{ issues possibles : } 36 = C_1^{36}$$

$$a) P = \frac{4}{36} = \frac{1}{9} \quad \text{car } E = \{A\heartsuit, A\diamondsuit, A\clubsuit, A\spadesuit\} \quad \#E = 4 \quad \text{ou } C_1^4$$

$$b) P = \frac{9}{36} = \frac{1}{4} \quad \text{car } E = \{A\diamondsuit, R\diamondsuit, \dots, 6\diamondsuit\} \quad \#E = 9 \quad \text{ou } C_1^9$$

$$c) P = \frac{1}{36} \quad \text{car } E = \{V\heartsuit\} \quad \#E = 1 \quad \text{ou } C_1^1$$

Ex 4.2.3

simultanément

$$\# \text{ issues possibles} = C_3^{36} = 7140$$

$$a) \frac{C_3^4}{C_3^{36}} = \frac{4}{7140} = \frac{1}{1785} \approx \underline{0,056\%}$$

$$b) \frac{C_2^4 \cdot C_1^4}{C_3^{36}} = \frac{6 \cdot 4}{7140} = \frac{2}{595} \approx \underline{0,336\%}$$

successivement

$$\# \text{ issues possibles} = A_3^{36} = 36 \cdot 35 \cdot 34$$

$$\frac{A_3^4}{A_3^{36}} = \frac{4 \cdot 3 \cdot 2}{36 \cdot 35 \cdot 34} = \dots$$

$$\frac{A_2^4 \cdot A_1^4 \cdot \overset{\text{place dame}}{3}}{A_3^{36}} = \frac{4 \cdot 3 \cdot 4 \cdot 3}{36 \cdot 35 \cdot 34} = \dots$$

c) au moins 1 valet \Leftrightarrow tout - aucun valet

$$\begin{aligned} 1 - \frac{C_3^{32}}{C_3^{36}} &= 1 - \frac{4960}{7140} \\ &= 1 - \frac{248}{357} = \frac{109}{357} \\ &\cong \underline{30,53\%} \end{aligned}$$

$$\begin{aligned} 1 - \frac{A_3^{32}}{A_3^{36}} &= 1 - \frac{32}{36} \cdot \frac{31}{35} \cdot \frac{30}{34} \\ &= \dots \end{aligned}$$

Ex 4.2.4

$$\# \text{ issues possibles} = 2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2} = \overline{A_4^2} = 16$$

a) $P = \frac{1}{16}$ car # issue favorable = 1 = $\frac{P P F F}{1 \cdot 1 \cdot 1 \cdot 1}$

b) $P = \frac{\overline{P_4(2,2)}}{2^4} = \frac{6}{16} = \frac{3}{8} = \frac{C_2^4}{2^4}$ $\begin{pmatrix} P P F F & P F F P \\ P F P F & F P P F \\ F P P F & F F P P \end{pmatrix}$

c) au plus 1 fois pile \Leftrightarrow 0 fois pile ou 1 fois pile
 $1 + C_1^4 = 1 + 4 = 5$
ou $1 + \overline{P_4(3)}$

$$\Rightarrow P = \frac{5}{16}$$

Ex 4.2.5

$$\# \text{ issues possibles} = 6^2 = 36$$

a) $P = \frac{6}{36} = \frac{1}{6}$ car $E = \{(1,1), (2,2), \dots, (6,6)\}$ #E = 6

b) $P = \frac{2}{36} = \frac{1}{18}$ car $E = \{(2R, 5B), (5R, 2B)\}$ #E = 2

$$c) P = \frac{1}{36} \quad \text{car } E = \{(2R, 5B)\} \quad \# E = 1$$

$$d) E = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\} \quad \# E = 6$$

$$\Rightarrow P = \frac{6}{36} = \frac{1}{6}$$

$$e) E = \{(1,1), (1,2), (2,1)\} \quad \# E = 3 \quad \Rightarrow P = \frac{3}{36} = \frac{1}{12}$$

f) somme au plus égale à 11 \Leftrightarrow somme = 2 ou 3 ou 4 ou ... ou 11

$$\Leftrightarrow \text{tout} - \text{somme} = 12$$

$$\Leftrightarrow 36 - 1 = 35$$

$$\Rightarrow P = \frac{35}{36}$$

Ex 4.2.6

simultanément

$$\frac{C_3^4 \cdot C_{10}^{48}}{C_{13}^{52}} \cong \underline{4,12\%}$$

successivement

$$\frac{A_3^4 \cdot A_{10}^{48} \cdot C_3^{13}}{A_{13}^{52}} \cong 4,12\% \quad \leftarrow \text{nbres de place pour les rois.}$$

Ex 4.2.8

$$\# \text{ issues possibles} = C_3^{36} = 7140$$

$$a) \frac{\overset{1 \text{ couleur}}{C_1^4} \cdot \overset{3 \text{ cartes par couleur}}{C_3^9}}{C_3^{36}} = \frac{336}{7140} = \frac{4}{85} \cong \underline{4,71\%}$$

$$b) \frac{C_3^4}{C_3^{36}} = \frac{4}{7140} = \frac{1}{1785} \cong \underline{0,06\%}$$

$$c) \frac{C_1^4 \cdot C_2^4}{C_3^{36}} = \frac{4 \cdot 6}{7140} = \frac{2}{595} \cong \underline{0,34\%}$$

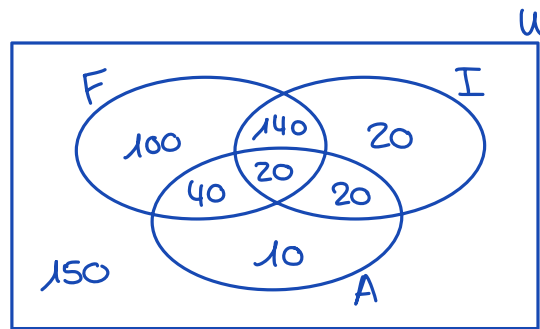
1 couleur 2 cartes 1 carte autre couleur

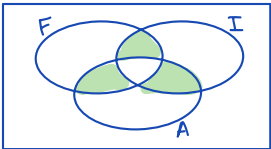
d) $\frac{C_1^4 \cdot C_2^9 \cdot C_1^{27}}{C_3^{36}} = \frac{4 \cdot 36 \cdot 27}{7140} = \frac{324}{595} \approx \underline{54,45\%}$

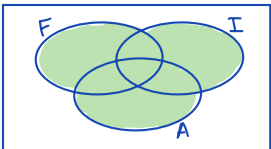
e) $\frac{C_2^{18} \cdot C_1^{18}}{C_3^{36}} = \frac{2754}{7140} = \frac{27}{70} \approx \underline{38,57\%}$

f) $\frac{C_1^4 \cdot C_1^4 \cdot C_1^4}{C_3^{36}} = \frac{4 \cdot 4 \cdot 4}{7140} = \frac{16}{1785} \approx \underline{0,896\%}$

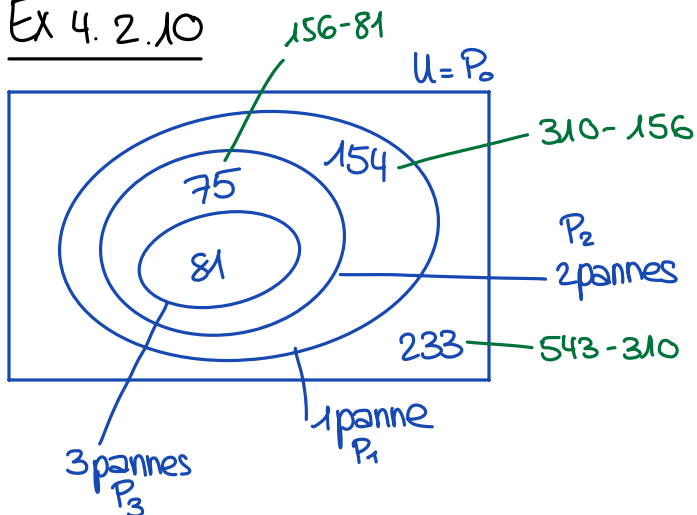
Ex 4.2.9



a)  $P = \frac{140 + 40 + 20}{500} = \frac{200}{500} = \frac{2}{5} = \underline{40\%}$

b)  $P = \frac{300 + 20 + 20 + 10}{500} = \frac{350}{500} = \frac{7}{10} = \underline{70\%}$

Ex 4.2.10



a) $P = \frac{154}{543} \approx \underline{28,36\%}$

b) $P = \frac{154 + 233}{543} = \frac{387}{543} \approx \underline{71,27\%}$

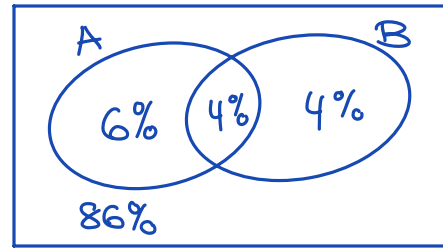
Ex 4.2.14

a) $P(A \cup B) = 6\% + 4\% + 4\% = \underline{14\%}$

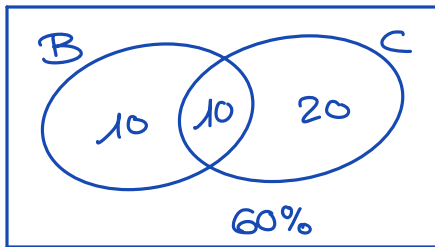
b) $P = \underline{6\%}$

c) $P = 6 + 4 = \underline{10\%}$

d) $P = 100\% - 6\% - 4\% - 4\% = \underline{86\%}$



Ex 4.2.15



$P(\overline{B \cup C}) = 60\% \Rightarrow P(B \cup C) = 100\% - 60\% = \underline{40\%}$

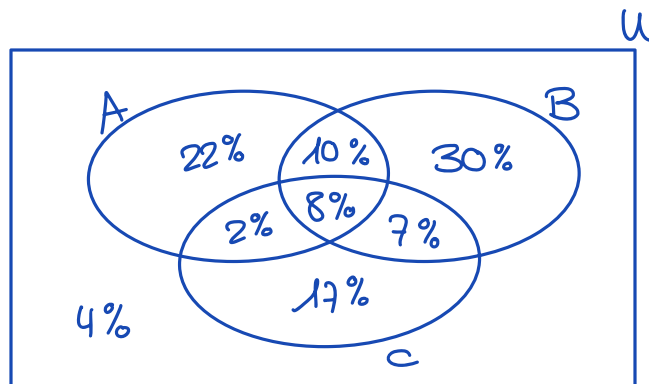
$P(B) = 20\%$ et $P(C) = 30\%$

Comme $P(B \cup C) = P(B) + P(C) - P(B \cap C) \Leftrightarrow 40 = 20 + 30 - P(B \cap C)$

$\Leftrightarrow 40 = 50 - P(B \cap C)$

$\Rightarrow P(B \cap C) = \underline{10\%}$

Ex 4.2.16



a) $P(A \cup B \cup C) = 42\% + 30\% + 7\% + 17\% = \underline{96\%}$

b) $P(\overline{A \cup B \cup C}) = 100\% - 96\% = \underline{4\%}$

c) $P = 10\% + 2\% + 7\% = \underline{19\%}$

d) $P = \underline{22\%}$

Ex 4.3.1

$$\#A = 5 \quad \#B = 30 \quad \#C = 18$$

$$P(A) = \frac{5}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/36}{30/36} = \frac{4}{30} = \frac{2}{15}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{1/36}{6/36} = \frac{1}{6}$$

$$P(A|\bar{C}) = \frac{P(A \cap \bar{C})}{P(\bar{C})} = \frac{3/36}{18/36} = \frac{3}{18} = \frac{1}{6}$$

Ex 4.3.2

$$\#A = 9 \quad \#B = 1 \quad \#C = 12$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/36}{9/36} = \frac{1}{9}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{9/36}{12/36} = \frac{9}{12} = \frac{3}{4}$$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{1/36}{12/36} = \frac{1}{12}$$

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{1/36}{1/36} = 1$$

Ex 4.3.7

$$a) p = \frac{C_2^4}{C_2^8} = \frac{6}{28} = \frac{3}{14} \cong 0,2143 = 21,43\%$$

$$b) p = \frac{C_2^2}{C_2^8} = \frac{1}{28} \cong 0,0357 = 3,57\%$$

$$c) p = \text{"tout-aucun"} = 1 - \frac{C_2^4}{C_2^8} = 1 - \frac{3}{14} = \frac{11}{14} \cong 0,7857 = 78,57\%$$

$$d) i) p = \frac{3/14}{11/14} = \frac{3}{11} \quad \frac{\text{"deux as et au moins un as"}}{\text{"au moins 1 as"}}$$

$$ii) p = \frac{\frac{1}{28} + \frac{C_1^2 C_1^2}{28}}{1 - \frac{C_2^6}{28}} = \frac{5/28}{1 - 15/28} = \frac{5/28}{13/28} = \frac{5}{13}$$

$$\frac{\text{"deux as et au moins un as rouge"}}{\text{"au moins 1 as rouge"}}$$

$$iii) p = \frac{\frac{C_1^1 \cdot C_1^3}{28}}{1 - \frac{C_2^7}{28}} = \frac{3/28}{1 - \frac{21}{28}} = \frac{3/28}{7/28} = \frac{3}{7}$$

$$\frac{\text{"deux as et au moins l'as de coeur"}}{\text{"au moins l'as de coeur"}}$$

Ex 4.3.8

A: somme obtenue sup. à 9

$$\#A = 6$$

B: 1^{er} dé a donné 5

$$\#B = 6$$

C: au moins 1 dé a donné 5

$$\#C = 11$$

$$a) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}$$

$$b) P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{3/36}{11/36} = \frac{3}{11}$$

D: "deux chiffres différents" $\#D = 30$

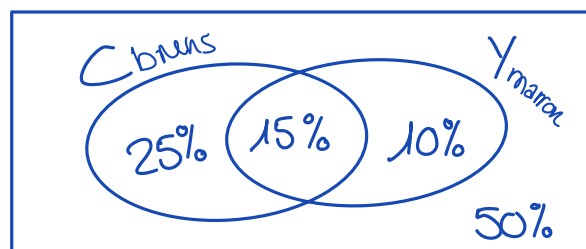
E: " $\Sigma = 6$ " $\#E = 5$

F: " $\Sigma < 5$ " $\#F = 6$

$$c) P(E|D) = \frac{P(E \cap D)}{P(D)} = \frac{4}{30} = \frac{2}{15}$$

$$d) P(F|D) = \frac{P(F \cap D)}{P(D)} = \frac{4}{30} = \frac{2}{15}$$

Ex 4.3.9



C: "cheveux bruns" $P(C) = 40\%$

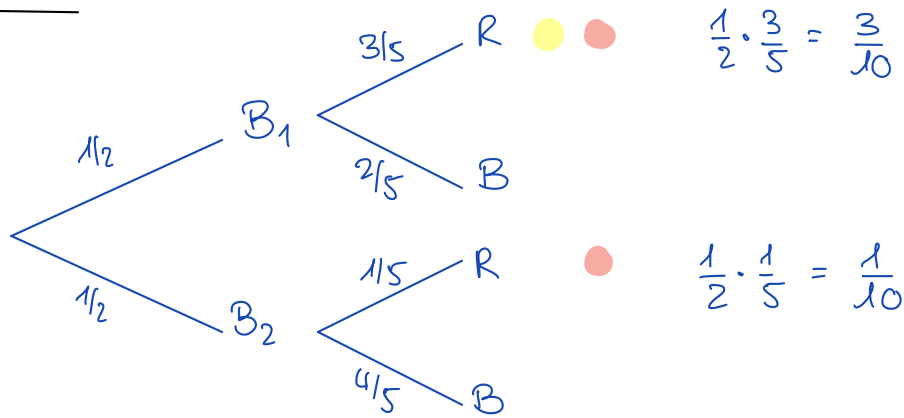
Y: "yeux marron" $P(Y) = 25\%$

$$a) P(Y|C) = \frac{P(Y \cap C)}{P(C)} = \frac{15\%}{40\%} = \frac{15}{40} = \frac{3}{8} = 0,375 = \underline{\underline{37,5\%}}$$

$$b) P(\bar{C}|Y) = \frac{P(\bar{C} \cap Y)}{P(Y)} = \frac{10}{25} = \frac{2}{5} = 0,4 = \underline{\underline{40\%}}$$

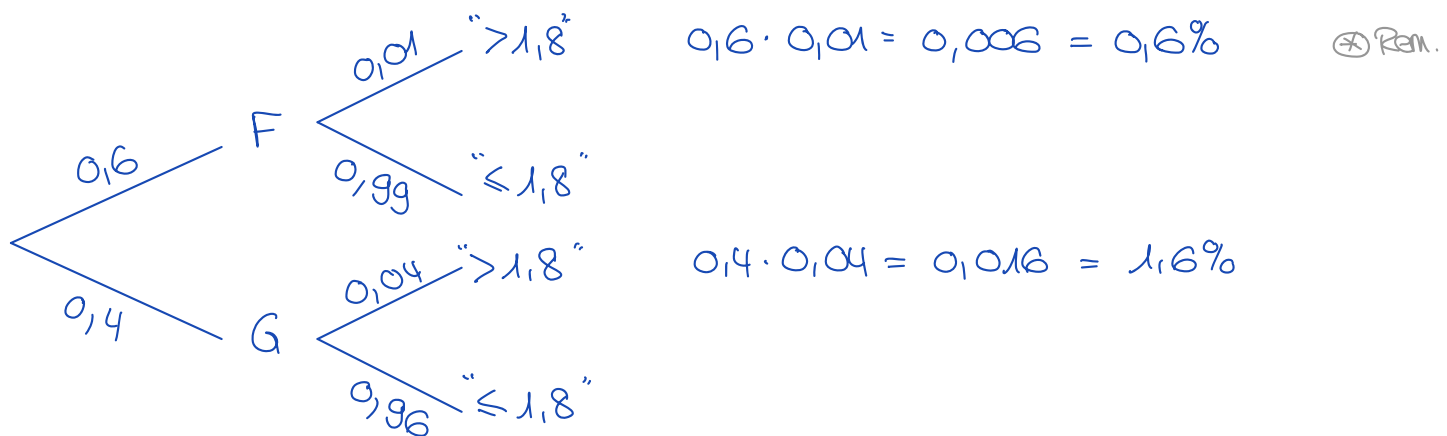
$$c) P(\bar{C} \cap \bar{Y}) = 100\% - 25\% - 15\% - 10\% = \underline{\underline{50\%}}$$

Ex 4.3.11



$$P(B_1|R) = \frac{P(B_1 \cap R)}{P(R)} = \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{10}} = \frac{\frac{3}{10}}{\frac{4}{10}} = \frac{3}{4} = 75\%$$

Ex 4.3.12



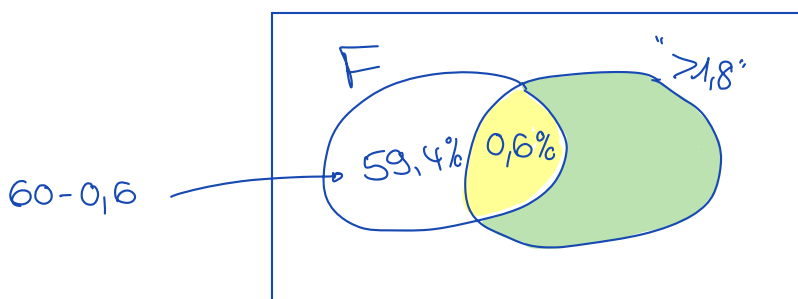
$$P(F|>1,8) = \frac{P(F \cap >1,8)}{P(>1,8)} = \frac{0,6\%}{0,6\% + 1,6\%} = \frac{0,6\%}{2,2\%} = \frac{0,6}{2,2} = \frac{6}{22} = \frac{3}{11} = \underline{\underline{27,27\%}}$$

⊗ Rem: Δ quand on multiplie les probabilités, ne pas le faire en % car le résultat obtenu n'est pas en % mais en ‰

exple : 1^{er} chemin : $60\% \cdot 1\% \neq 60\% = \frac{60}{100}$

mais $60\% \cdot 1\% = \frac{60}{100} \cdot \frac{1}{100} = \frac{60}{10000} = 0,006 = 0,6\% \left(= \frac{0,6}{100} \right)$

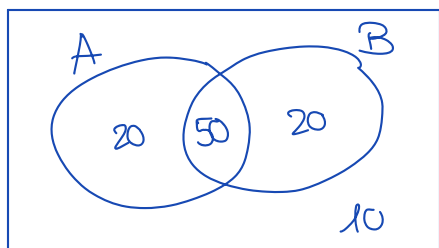
Variante avec diagramme de Venn :



$P(F \cap \text{">1,8"}) = 0,6 \cdot 0,01 = 0,6\%$
 ↑
 thm de multiplic.

$P(G \cap \text{">1,8"}) = 0,4 \cdot 0,04 = 1,6\%$

Ex 4.3.13



A : 1^{er} salle occupée
 B : 2^{er} salle occupée

$P(A) = P(B)$

$P(A \cup B) = 90\%$

$P(A \cap B) = 50\%$

a) $P(\bar{A}) = 20 + 10 = 30\%$

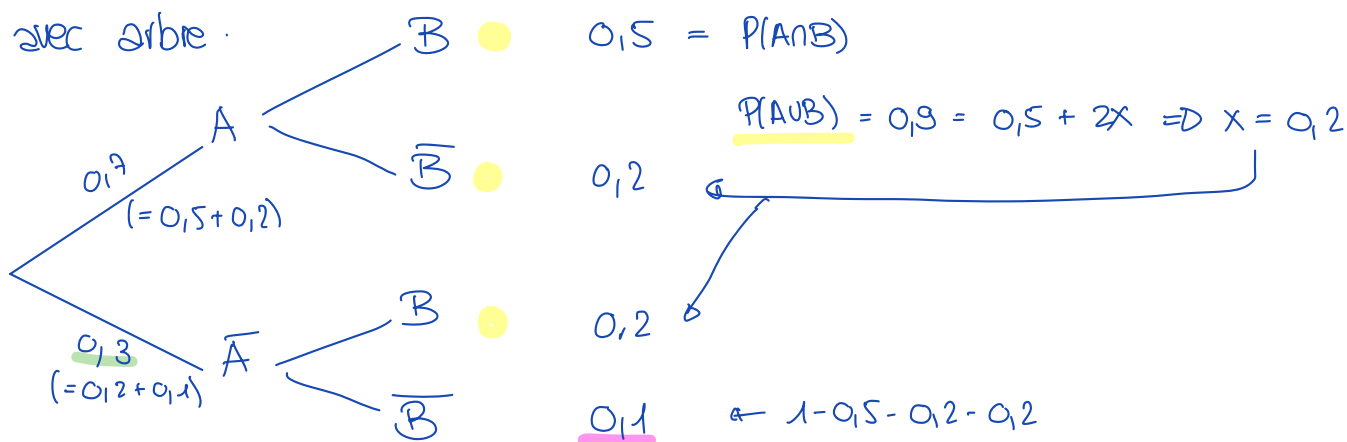
b) $P(\bar{A} \cap \bar{B}) = 100 - 90 = 10\%$

c) $P(\bar{A} \cup \bar{B}) = 20 + 20 + 10 = 50\%$

d) $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 20 + 20 = 40\%$

e) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{20}{70} = \frac{2}{7} \cong 28,6\%$

Variante avec arbre :

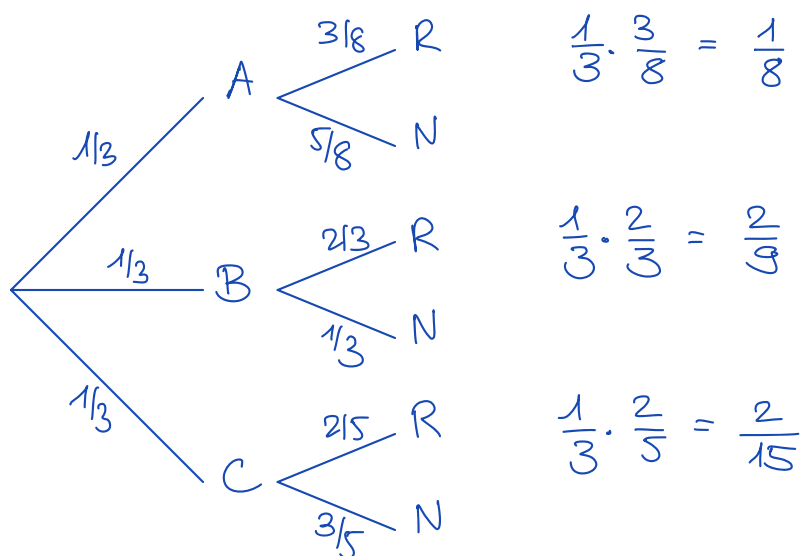


a) $P(\bar{A}) = 0,3$ b) $P(\bar{A} \cap \bar{B}) = 0,1$ c) $P(\bar{A} \cup \bar{B}) = 0,3 + 0,2 = 0,5$

d) $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 0,2 + 0,2 = 0,4$

e) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{20}{70} = \frac{2}{7} \cong 28,6\%$

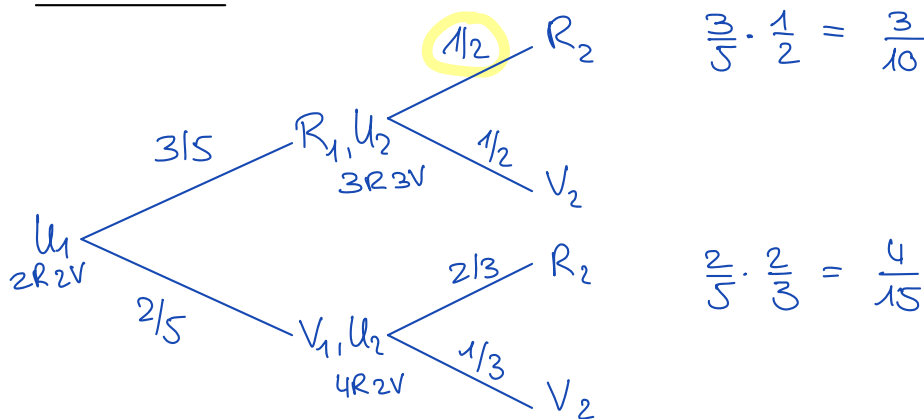
Ex 4.3.14



$$a) P(R) = \frac{1}{8} + \frac{2}{9} + \frac{2}{15} = \frac{173}{360} = \underline{48,05\%}$$

$$b) P(R|A) = \frac{\frac{1}{8}}{\frac{173}{360}} = \frac{1}{8} \cdot \frac{360}{173} = \frac{45}{173} \approx \underline{26,01\%}$$

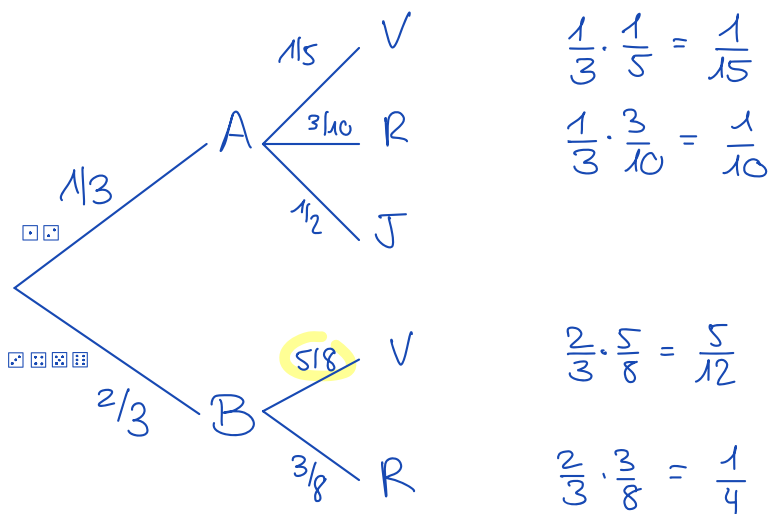
Ex 4.3.15



$$a) P(R_2) = \frac{3}{10} + \frac{4}{15} = \frac{17}{30} = \underline{56,6\%}$$

$$b) P(R_2 | R_1) = \frac{1}{2} = \underline{50\%} \quad (\text{directement de l'arbre}) \quad \text{ou avec def: } \frac{3/10}{3/5} = \frac{1}{2}$$

$$c) P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{3/10}{17/30} = \frac{3}{10} \cdot \frac{30}{17} = \frac{9}{17} \approx \underline{52,94\%}$$

Ex 4.3.17

$$a) P(V) = \frac{1}{15} + \frac{5}{12} = \frac{29}{60} = \underline{\underline{48,3\%}}$$

$$b) P(V|B) = \frac{5}{8} \text{ (direct. ds l'arbre)}$$

$$\text{ou avec déf: } \frac{5/12}{2/3} = \frac{5}{8}$$

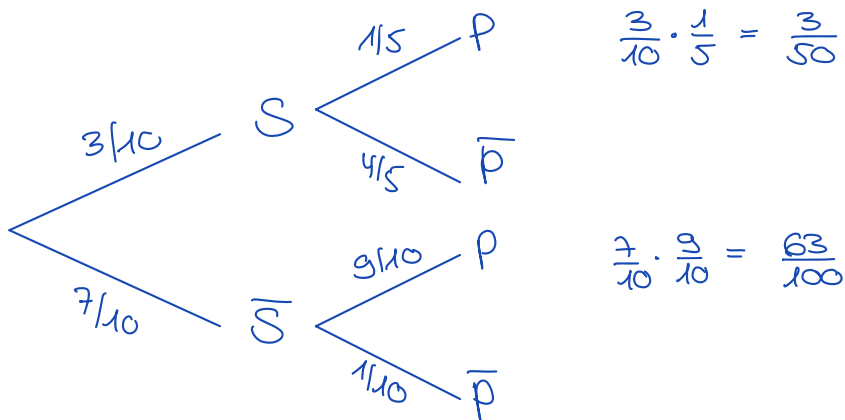
$$c) P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{1/10}{1/10 + 1/4} = \frac{1/10}{7/20} = \frac{1}{10} \cdot \frac{20}{7} = \frac{2}{7} \approx \underline{\underline{28,57\%}}$$

$$d) P(B|J) = 0 \text{ (Événement impossible)}$$

Ex 4.3.18

S: course encadrée

P: être parmi les 10 premiers



$$P(\bar{S}|P) = \frac{P(\bar{S} \cap P)}{P(P)} = \frac{63/100}{63/100 + 3/50} = \frac{63/100}{69/100} = \frac{63}{69} = \frac{21}{23} \approx \underline{\underline{91,3\%}}$$

