Étude complète d'une parabole :

Étudier la fonction $f(x) = -x^2 + 3x + 4$ $\quad a = -1\ ,\ b = 3\ ,\ c = 4$

1° Convexe ou concave : concave car $a = -1$

2° Ordonnée à l'origine : 4

3° Zéro(s) : $\Delta = 9 - 4 \cdot (-1) \cdot 4 = 25$

$$x_{1,2} = \frac{-3 \pm 5}{-2}$$

$$x_{1,2} = \begin{cases} \frac{2}{-2} = -1 \\ \frac{-8}{-2} = 4 \end{cases}$$

4° Sommet : $S\left(-\frac{b}{2a} ; f\left(-\frac{b}{2a}\right)\right)$

$$x = \frac{-b}{2a} = \frac{-3}{-2} = \frac{3}{2} = 1{,}5$$

$$f\left(\frac{3}{2}\right) = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 4 = -\frac{9}{4} + \frac{9}{2} + 4$$

$$= \frac{-9 + 18 + 16}{4} = \frac{25}{4} = 6{,}25$$

$\Rightarrow S(1{,}5 ; 6{,}25)$

5° Axe de symétrie : $x = 1{,}5$

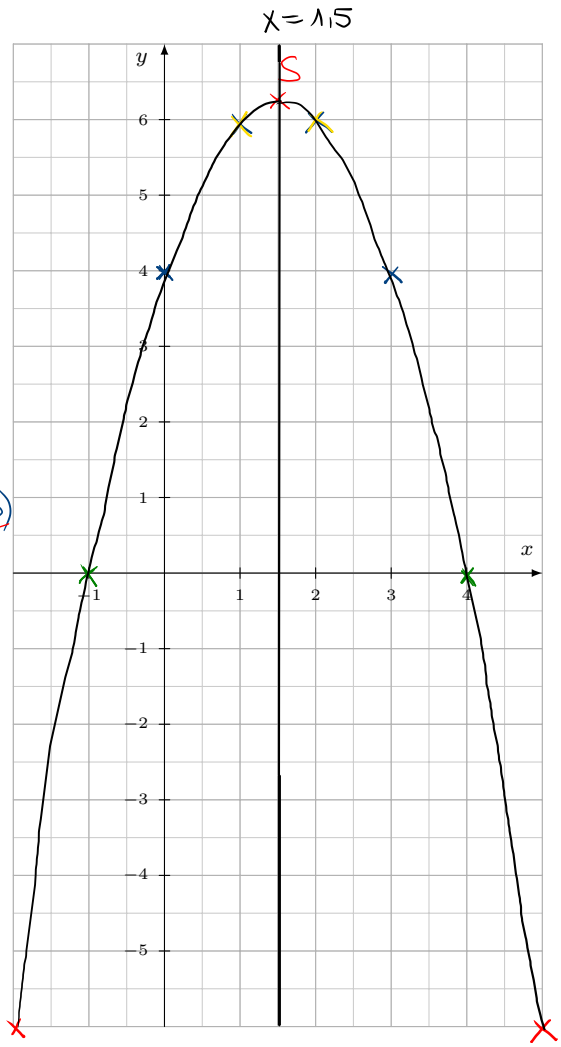
$$x = \frac{-b}{2a}$$

6° Quelques points :

$f(1) = -1 + 3 + 4 = 6 = f(2)$ par symétrie

$f(-2) = -4 - 6 + 4 = -6 = f(5)$

7° Graphe :



Questions complémentaires :

- Le point $(2; 5)$ est-il situé sur la parabole d'équation $y = -x^2 + 3x + 4$?

  non car $f(2) = 6 \neq 5$

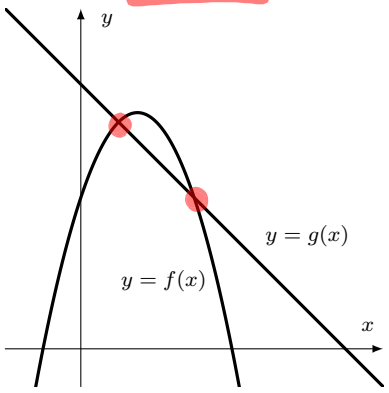
- Compléter les coordonnées du point suivant appartenant à la parabole f : $(2; \ldots 6 \ldots)$

- Compléter les coordonnées du point suivant appartenant à la parabole f : $(\ldots -2 \ldots ; -6)$

  ou $(5 ; -6)$

Position relative d'une droite et d'une parabole

Sécantes



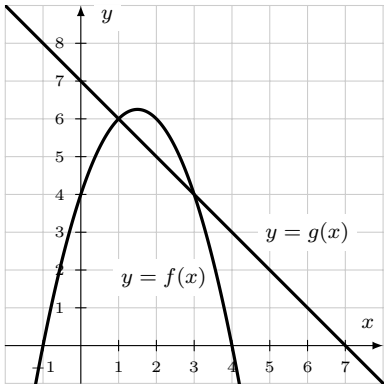
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta > 0$

Deux points d'intersection

Exemple :



$f(x) = -x^2 + 3x + 4$ et $g(x) = -x + 7$

$-x^2+3x+4 = -x+7$

$-x^2+4x-3 = 0 \quad |\cdot(-1)$

$x^2-4x+3 = 0$

$\Delta = 16-4\cdot 3 = 4 > 0$

$\Rightarrow x_{1,2} = \frac{4 \pm 2}{2} \begin{cases} \frac{6}{2} = 3 \\ \frac{2}{2} = 1 \end{cases}$

(1ère coords des points d'intersection)

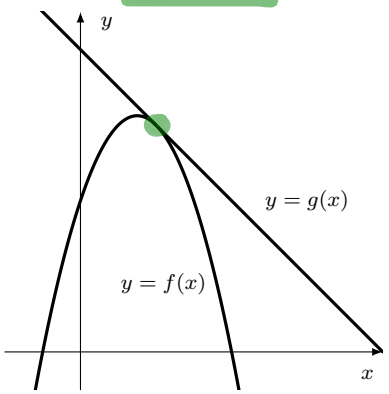
Si $x=3 \Rightarrow g(3) = -3+7 = 4 = y$

$\Rightarrow I_1(3;4)$

Si $x=1 \Rightarrow g(1) = -1+7 = 6 = y$

$\Rightarrow I_2(1;6)$

Tangentes



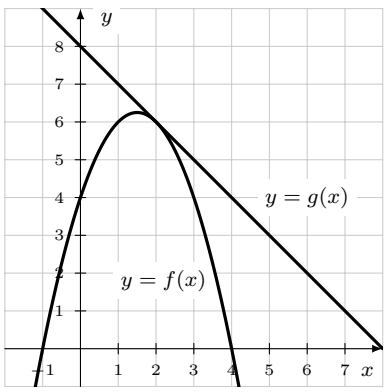
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta = 0$

Un point de tangence

Exemple :



$f(x) = -x^2 + 3x + 4$ et $g(x) = -x + 8$

$-x^2+3x+4 = -x+8$

$-x^2+4x-4 = 0$

$x^2-4x+4 = 0$

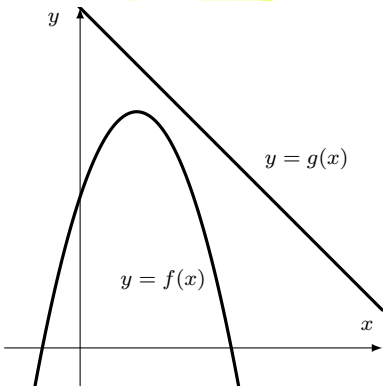
$\Delta = 16-4\cdot 4 = 0$

$\Rightarrow x = \frac{+4}{2} = 2$

Si $x=2 \Rightarrow g(2) = -2+8 = 6$

$\Rightarrow I(2;6) = T(2;6)$

Disjointes



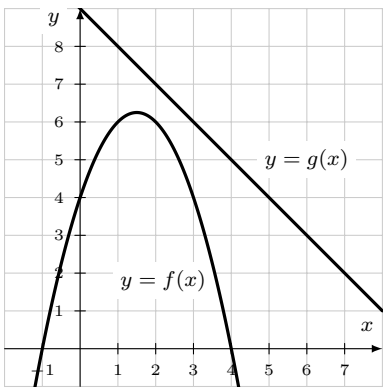
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta < 0$

Aucun point d'intersection

Exemple :



$f(x) = -x^2 + 3x + 4$ et $g(x) = -x + 9$

$-x^2+3x+4 = -x+9$

$-x^2+4x-5 = 0$

$x^2-4x+5 = 0$

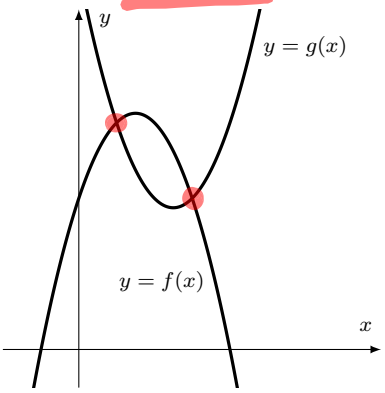
$\Delta = 16-4\cdot 5 = -4 < 0$

$\Rightarrow$ pas de point d'intersection

Position relative de deux paraboles

Note : on se contentera d'étudier 3 cas :

Sécantes



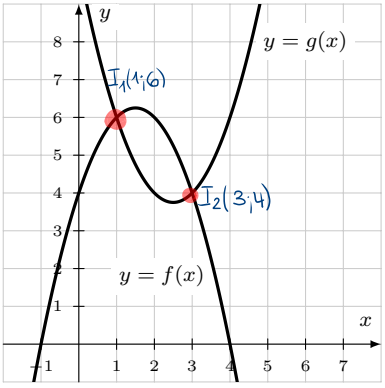
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta > 0$

Deux points d'intersection

Exemple :



$f(x) = -x^2 + 3x + 4$

$g(x) = x^2 - 5x + 10$

$-x^2+3x+4 = x^2-5x+10$

$-2x^2+8x-6 = 0 \quad | :(-2)$

$x^2-4x+3 = 0$

$\Delta = 16-4\cdot 3 = 4$

$x_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} 1 \\ 3 \end{cases}$

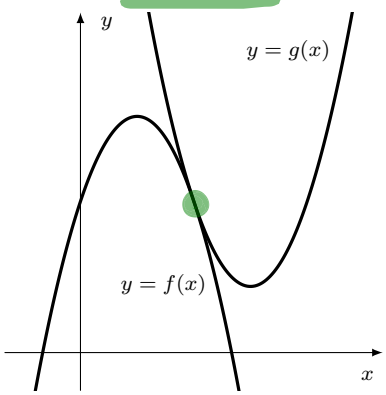
si $x=1 \Rightarrow y = f(1) = -1+3+4 = 6$

$I_1(1;6)$

Si $x=3 \Rightarrow y = f(3) = -9+9+4 = 4$

$I_2(3;4)$

Tangentes



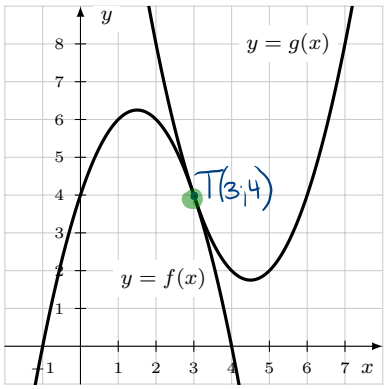
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta = 0$

Un point de tangence

Exemple :



$f(x) = -x^2 + 3x + 4$

$g(x) = x^2 - 9x + 22$

$-x^2+3x+4 = x^2-9x+22$

$-2x^2+12x-18 = 0 \quad | \div(-2)$

$x^2-6x+9 = 0$

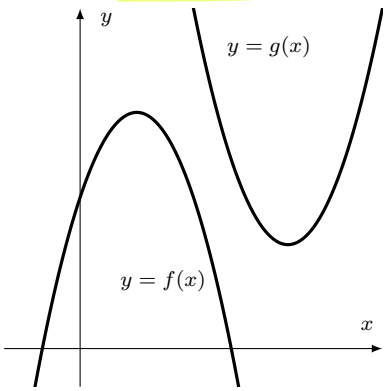
$\Delta = 36-4\cdot 1\cdot 9 = 0$

$x = +\frac{+6}{2} = 3$

si $x=3 \Rightarrow y = f(3) = -9+9+4 = 4$

$\Rightarrow T(3;4)$

Disjointes



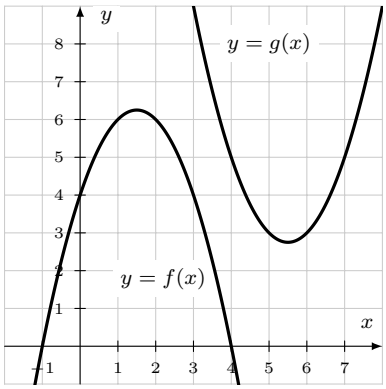
Résoudre $f(x) = g(x)$

$\vdots$

$\Delta < 0$

Aucun point d'intersection

Exemple :



$f(x) = -x^2 + 3x + 4$

$g(x) = x^2 - 11x + 33$

$-x^2+3x+4 = x^2-11x+33$

$-2x^2+14x-29 = 0$

$2x^2-14x+29 = 0$

$\Delta = (-14)^2-4\cdot 2\cdot 29$

$= 196-232 = -36 < 0$

pas de pt d'intersection.