

Ex 2.2.1

a) $\rightarrow$ i) et k) l) voir solutions brochures

$$\begin{aligned} \text{j) } \underbrace{(2a+3b)(2x+y)}_{2 \text{ termes}} + \underbrace{(3a+5b)(2x+y)}_{2 \text{ termes}} &= (2x+y) \left[\underbrace{(2a+3b)}_{2 \text{ termes}} + \underbrace{(3a+5b)}_{2 \text{ termes}} \right] \\ &= \underline{(2x+y)(5a+8b)} \end{aligned}$$

$$\begin{aligned} \text{m) } \underbrace{(x-3)(x+1)}_{3 \text{ termes}} + \underbrace{2(x-3)^2}_{3 \text{ termes}} - \underbrace{(x-3)}_{3 \text{ termes}} &= (x-3) \left[\underbrace{(x+1)}_{3 \text{ termes}} + \underbrace{2(x-3)}_{3 \text{ termes}} - \underbrace{1}_{3 \text{ termes}} \right] \\ &= (x-3)(x+1+2x-6-1) \\ &= (x-3) \underbrace{(3x-6)}_{3(x-2)} = \underline{3(x-3)(x-2)} \end{aligned}$$

$$\begin{aligned} \text{n) } \underbrace{(u+v)^3}_{2 \text{ termes}} - \underbrace{(u+v)^2}_{2 \text{ termes}} &= (u+v)^2 \left[\underbrace{(u+v)}_{2 \text{ termes}} - \underbrace{1}_{2 \text{ termes}} \right] = (u+v)^2(u+v-1) \end{aligned}$$

$$\begin{aligned} \text{o) } \underbrace{2a(a-b)}_{2 \text{ termes}} - \underbrace{(a-b)^2}_{2 \text{ termes}} &= (a-b) \left[\underbrace{2a}_{2 \text{ termes}} - \underbrace{(a-b)}_{2 \text{ termes}} \right] = (a-b)(2a-a+b) \\ &= \underline{(a-b)(a+b)} \end{aligned}$$

Ex 2.2.2

a) $\rightarrow$ c) , m) $\rightarrow$ r) voir solutions brochures

$$d) \quad \underline{(a+b)^2 - x^2} = [(a+b)+x][(a+b)-x] = \underline{(a+b+x)(a+b-x)}$$

$$\text{différence de 2 carrés} \quad \underline{A^2 - B^2} = (A+B)(A-B)$$

$$e) \quad \underline{(ax+2y)^2 - (2x-3y)^2} = [(ax+2y)+(2x-3y)][(ax+2y)-(2x-3y)]$$

$$\underline{A^2 - B^2} = (ax+2x-y)(ax+2y-2x+3y)$$

$$= \underline{(ax+2x-y)(ax-2y+5y)}$$

$$f) \quad \underline{(a-b)^2 - 1} = \underline{(a-b+1)(a-b-1)}$$

$$\underline{A^2 - B^2}$$

$$g) \quad 3a^2 - 3 \stackrel{\text{Hee}}{=} 3(a^2 - 1) \stackrel{\text{PR}}{=} \underline{3(a+1)(a-1)}$$

$$h) \quad 4x^5y^2 - 9x^3 \stackrel{\text{Hee}}{=} x^3(4x^2y^2 - 9) \stackrel{\text{PR}}{=} \underline{x^3(2xy+3)(2xy-3)}$$

$$i) \quad a^4 - b^4 \stackrel{\text{PR}}{=} (a^2 + b^2)(a^2 - b^2) \stackrel{\text{PR}}{=} \underline{(a^2 + b^2)(a+b)(a-b)}$$

$$j) \quad a^5 - a \stackrel{\text{Hee}}{=} a(a^4 - 1) \stackrel{\text{PR}}{=} a(a^2 + 1)(a^2 - 1) \stackrel{\text{PR}}{=} \underline{a(a^2 + 1)(a+1)(a-1)}$$

$$k) \quad \frac{u^4}{625} - \frac{v^4}{81} \stackrel{\text{PR}}{=} \left(\frac{u^2}{25} + \frac{v^2}{9}\right)\left(\frac{u^2}{25} - \frac{v^2}{9}\right)$$

$$\stackrel{\text{PR}}{=} \underline{\left(\frac{u^2}{25} + \frac{v^2}{9}\right)\left(\frac{u}{5} + \frac{v}{3}\right)\left(\frac{u}{5} - \frac{v}{3}\right)}$$

$$l) \quad x^5y^4 - x \stackrel{\text{Hee}}{=} x(x^4y^4 - 1) \stackrel{\text{PR}}{=} x(x^2y^2 + 1)(x^2y^2 - 1)$$

$$\stackrel{\text{PR}}{=} \underline{x(x^2y^2 + 1)(xy + 1)(xy - 1)}$$

$$s) \quad (a+b)^2 - 2(a+b)c + c^2 \stackrel{\text{PR}}{=} \underline{(a+b-c)^2}$$

$$\underline{A^2 - 2AB + B^2} = (A-B)^2$$

$$t) \quad 5x^2 - 10x + 5 \stackrel{\text{Hee}}{=} 5(x^2 - 2x + 1) \stackrel{\text{PR}}{=} 5(x-1)^2$$

$$u) \quad \underline{x^2(a+b)} + \underline{2(a+b)x} + \underline{(a+b)} \stackrel{\text{Hee}}{=} (a+b)(\underline{x^2 + 2x + 1}) \stackrel{\text{PR}}{=} \underline{(a+b)(x+1)^2}$$

Ex 2.2.3

a) , c) $\rightarrow$ h) voir solutions brochures

$$b) a^4 - \frac{8ab^3}{27} \stackrel{\text{Hee}}{=} a \left(a^3 - \frac{8b^3}{27} \right) \stackrel{\text{PR}}{=} \underline{a \left(a - \frac{2b}{3} \right) \left(a^2 + \frac{2ab}{3} + \frac{4b^2}{9} \right)}$$

$$\begin{aligned} i) 12a^3 + \frac{9ab^2}{4} + \frac{3b^3}{16} + 3a^2b &\stackrel{\text{Hee}}{=} 12 \left(a^3 + \frac{3ab^2}{4 \cdot 12_4} + \frac{3b^3}{16 \cdot 12_4} + \frac{3a^2b}{12_4} \right) \\ &= 12 \left(a^3 + \frac{3ab^2}{16} + \frac{b^3}{64} + \frac{3a^2b}{4} \right) \\ &= 12 \left(a^3 + 3a^2 \frac{b}{4} + 3a \frac{b^2}{16} + \frac{b^3}{64} \right) \\ &\stackrel{\text{PR}}{=} \underline{12 \left(a + \frac{b}{4} \right)^3} \end{aligned}$$

Ex 2.2.4

a) $\rightarrow$ d) g) j) n) voir solutions brochures

$$e) 9x^2 + 6x + 1 \stackrel{\text{PR}}{=} \underline{(3x+1)^2} \quad (\Delta = 0)$$

$$f) 4z^2 + 5z + 1 = 4 \left(z + \frac{1}{4} \right) (z+1) = \underline{(4z+1)(z+1)}$$

$$\Delta = 25 - 16 = 9$$

$$z_{1,2} = \frac{-5 \pm 3}{8} = \begin{cases} -\frac{2}{8} = -\frac{1}{4} \\ -\frac{8}{8} = -1 \end{cases}$$

$$\begin{aligned} h) 3y^2 + 7y + 3 &= 3 \left(y - \frac{-7+\sqrt{13}}{6} \right) \left(y + \frac{-7-\sqrt{13}}{6} \right) \\ \Delta &= 49 - 36 = 13 \\ y_{1,2} &= \frac{-7 \pm \sqrt{13}}{6} \\ &= 3 \left(y + \frac{7-\sqrt{13}}{6} \right) \left(y - \frac{7+\sqrt{13}}{6} \right) \end{aligned}$$

$$i) 6x^2 + 5x + 1 = 6 \left(x + \frac{1}{2} \right) \left(x + \frac{1}{3} \right) = \underline{(2x+1)(3x+1)}$$

$$\Delta = 25 - 24 = 1$$

$$x_{1,2} = \frac{-5 \pm 1}{12} = \begin{cases} -\frac{6}{12} = -\frac{1}{2} \\ -\frac{4}{12} = -\frac{1}{3} \end{cases}$$

k) $x^2 + x + 1$ pas factorisable

$$\Delta = 1 - 4 = -3 < 0$$

l) $16u^2 - 72u + 81 \stackrel{PR}{=} (4u - 9)^2 \quad (\Delta = 0)$

m) $40x^2 + 3x - 28 = 40\left(x - \frac{4}{5}\right)\left(x + \frac{7}{8}\right) = \underline{(5x - 4)(8x + 7)}$

$$\Delta = 9 + 4 \cdot 40 \cdot 28 = 4489 = 67^2$$

$$x_{1,2} = \frac{-3 \pm 67}{80} = \begin{cases} \frac{64}{80} = \frac{4}{5} \\ -\frac{70}{80} = -\frac{7}{8} \end{cases}$$

o) $2x^2 - 5x - 2 = 2\left(x - \frac{5 + \sqrt{41}}{4}\right)\left(x - \frac{5 - \sqrt{41}}{4}\right)$

$$\Delta = 25 + 16 = 41$$

$$x_{1,2} = \frac{5 \pm \sqrt{41}}{4}$$

p) $4m^2 + 25m - 21 = 4\left(m - \frac{3}{4}\right)(m + 7) = \underline{(4m - 3)(m + 7)}$

$$\Delta = 625 + 4 \cdot 4 \cdot 21 = 961 = 31^2$$

$$m_{1,2} = \frac{-25 \pm 31}{8} = \begin{cases} \frac{6}{8} = \frac{3}{4} \\ -7 \end{cases}$$

Ex 2.2.5

a) $x^4 - 13x^2 + 36 \stackrel{SP}{=} (x^2 - 9)(x^2 - 4) \stackrel{PR}{=} \underline{(x+3)(x-3)(x+2)(x-2)}$

b) $a^6 + 19a^3 - 216 \stackrel{SP}{=} (a^3 + 27)(a^3 - 8) \stackrel{PR}{=} \underline{(a+3)(a^2 - 3a + 9)(a-2)(a^2 + 2a + 4)}$

Variante : $\Delta = 1225 = 35^2$

$$a_{1,2} = \frac{-19 \pm 35}{2} = \begin{cases} 8 \\ -27 \end{cases}$$

c) $x^8 - 257x^4 + 256 \stackrel{SP}{=} (x^4 - 1)(x^4 - 256) \stackrel{PR}{=} (x^2 + 1)(x^2 - 1)(x^2 + 16)(x^2 - 16)$

$$= \underline{(x^2 + 1)(x + 1)(x - 1)(x^2 + 16)(x + 4)(x - 4)}$$

d) $7x^4 - 61x^2 - 18 \stackrel{\Delta}{=} 7\left(x^2 - 9\right)\left(x^2 + \frac{2}{7}\right) \stackrel{PR}{=} \underline{(x+3)(x-3)(7x^2 + 2)}$

$$\Delta = 4225 = 65^2$$

$$x_{1,2}^2 = \frac{61 \pm 65}{14} = \begin{cases} 9 \\ -\frac{4}{14} = -\frac{2}{7} \end{cases}$$

$$e) \quad 64x^6 - 91x^3 + 27 \stackrel{\Delta}{=} 64(x^3-1)(x^3-\frac{27}{64}) = (x^3-1)(64x^3-27)$$

$$\Delta = 1369 = 37^2$$

$$x_{1,2}^3 = \frac{91 \pm 37}{128} = \begin{cases} 1 \\ \frac{54}{128} = \frac{27}{64} \end{cases}$$

$$\stackrel{PR}{=} (x-1)(x^2+x+1)(4x-3)(16x^2+12x+9)$$

$$f) \quad 6x^4 + 7x^2 - 3 \stackrel{\Delta}{=} 6(x^2 + \frac{3}{2})(x^2 - \frac{1}{3}) = (2x^2+3)(3x^2-1)$$

$$\Delta = 121$$

$$x_{1,2}^2 = \frac{-7 \pm 11}{12} = \begin{cases} -\frac{18}{12} = -\frac{3}{2} \\ \frac{4}{12} = \frac{1}{3} \end{cases}$$

$$\stackrel{PR}{=} (2x^2+3)(\sqrt{3}x+1)(\sqrt{3}x-1)$$

$$g) \quad 16x^8 - 641x^4 + 625$$

$$\stackrel{\Delta}{=} 16(x^4 - \frac{625}{16})(x^4 - 1)$$

$$= (16x^4 - 625)(x^4 - 1)$$

$$\stackrel{PR}{=} (4x^2 + 25)(4x^2 - 25)(x^2 + 1)(x^2 - 1)$$

$$\stackrel{PR}{=} (4x^2 + 25)(2x+5)(2x-5)(x^2+1)(x+1)(x-1)$$

$$\Delta = 370'881 = 609^2$$

$$x_{1,2}^4 = \frac{641 \pm 609}{32} = \begin{cases} \frac{1250}{32} = \frac{625}{16} \\ 1 \end{cases}$$

$$h) \quad 81z^4 + 80z^2 - 1 \stackrel{\Delta}{=} 81(z^2+1)(z^2-\frac{1}{81}) = (z^2+1)(81z^2-1)$$

$$\Delta = 6724 = 82^2$$

$$z_{1,2}^2 = \frac{-80 \pm 82}{162} = \begin{cases} -1 \\ \frac{1}{81} \end{cases}$$

$$\stackrel{PR}{=} (z^2+1)(9z+1)(9z-1)$$

Ex 2.2.6

$$a) \quad \underline{ax+bx} + \underline{ay+by} = x(a+b) + y(a+b) = \underline{(a+b)(x+y)}$$

$$b) \quad \underline{a+b} + \underline{ax+bx} + \underline{ay+by} = 1(a+b) + x(a+b) + y(a+b) = \underline{(a+b)(1+x+y)}$$

$$\text{ou } \underline{a+b} + \underline{ax+bx} + \underline{ay+by} = a(1+x+y) + b(1+x+y) = \underline{(1+x+y)(a+b)}$$

$$c) \quad \underline{ax-bx} - \underline{ay+by} = x(a-b) - y(a-b) = \underline{(a-b)(x-y)}$$

$$d) \quad \underline{ax-4x} + \underline{4y-ay} = x(a-4) + y(4-a) = x(a-4) - y(-4+a) \\ = x(a-4) - y(a-4) = \underline{(a-4)(x-y)}$$

$$e) \quad \underline{ax+x} - \underline{a-1} = x(a+1) - 1(a+1) = \underline{(a+1)(x-1)}$$

$$f) \quad \underline{x^3+x} - \underline{x^2-1} = x(x^2+1) - 1(x^2+1) = \underline{(x^2+1)(x-1)}$$

↑
pas factorisable (somme de deux carrés)

$$g) \quad \underline{\frac{xy}{2} - \frac{x}{4}} + \underline{\frac{yz}{3} - \frac{z}{6}} = \frac{x}{2} \left(y - \frac{1}{2}\right) + \frac{z}{3} \left(y - \frac{1}{2}\right) = \underline{\left(y - \frac{1}{2}\right) \left(\frac{x}{2} + \frac{z}{3}\right)}$$

$$h) \quad \underline{10xz-10z} - \underline{x^2+x} = 10z(x-1) - x(x-1) = \underline{(x-1)(10z-x)}$$

$$i) \quad (\underline{a^2-2ab+b^2})-1 = (a-b)^2-1 \stackrel{\text{P.R.}}{=} [(a-b)+1][(a-b)-1] = \underline{(a-b+1)(a-b-1)}$$

$$j) \quad \underline{4x^2+2x-9y^2-3y} = (4x^2-9y^2) + (2x-3y) \stackrel{\text{P.R.}}{=} (2x+3y)(2x-3y) + (2x-3y) \\ = \underline{(2x-3y)(2x+3y+1)}$$

$$k) \quad \underline{1+x+x^2} + \underline{x^3+x^4+x^5} = 1(1+x+x^2) + x^3(1+x+x^2) = (1+x+x^2)(1+x^3) \\ \stackrel{\text{P.R.}}{=} \underline{(1+x+x^2)(1+x)(1-x+x^2)}$$

$$l) \quad \underline{8y^4-8y^3} + \underline{y-1} = 8y^3(y-1) + 1(y-1) = (y-1)(8y^3+1) \stackrel{\text{P.R.}}{=} \underline{(y-1)(2y+1)(4y^2-2y+1)}$$

$$m) \quad \underline{x^3+x} - \underline{x^2-1} = x(x^2+1) - 1(x^2+1) = \underline{(x^2+1)(x-1)}$$

$$n) \quad \underline{2a^4-3-2a^3+3a} = a(2a^3+3) - 1(2a^3+3) = \underline{(2a^3+3)(a-1)}$$

$$o) \quad \underline{6x^2+xy} + \underline{18xz+3yz} = x(6x+y) + 3z(6x+y) = \underline{(6x+y)(x+3z)}$$