

Ex 2.5.2

Résoudre $2x^2 + 7x - 15 = 0$

$$\Delta = 49 - 4 \cdot 2 \cdot (-15) = 49 + 120 = 169$$

$$x_{1,2} = \frac{-7 \pm 13}{4} = \begin{matrix} \frac{3}{2} \\ -5 \end{matrix} \Rightarrow \underline{S = \left\{ -5; \frac{3}{2} \right\}}$$

Factoriser $2x^2 + 7x - 15 = 2\left(x - \frac{3}{2}\right)(x + 5) = \underline{(2x - 3)(x + 5)}$

a) $2x^2 - 7x - 4$

résoudre : $2x^2 - 7x - 4 = 0$ $\Delta = 49 - 4 \cdot 2 \cdot (-4) = 81$

$$x_{1,2} = \frac{7 \pm 9}{4} = \begin{matrix} 4 \\ -\frac{1}{2} \end{matrix} \Rightarrow \underline{S = \left\{ -\frac{1}{2}; 4 \right\}}$$

factoriser : $2x^2 - 7x - 4 = 2\left(x - 4\right)\left(x + \frac{1}{2}\right) = \underline{(x - 4)(2x + 1)}$

c) $6x^2 - 25x - 25$

résoudre : $6x^2 - 25x - 25 = 0$ $\Delta = 25^2 - 4 \cdot 6 \cdot (-25) = 1225 = 35^2$

$$x_{1,2} = \frac{25 \pm 35}{12} = \begin{matrix} 5 \\ -\frac{10}{12} = -\frac{5}{6} \end{matrix} \Rightarrow \underline{S = \left\{ -\frac{5}{6}; 5 \right\}}$$

factoriser : $6x^2 - 25x - 25 = 6\left(x - 5\right)\left(x + \frac{5}{6}\right) = \underline{(x - 5)(6x + 5)}$

d) $6x^2 - 20x + 20$

résoudre : $6x^2 - 20x + 20 = 0$ $\Delta = 20^2 - 4 \cdot 6 \cdot 20 = -80 < 0$

factoriser : $6x^2 - 20x + 20$ n'est pas factorisable $\Rightarrow \underline{S = \emptyset}$

e) $12x^2 + 23x - 24$

résoudre : $12x^2 + 23x - 24 = 0$

$\Delta = 23^2 - 4 \cdot 12 \cdot (-24) = 1681 = 41^2$

$$x_{1,2} = \frac{-23 \pm 41}{24} = \begin{cases} \frac{18}{24} = \frac{3}{4} \\ -\frac{64}{24} = -\frac{8}{3} \end{cases} \Rightarrow \underline{S = \left\{ \frac{3}{4}; -\frac{8}{3} \right\}}$$

factoriser : $12x^2 + 23x - 24 = \underbrace{12}_{4 \cdot 3} \left(x - \frac{3}{4} \right) \left(x + \frac{8}{3} \right) = \underline{(4x-3)(3x+8)}$

f) $5x^2 + \frac{29}{3}x - \frac{14}{3}$

résoudre : $5x^2 + \frac{29}{3}x - \frac{14}{3} = 0 \quad | \cdot 3$

$15x^2 + 29x - 14 = 0$

$\Delta = 29^2 - 4 \cdot 15 \cdot (-14) = 1681 = 41^2$

$$x_{1,2} = \frac{-29 \pm 41}{30} = \begin{cases} -\frac{7}{3} \\ \frac{12}{30} = \frac{2}{5} \end{cases} \Rightarrow \underline{S = \left\{ -3; \frac{2}{5} \right\}}$$

factoriser : $5x^2 + \frac{29}{3}x - \frac{14}{3} = 5 \left(x - \frac{7}{3} \right) \left(x + \frac{2}{5} \right) = \underline{(5x-7)(x+\frac{2}{5})}$

Ex 2.5.3

$$b) \quad 4x^2 - 1 = 0 \quad \Leftrightarrow \quad (2x+1)(2x-1) = 0 \quad \Rightarrow \quad \underline{S = \left\{ -\frac{1}{2}; \frac{1}{2} \right\}}$$

$$d) \quad (x^2 - x - 6)(x+5) = 0 \quad \Leftrightarrow \quad (x-3)(x+2)(x+5) = 0 \quad \Rightarrow \quad \underline{S = \{-5; -2; 3\}}$$

$$f) \quad (x-1)(\underbrace{x^2+1}_{\Delta < 0}) = 0 \quad \Rightarrow \quad \underline{S = \{1\}}$$

(somme de 2 carrés)

$$j) \quad x^3 + 2x^2 - x - 2 = 0$$

$$\Leftrightarrow x^2(x+2) - (x+2) = 0$$

$$\Leftrightarrow (x+2)(x^2-1) = 0 \quad \Leftrightarrow \quad (x+2)(x+1)(x-1) = 0 \quad \Rightarrow \quad \underline{S = \{-2; -1; 1\}}$$

Ex 2.5.4

$$a) \quad (x^2 - 8x + 12)(x+2)^3 = 0 \quad \Leftrightarrow \quad (x-6)(x-2)(x+2)^3 = 0 \quad \Rightarrow \quad \underline{S = \{-2; 2; 6\}}$$

$$c) \quad x^3 + 2x^2 - 4x = 8$$

$$\Leftrightarrow x^3 + 2x^2 - 4x - 8 = 0$$

$$\Leftrightarrow x^2(x+2) - 4(x+2) = 0$$

$$\Leftrightarrow (x+2)(x^2-4) = 0 \quad \Leftrightarrow \quad (x+2)^2(x-2) = 0 \quad \Rightarrow \quad \underline{S = \{-2; 2\}}$$

$$g) \quad x^3 + 3x^2 = 9x + 27$$

$$\Leftrightarrow x^3 + 3x^2 - 9x - 27 = 0$$

$$\Leftrightarrow x^2(x+3) - 9(x+3) = 0$$

$$\Leftrightarrow (x+3)(x^2-9) = 0 \quad \Leftrightarrow \quad (x+3)^2(x-3) = 0 \quad \Rightarrow \quad \underline{S = \{-3; 3\}}$$

Ex 2.5.6

$$\begin{aligned} \text{a)} \quad x^4 - 13x^2 + 36 &= 0 \quad \Leftrightarrow (x^2 - 9)(x^2 - 4) = 0 \\ &\Leftrightarrow (x+3)(x-3)(x+2)(x-2) = 0 \quad \Rightarrow \underline{S = \{-3; -2; 2; 3\}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad x^4 - 1 &= 0 \quad \Leftrightarrow \underbrace{(x^2 + 1)}_{\Delta < 0} (x+1)(x-1) = 0 \quad \Rightarrow \underline{S = \{\pm 1\}} \\ &\quad (\Sigma \text{ de 2 casés}) \end{aligned}$$

$$\begin{aligned} \text{c)} \quad x^4 + 2x^2 + 1 &= 0 \quad \Leftrightarrow \underbrace{(x^2 + 1)^2}_{\Delta < 0} = 0 \quad \Rightarrow \underline{S = \emptyset} \\ &\quad (\Sigma \text{ de 2 casés}) \end{aligned}$$

$$\begin{aligned} \text{d)} \quad x^6 - 7x^3 - 8 &= 0 \quad \Leftrightarrow (x^3 - 8)(x^3 + 1) = 0 \\ &\Leftrightarrow (x-2) \underbrace{(x^2 + 2x + 4)}_{\Delta < 0} (x+1) \underbrace{(x^2 - x + 1)}_{\Delta < 0} = 0 \quad \Rightarrow \underline{S = \{-1; 2\}} \end{aligned}$$

Ex 2.5.7

$$\text{b)} \quad \frac{3x-7}{5} + \frac{x^2-9}{7} = 2 \quad | \cdot 35$$

$$\Leftrightarrow 21x - 49 + 5x^2 - 45 = 70$$

$$\Leftrightarrow 5x^2 + 21x - 164 = 0 \quad \Delta = 3721 = 61^2$$

$$\Rightarrow x_{1,2} = \frac{-21 \pm 61}{10} = \begin{matrix} < -\frac{41}{5} \\ > 4 \end{matrix} \quad \Rightarrow \underline{S = \{-\frac{41}{5}; 4\}}$$

$$\text{d)} \quad \frac{x^2-3}{2} - \frac{x^2+1}{3} = \frac{x^2-11}{6} \quad | \cdot 6$$

$$\Leftrightarrow 3x^2 - 9 - 2x^2 - 2 = x^2 - 11$$

$$\Leftrightarrow 0 = 0 \quad \checkmark \text{ tjs vrai } \forall x \in \mathbb{R} \quad \Rightarrow \underline{S = \mathbb{R}}$$

$$e) \quad \frac{3x+1}{8} - \frac{x^2+5}{4} = \frac{55}{2} \quad | \cdot 8$$

$$\Leftrightarrow 3x+1 - 2x^2-10 = 220$$

$$\Leftrightarrow 2x^2 - 3x + 229 = 0 \quad \Delta = -1823 \quad \Rightarrow \underline{S = \emptyset}$$

$$j) \quad \frac{5-4x}{2} + \frac{3x^2-1}{3} = \frac{2x^2+5}{6} \quad | \cdot 6$$

$$\Leftrightarrow 15-12x+6x^2-2 = 2x^2+5$$

$$\Leftrightarrow 4x^2-12x+8 = 0 \quad | :4$$

$$\Leftrightarrow x^2-3x+2 = 0 \quad \Leftrightarrow (x-2)(x-1) = 0 \quad \Rightarrow \underline{S = \{1, 2\}}$$

Ex 2.5.11

$$a) \frac{x-1}{2x-1} = \frac{3x-5}{\underbrace{4x-2}_{2(2x-1)}} \quad | \cdot 2(2x-1) \quad ED = \mathbb{R} - \left\{ \frac{1}{2} \right\}$$

$$2x-2 = 3x-5$$

$$x = 3 \in ED \Rightarrow \underline{S = \{3\}}$$

$$b) \frac{x^2+x+1}{2x+2} = x \quad | \cdot 2x+2 \quad ED = \mathbb{R} - \{-1\}$$

$$x^2+x+1 = 2x^2+2x$$

$$x^2+x-1 = 0 \quad \Delta = 5$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \in ED \Rightarrow \underline{S = \left\{ \frac{-1 \pm \sqrt{5}}{2} \right\}}$$

$$c) \frac{1}{x+1} + \frac{1}{x+3} + \frac{3}{4} = 0 \quad | \cdot 4(x+1)(x+3) \quad ED = \mathbb{R} - \{-1, -3\}$$

$$4x+12 + 4x+4 + 3(x+1)(x+3) = 0$$

$$8x+16 + 3x^2+12x+9 = 0$$

$$3x^2+20x+25 = 0 \quad \Delta = 100$$

$$x_{1,2} = \frac{-20 \pm 10}{6} = \begin{cases} -5 \in ED \\ -\frac{5}{3} \in ED \end{cases} \Rightarrow \underline{S = \left\{ -5, -\frac{5}{3} \right\}}$$

$$f) \frac{x}{x-6} - \frac{1}{2} = \frac{x}{6} + \frac{x+6}{\underbrace{6-x}_{-(x-6)}} \quad | \cdot 6(x-6) \quad ED = \mathbb{R} - \{6\}$$

$$6x - 3(x-6) = x(x-6) - 6(x+6)$$

$$6x - 3x + 18 = x^2 - 6x - 6x - 36$$

$$x^2 - 15x - 54 = 0$$

$$(x-18)(x+3) = 0$$

$$\begin{matrix} \downarrow & \downarrow \\ 18 & -3 \end{matrix} \in ED \Rightarrow \underline{S = \{-3, 18\}}$$

Ex 2.5.12

$$e) \frac{z}{z-3} - \frac{2}{\underbrace{2-z}_{-(z-2)}} = \frac{3}{\underbrace{z^2-5z+6}_{(z-3)(z-2)}}$$

$$| \cdot (z-3)(z-2)$$

$$ED = \mathbb{R} - \{2; 3\}$$

$$z(z-2) + 2(z-3) = 3$$

$$z^2 - 2z + 2z - 6 = 3$$

$$z^2 - 9 = 0$$

$$(z+3)(z-3) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \\ -3 & 3 & \notin ED \\ \in ED & & \end{array}$$

$$\Rightarrow \underline{S = \{-3\}}$$

$$h) \frac{x+4}{x} - \frac{1}{x+4} = \frac{4}{\underbrace{x^2+4x}_{x(x+4)}}$$

$$| \cdot x(x+4)$$

$$ED = \mathbb{R}^* - \{-4\}$$

$$(x+4)^2 - x = 4$$

$$x^2 + 8x + 16 - x = 4$$

$$x^2 + 7x + 12 = 0$$

$$(x+4)(x+3) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \\ -4 & -3 & \in ED \\ \notin ED & & \end{array}$$

$$\Rightarrow \underline{S = \{-3\}}$$

$$i) \frac{1}{\underbrace{x^2-x}_{x(x-1)}} + \frac{5}{\underbrace{x^2+x}_{x(x+1)}} = \frac{4}{\underbrace{x^2-1}_{(x+1)(x-1)}}$$

$$| \cdot x(x+1)(x-1)$$

$$ED = \mathbb{R}^* - \{-1; 1\}$$

$$x+1 + 5(x-1) = 4x$$

$$x+1 + 5x-5 = 4x$$

$$2x-4 = 0$$

$$x = 2 \in ED$$

$$\Rightarrow \underline{S = \{2\}}$$

Ex 2.5.13

$$a) \quad 1 = \frac{3x}{\underbrace{x^2-9}_{(x+3)(x-3)}} - \frac{x}{\underbrace{2x-6}_{2(x-3)}} \quad | \cdot 2(x+3)(x-3) \quad ED = \mathbb{R} - \{\pm 3\}$$

$$2(x+3)(x-3) = 3x \cdot 2 - x(x+3)$$

$$2x^2 - 18 = 6x - x^2 - 3x$$

$$3x^2 - 3x - 18 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

↓

3
∉ ED

↓

-2 ∈ ED

$$\Rightarrow \underline{S = \{-2\}}$$

$$c) \quad \frac{2-x}{x+1} - \frac{5}{3} = \frac{2x+1}{3-2x} \quad | \cdot 3(x+1)(3-2x) \quad ED = \mathbb{R} - \{-1, \frac{3}{2}\}$$

$$3(2-x)(3-2x) - 5(x+1)(3-2x) = 3(2x+1)(x+1)$$

$$6x^2 - 21x + 18 + 10x^2 - 5x - 15 = 6x^2 + 9x + 3$$

$$10x^2 - 35x = 0$$

$$5x(2x-7) = 0$$

↓

0

∈ ED

↓

$\frac{7}{2}$

∈ ED

$$\Rightarrow \underline{S = \{0, \frac{7}{2}\}}$$

$$d) \quad \frac{10x-2}{\underbrace{6x-3}_{3(2x-1)}} + \frac{3x+5}{\underbrace{4x^2-1}_{(2x-1)(2x+1)}} = \frac{x-1}{2x+1} \quad | \cdot 3(2x-1)(2x+1) \quad ED = \mathbb{R} - \{\pm \frac{1}{2}\}$$

$$(10x-2)(2x+1) + 3(3x+5) = 3(x-1)(2x-1)$$

$$20x^2 + 6x - 2 + 9x + 15 = 6x^2 - 9x + 3$$

$$14x^2 + 24x + 10 = 0$$

$$7x^2 + 12x + 5 = 0$$

$$\Delta = 4$$

$$x_{1,2} = \frac{-12 \pm 2}{14} = \begin{cases} -1 & \in ED \\ -\frac{5}{7} & \in ED \end{cases}$$

$$\Rightarrow \underline{S = \{-1, -\frac{5}{7}\}}$$

Ex 2.5.14

$$a) \sqrt{7-x} = x-5 \quad | ()^2$$

$$7-x = x^2 - 10x + 25$$

$$x^2 - 9x + 18 = 0$$

$$(x-3)(x-6) = 0$$

$$x = \begin{cases} 3 \\ 6 \end{cases} \quad \text{vérif: } \begin{array}{l} \sqrt{4} \stackrel{?}{=} -2 \Leftrightarrow 2 \neq -2 \quad \text{X} \\ \sqrt{1} \stackrel{?}{=} 1 \Leftrightarrow 1 = 1 \quad \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{6\}}$$

$$b) \quad x = 4 + \sqrt{4x-19}$$

$$x-4 = \sqrt{4x-19} \quad | ()^2$$

$$x^2 - 8x + 16 = 4x - 19$$

$$x^2 - 12x + 35 = 0$$

$$(x-5)(x-7) = 0$$

$$x = \begin{cases} 5 \\ 7 \end{cases} \quad \text{vérif: } \begin{array}{l} 5 \stackrel{?}{=} 4 + \sqrt{1} \Leftrightarrow 5 = 5 \quad \checkmark \\ 7 \stackrel{?}{=} 4 + \sqrt{9} \Leftrightarrow 7 = 7 \quad \checkmark \end{array}$$

$$\Rightarrow \underline{S = \{5, 7\}}$$

$$c) \quad \sqrt{x+1} - x = x+2$$

$$\sqrt{x+1} = 2x+2 \quad | ()^2$$

$$x+1 = 4x^2 + 8x + 4$$

$$4x^2 + 7x + 3 = 0$$

$$\Delta = 1$$

$$x_{1,2} = \frac{-7 \pm 1}{8} = \begin{cases} -1 \\ -\frac{3}{4} \end{cases}$$

$$\text{vérif: } \sqrt{0} + 1 \stackrel{?}{=} 1 \quad \checkmark$$

$$\sqrt{\frac{1}{4}} + \frac{3}{4} \stackrel{?}{=} \frac{5}{4}$$

$$\frac{1}{2} + \frac{3}{4} \stackrel{?}{=} \frac{5}{4} \quad \checkmark$$

$$\Rightarrow \underline{S = \{-1, -\frac{3}{4}\}}$$

$$e) \quad \sqrt{7-2x} - \sqrt{5+x} = \sqrt{4+3x} \quad | ()^2$$

$$7-2x - 2\sqrt{7-2x}\sqrt{5+x} + 5+x = 4+3x$$

$$-2\sqrt{7-2x}\sqrt{5+x} = 4x-8 \quad | ()^2$$

$$4(7-2x)(5+x) = 16x^2 - 64x + 64$$

$$-8x^2 - 12x + 140 = 16x^2 - 64x + 64$$

$$24x^2 - 52x - 76 = 0$$

$$\Delta = 10000$$

$$x_{1,2} = \frac{52 \pm 100}{48} = \begin{cases} 19/6 \\ -1 \end{cases}$$

$$\text{vérif: } \sqrt{\frac{2}{3}} - \sqrt{\frac{49}{6}} \stackrel{?}{=} \sqrt{\frac{27}{2}} \quad \text{à la m.â.c. } \times$$

$$\sqrt{3} - \sqrt{4} \stackrel{?}{=} \sqrt{1} \quad \checkmark$$

$$\Rightarrow S = \{-1\}$$

$$f) \quad \sqrt{11+8x} + 1 = \sqrt{9+4x} \quad | ()^2$$

$$11+8x + 2\sqrt{11+8x} + 1 = 9+4x$$

$$2\sqrt{11+8x} = -4x-3 \quad | ()^2$$

$$4(11+8x) = 16x^2 + 24x + 9$$

$$16x^2 - 8x - 35 = 0$$

$$\Delta = 2304 = 48^2$$

$$x_{1,2} = \frac{8 \pm 48}{32} = \begin{cases} 7/4 \\ -5/4 \end{cases}$$

$$\text{vérif: } \sqrt{25} + 1 \stackrel{?}{=} \sqrt{16} \Leftrightarrow 6 \neq 4 \quad \times$$

$$\sqrt{1} + 1 \stackrel{?}{=} \sqrt{4} \Leftrightarrow 2 = 2 \quad \checkmark$$

$$\Rightarrow S = \{-5/4\}$$

Ex 2.5.15

$$a) \quad 3\sqrt{2x-3} + 2\sqrt{7-x} = 11 \quad | ()^2$$

$$9(2x-3) + 12\sqrt{2x-3}\sqrt{7-x} + 4(7-x) = 121$$

$$12\sqrt{2x-3}\sqrt{7-x} = -14x + 120 \quad | \div 2 \quad \otimes$$

$$6\sqrt{2x-3}\sqrt{7-x} = -7x + 60 \quad | ()^2$$

$$36(2x-3)(7-x) = 49x^2 - 840x + 3600$$

$$121x^2 - 1452x + 4356 = 0 \quad \Delta = 0 \quad (\text{P.R.})$$

$$(11x - 66)^2 = 0$$

$$x = 6 \quad \text{vérif: } 3\sqrt{9} + 2\sqrt{1} \stackrel{?}{=} 11 \Leftrightarrow 9+2=11 \quad \checkmark \Rightarrow S = \{6\}$$

$$\textcircled{*} \text{ variante sans } \div 2 : 144(2x-3)(7-x) = 196x^2 - 3360x + 14'400$$

$$484x^2 - 5808x + 17'424 = 0 \quad \Delta = 0 \text{ (P.R.)}$$

$$(22x - 132)^2 = 0 \Leftrightarrow x = 6$$

Ex 2.5.16

$$a) \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$$

$$\sqrt{x+1} + \sqrt{x-1} = 3(\sqrt{x+1} - \sqrt{x-1})$$

$$\begin{array}{lcl} 4\sqrt{x-1} & = & 2\sqrt{x+1} \\ 2\sqrt{x-1} & = & \sqrt{x+1} \end{array} \quad \left| \begin{array}{l} \div 2 \\ ()^2 \end{array} \right.$$

$$4(x-1) = x+1$$

$$3x = 5$$

$$x = \frac{5}{3}$$

$$\text{vérif: } \frac{\sqrt{\frac{8}{3}} + \sqrt{\frac{2}{3}}}{\sqrt{\frac{8}{3}} - \sqrt{\frac{2}{3}}} \stackrel{?}{=} 3 \quad \text{à la m.à.c. } \checkmark$$

$$\Rightarrow S = \left\{ \frac{5}{3} \right\}$$

Ex 2.5.21

pour éliminer
pour z

$$o) \begin{cases} 2x + y - z = 1 \\ x + 2y + z = 8 \\ 3x - y + 2z = 7 \end{cases} \quad \left| \begin{array}{c|c} 1 & 2 \\ 1 & \\ & 1 \end{array} \right| \Leftrightarrow \begin{cases} 2x + y - z = 1 \\ 3x + 3y = 9 \\ 7x + y = 9 \end{cases} \quad \left| \begin{array}{c} \\ -\frac{1}{3} \\ 1 \end{array} \right|$$

$$\begin{cases} 2x + y - z = 1 \\ 7x + y = 9 \\ 6x = 6 \end{cases} \Leftrightarrow \begin{cases} z = 2 + 2 - 1 = 3 \\ y = 9 - 7 = 2 \\ x = 1 \end{cases} \Rightarrow S = \{1; 2; 3\}$$

$$v) \begin{cases} 2x + y + 3z = 3 & | \text{ 1 } \\ 3x - y + 4z = 2 & | \text{ 1 } \\ 4x + y - z = 5 & | \text{ 1 } \\ x + y + z = 4 \end{cases}$$

4 équations mais 3 inconnues

(on résout 3x3 puis on vérifie dans la 4^e équation

si ok ...
si non $S = \emptyset$)

$$\Leftrightarrow \begin{cases} 2x + y + 3z = 3 \\ 5x + 7z = 5 & | \text{ 3 } \\ 7x + 3z = 7 & | \text{ -7 } \\ x + y + z = 4 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x + y + 3z = 3 \\ 5x + 7z = 5 \\ -34x = -34 \\ x + y + z = 4 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = -2 \cdot 1 + 3 = 1 \\ 7z = 5 - 5 = 0 \Leftrightarrow z = 0 \\ x = 1 \\ 1 + 1 + 0 \neq 4 \end{cases} \quad \left. \begin{array}{l} \nearrow \\ \nearrow \end{array} \right\} \text{on vérifie}$$

$$\Rightarrow \underline{S = \emptyset}$$

Ex 2.5.22

$$a) \begin{cases} 2xy - 3y = 3 \\ y^2 - 4xy = -15 \end{cases} \Leftrightarrow \begin{cases} 2xy = 3y + 3 \\ y^2 - 2(3y + 3) = -15 \end{cases} \quad \text{substitution}$$

$$\Leftrightarrow \begin{cases} 2xy = 3y + 3 \\ y^2 - 6y + 9 = 0 \end{cases} \Leftrightarrow (y - 3)^2 = 0$$

$$\Leftrightarrow \begin{cases} 2x \cdot 3 = 3 \cdot 3 + 3 \\ y = 3 \end{cases} \Leftrightarrow 6x = 12 \Leftrightarrow x = 2$$

$$\Rightarrow S = \{(2; 3)\}$$

$$b) \begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\ x + y = 30 \end{cases} \quad \text{ED} = \mathbb{R}^* \times \mathbb{R}^*$$

$$\Leftrightarrow \begin{cases} 6x^2 - 6y^2 = 5xy \\ y = 30 - x \end{cases} \Leftrightarrow \begin{cases} 6x^2 - 6(30 - x)^2 = 5x(30 - x) \\ y = 30 - x \end{cases} \quad (1)$$

$$(1) \Leftrightarrow 6x^2 - 6(900 - 60x + x^2) = 150x - 5x^2$$

$$\Leftrightarrow 6x^2 - 5400 + 360x - 6x^2 = 150x - 5x^2$$

$$\Leftrightarrow 5x^2 + 210x - 5400 = 0$$

$$\Leftrightarrow x^2 + 42x - 1080 = 0 \quad \Delta = 6084 = 78^2$$

$$\Leftrightarrow x_{1,2} = \frac{-42 \pm 78}{2} = \begin{cases} 18 \\ -60 \end{cases} \Rightarrow \begin{cases} y_1 = 30 - 18 = 12 \\ y_2 = 30 + 60 = 90 \end{cases}$$

$$\Rightarrow S = \{(18; 12); (-60; 90)\}$$

$$d) \begin{cases} (1) x^2 + y^2 = 34 \\ (2) xy - (x+y) = -13 \end{cases} \Leftrightarrow \begin{cases} x^2 + 2xy + y^2 = 34 + 2xy \\ (x+y) = xy + 13 \end{cases}$$

$$\Leftrightarrow \begin{cases} (x+y)^2 = 34 + 2xy \\ x+y = xy + 13 \end{cases} \quad \text{substitution} \Leftrightarrow \begin{cases} (xy+13)^2 = 34 + 2xy & (3) \\ x+y = xy + 13 & (4) \end{cases}$$

$$(3) \Leftrightarrow (xy)^2 + 26xy + 169 = 34 + 2xy$$

$$\Leftrightarrow (xy)^2 + 24xy + 135 = 0$$

$$\text{On pose } t = xy \Rightarrow t^2 + 24t + 135 = 0 \quad \Delta = 36$$

$$\Rightarrow t_{1,2} = \frac{-24 \pm 6}{2} = \begin{cases} -15 \\ -9 \end{cases}$$

$$t_1 = xy$$

$$\Rightarrow xy = -15 \quad (4) \Rightarrow x+y = -15 + 13 = -2 \Leftrightarrow y = -2-x$$

$$(1)$$

$$\Rightarrow x^2 + (-2-x)^2 = 34 \Leftrightarrow x^2 + 4 + 4x + x^2 = 34$$

$$\Leftrightarrow 2x^2 + 4x - 30 = 0$$

$$\Leftrightarrow x^2 + 2x - 15 = 0$$

$$\Leftrightarrow (x+5)(x-3) = 0$$

$$\Rightarrow x_{1,2} = \begin{cases} -5 \\ 3 \end{cases} \Rightarrow \begin{aligned} y_1 &= -2+5 = 3 \\ y_2 &= -2-3 = -5 \end{aligned}$$

$$t_2 = xy$$

$$\Rightarrow xy = -9 \quad (4) \Rightarrow x+y = -9 + 13 = 4 \Leftrightarrow y = 4-x$$

$$(1)$$

$$\Rightarrow x^2 + (4-x)^2 = 34 \Leftrightarrow x^2 + 16 - 8x + x^2 = 34$$

$$\Leftrightarrow 2x^2 - 8x - 18 = 0$$

$$\Leftrightarrow x^2 - 4x - 9 = 0 \quad \Delta = 52$$

$$\Rightarrow x_{3,4} = \frac{4 \pm \sqrt{52}}{2} = \begin{cases} 2+\sqrt{13} \Rightarrow y = 4-(2+\sqrt{13}) = 2-\sqrt{13} \\ 2-\sqrt{13} \Rightarrow y = 4-(2-\sqrt{13}) = 2+\sqrt{13} \end{cases}$$

$$\Rightarrow S = \{(-5, 3); (3, -5); (2 + \sqrt{13}, 2 - \sqrt{13}); (2 - \sqrt{13}, 2 + \sqrt{13})\}$$