Ex 2.3.1

a) $x^3 - 8x^2 + 16x - 5$ | $x-5$

$-x^3 + 5x^2$ | $x^2 - 3x + 1$

$-3x^2 + 16x - 5$

$+3x^2 \mp 15x$

$x - 5$

$x - 5$

$/$

$\Rightarrow x^3 - 8x^2 + 16x - 5 = (x-5)(x^2-3x+1)$

b) $x^5 - x^4 - 4x^3 + 2x^2 + 3x - 1$ | $x^2 + 2x - 1$

$-x^5 \mp 2x^4 + x^3$ | $x^3 - 3x^2 + 3x - 7$

$-3x^4 - 3x^3 + 2x^2 + 3x - 1$

$+3x^4 + 6x^3 \mp 3x^2$

$3x^3 - x^2 + 3x - 1$

$-3x^3 \mp 6x^2 + 3x$

$-7x^2 + 6x - 1$

$+7x^2 + 14x \mp 7$

$20x - 8$

$\Rightarrow x^5 - x^4 - 4x^3 + 2x^2 + 3x - 1 = (x^2+2x-1)(x^3-3x^2+3x-7) + 20x - 8$

c) $x^4 - 3x^3 + x - 5$ | $x^2 - 3$

$-x^4 + 3x^2$ | $x^2 - 3x + 3$

$-3x^3 + 3x^2 + x - 5$

$+3x^3 \mp 9x$

$3x^2 - 8x - 5$

$-3x^2 + 9$

$-8x + 4$

$\Rightarrow x^4 - 3x^3 + x - 5 = (x^2-3)(x^2-3x+3) - 8x + 4$

d)

$$\begin{array}{r|l} 35x^3+47x^2+13x+1 & 5x+1 \\ \hline -35x^3 \mp 7x^2 & 7x^2+8x+1 \\ \hline \quad 40x^2+13x+1 & \\ \quad -40x^2 \mp 8x & \\ \hline \qquad\quad 5x+1 & \\ \qquad\quad -5x \mp 1 & \\ \hline \qquad\qquad 0 & \end{array}$$

$$\Rightarrow \underline{35x^3+47x^2+13x+1=(5x+1)(7x^2+8x+1)}$$

h)

$$\begin{array}{r|l} x^5-3x^3+2x^2+5x & x+2 \\ \hline -x^5 \mp 2x^4 & x^4-2x^3+x^2+5 \\ \hline \quad -2x^4-3x^3+2x^2+5x & \\ \quad +2x^2+4x^3 & \\ \hline \qquad\quad x^3+2x^2+5x & \\ \qquad\quad -x^3 \mp 2x^2 & \\ \hline \qquad\qquad\quad 5x & \\ \qquad\qquad\quad -5x \mp 10 & \\ \hline \qquad\qquad\qquad -10 & \end{array}$$

$$\Rightarrow \underline{x^5-3x^3+2x^2+5x=(x+2)(x^4-2x^3+x^2+5)-10}$$

avec schéma de Horner :

	1	0	-3	2	5	0
-2		-2	4	-2	0	-10
	1	-2	1	0	5	-10

Ex 2.3.2

b)

$$\begin{array}{rl|l}
12x^4+47x^3+10x^2 & +12 & -3x^2-8x+6 \\
\hline
-12x^4\mp 32x^3+24x^2 & & -4x^2-5x+2 \\
\hline
15x^3+34x^2 & +12 & \\
-15x^3\mp 40x^2+30x & & \\
\hline
-6x^2+30x & +12 & \\
+6x^2+16x\mp 12 & & \\
\hline
46x & &
\end{array}$$

$$12x^4+47x^3+10x^2+12 = (-3x^2-8x+6)(-4x^2-5x+2)+46x$$

Ex 2.3.3

$$(x-5)\cdot p = x^3-3x^2-4x-30$$

$$p = (x^3-3x^2-4x-30)\div(x-5)$$

Avec le schéma de Horner :

	1	-3	-4	-30
5		5	10	30
	1	2	6	0

$$\Rightarrow \quad p = x^2+2x+6$$

Ex 2.3.4

$$\begin{aligned}
p &= (2x^2+1)(5x^2-3x+1)+1-x \\
&= 10x^4-6x^3+2x^2+5x^2-3x+1+1-x \\
&= 10x^4-6x^3+7x^2-4x+2
\end{aligned}$$

Ex 2.3.5

$p(x) = 2x^3 - 3x^2 + 5x - 7$

$p(1) = 2 - 3 + 5 - 7 = -3$

$p(3) = 2 \cdot 27 - 3 \cdot 9 + 15 - 7 = 35$

$p(0) = -7$

$p(-2) = 2(-8) - 3 \cdot 4 - 5 \cdot 2 - 7 = -16 - 12 - 10 - 7 = -45$

$p(-3) = 2 \cdot (-27) - 3 \cdot 9 + 5(-3) - 7 = -54 - 27 - 15 - 7 = -103$

$p\left(\frac{1}{3}\right) = 2 \cdot \frac{1}{27} - 3 \cdot \frac{1}{9} + 5 \cdot \frac{1}{3} - 7 = \frac{2}{27} - \frac{1}{3} + \frac{5}{3} - 7 = -\frac{151}{27}$

$p\left(-\frac{1}{2}\right) = 2 \cdot \left(-\frac{1}{8}\right) - 3 \cdot \frac{1}{4} + 5\left(-\frac{1}{2}\right) - 7 = -\frac{1}{4} - \frac{3}{4} - \frac{5}{2} - 7 = -\frac{21}{2}$

Ex 2.3.6

$$
\begin{array}{l|l}
t^5 - 7t^4 \quad -t^2 - 9t + 9 & t^2 - 5t + 4 \\
\cline{2-2}
-t^5 + 5t^4 - 4t^3 & t^3 - 2t^2 - 14t - 63 \\
\cline{1-1}
\quad -2t^4 - 4t^3 - t^2 & \\
\quad +2t^4 - 10t^3 + 8t^2 & \\
\cline{1-1}
\quad\quad -14t^3 + 7t^2 - 9t & \\
\quad\quad +14t^3 - 70t^2 + 56t & \\
\cline{1-1}
\quad\quad\quad -63t^2 + 47t + 9 & \\
\quad\quad\quad +63t^2 - 315t + 252 & \\
\cline{1-1}
\quad\quad\quad\quad -268t + 261 &
\end{array}
$$

$\Rightarrow$ égalité fondamentale :

$$t^5 - 7t^4 - t^2 - 9t + 9 = (t^2 - 5t + 4)(t^3 - 2t^2 - 14t - 63) - 268t + 261$$

Variante en utilisant 2 fois Horner : $t^2-5t+4 = (t-4)(t-1)$

	1	−7	0	−1	−9	9
4		4	−12	−48	−196	−820
	1	−3	−12	−49	−205	−811
1		1	−2	−14	−63	
	1	−2	−14	−63	−268	

$$t^5-7t^4-t^2-9t+9 = (t-4)(t^4-3t^3-12t^2-49t-205) - 811$$
$$= (t-4)\left[(t-1)(t^3-2t^2-14t-63) - 268\right] - 811$$
$$= (t-4)(t-1)(t^3-2t^2-14t-63) - (t-4)(268) - 811$$
$$= (t-4)(t-1)(t^3-2t^2-14t-63) - 268t + 261$$

Ex 2.3.7

$P(x) = ax^2 + bx + c$

$P(1) = -2 \iff a+b+c = -2$

$P(-2) = 3 \iff 4a - 2b + c = 3$

$P(0) = -1 \iff c = -1$

$$\Rightarrow \begin{cases} a+b+c = -2 \\ 4a-2b+c = 3 \\ c = -1 \end{cases}$$

$$\iff \begin{cases} a+b-1 = -2 \\ 4a-2b-1 = 3 \\ c = -1 \end{cases} \iff \begin{cases} a+b = -1 \\ 4a-2b = 4 \\ c = -1 \end{cases} \iff \begin{cases} a+b = -1 \quad \mid 1 \\ 2a-b = 2 \quad \mid 1 \\ c = -1 \end{cases}$$

$$\iff \begin{cases} a+b = -1 \\ 3a = 1 \\ c = -1 \end{cases} \iff \begin{cases} a = 1/3 \\ b = -1 - 1/3 = -4/3 \\ c = -1 \end{cases} \Rightarrow P(x) = \frac{1}{3}x^2 - \frac{4}{3}x - 1$$

Ex 2.3.8

a) $r(x) = a(1) = 4 - 10 + 11 - 5 = 0$

b) $r(x) = a(-2) = 9 \cdot 16 - 8 - 4 - 2 + 2 = 132$

c) $r(x) = a(0) = -7$

Ex 2.3.9

a) $r(x) = A(3) = 2 \cdot 27 - 9 + 15 - 1 = 59$

b) $r(x) = A(-2) = 16 + 2 + 1 = 19$

c) $r(x) = A(3) = 27 - 27 = 0$

Ex 2.3.10

a)

	1	−3	0	4
2		2	−2	−4
	1	−1	−2	0
−1		−1	2	
	1	−2	0	

$\Rightarrow$ quotient : $x - 2$

b)

	9	9	−7	−9	−2
−1		−9	0	7	2
	9	0	−7	−2	0
−1/3		−3	1	2	
	9	−3	−6	0	

$\Rightarrow 9x^4 + 9x^3 - 7x^2 - 9x - 2$

$= (x+1)(x+\frac{1}{3})(9x^2 - 3x - 6)$

$= (x+1)(3x+1)(3x^2 - x - 2)$

$\Rightarrow$ quotient : $3x^2 - x - 2$

Ex 2.3.11

$$P(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$$

a) $P(1) = 1 + 2 - 16 - 2 + 15 = 0 \Rightarrow$ $P(x)$ est divisible par $x-1$

b) $P(-4) \neq 0$ car -4 n'est pas un diviseur de 15

$\Rightarrow$ $P(x)$ n'est pas divisible par $x+4$

c) $P\left(-\frac{1}{2}\right) = \frac{1}{16} - 2\cdot\frac{1}{8} - 16\cdot\frac{1}{4} + 2\cdot\frac{1}{2} + 15 = \frac{1}{16} - \frac{1}{4} - 4 + 1 + 15 = \frac{189}{16} \neq 0$

$\Rightarrow$ $P(x)$ n'est pas divisible par $x+\frac{1}{2}$

d) $P(-1) = 1 - 2 - 16 + 2 + 15 = 0 \Rightarrow$ $P(x)$ est divisible par $x+1$

e) $P(-5) = 625 - 2\cdot 125 - 16\cdot 25 + 10 + 15 = 0 \Rightarrow$ $P(x)$ est divisible par $x+5$

f) $P(3) = 81 + 2\cdot 27 - 16\cdot 9 - 2\cdot 3 + 15 = 0 \Rightarrow$ $P(x)$ est divisible par $x-3$

$\Rightarrow$ $P(x) = (x-1)(x+1)(x+5)(x-3)$

Ex 2.3.12

a) $P(x) = 2x^3 - 14x + 12$ candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$P(1) = 2 - 14 + 12 = 0 \Rightarrow$ 1 est un zéro

$P(2) = 16 - 28 + 12 = 0 \Rightarrow$ 2 est un zéro

$P(3) = 54 - 42 + 12 \neq 0$

$P(-3) = -54 + 42 + 12 = 0 \Rightarrow$ -3 est un zéro

b) $P(x) = x^4 - 6x^3 + x - 6$ candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 1 - 6 + 1 - 6 \neq 0$

$P(-1) = 1 + 6 - 1 - 6 = 0 \Rightarrow$ -1 est un zéro

$P(2) = 16 - 48 + 2 - 6 \neq 0$

$P(-2) = 16 + 48 - 2 - 6 \neq 0$

$P(3) = 81 - 162 + 3 - 6 \neq 0$

$P(-3) \neq 0$

$P(6) = 1296 - 1296 + 6 - 6 = 0 \Rightarrow$ 6 est un zéro

$P(-6) \neq 0$

Variante : $x^4 - 6x^3 + x - 6 \overset{GR}{=} x^3(x-6) + (x-6) = (x-6)(x^3+1)$

$= (x-6)(x+1)(x^2-x+1)$

$6 \quad -1 \quad \Delta < 0$

Ex 2.3.13

a) $x^3 + mx + n$ est divisible par $(x-1)(x+2)$

$\Leftrightarrow p(1) = 0$ et $p(-2) = 0$

$\Leftrightarrow \begin{cases} 1 + m + n = 0 \\ -8 - 2m + n = 0 \end{cases} \Leftrightarrow + \begin{cases} m + n = -1 \\ 2m - n = -8 \end{cases}$

$3m = -9$

$m = -3 \Rightarrow n = -1 - (-3) = 2$

b) $x^3 + mx^2 + n$ est divisible par $x^2 - x - 6 = (x-3)(x+2)$

$\Leftrightarrow p(3) = 0$ et $p(-2) = 0$

$\Leftrightarrow \begin{cases} 27 + 9m + n = 0 \\ -8 + 4m + n = 0 \end{cases} \Leftrightarrow + \begin{cases} 9m + n = -27 \\ -4m - n = -8 \end{cases}$

$5m = -35$

$m = -7$

$\Rightarrow n = 8 - 4 \cdot (-7) = 36$

2.3.14 Je suis un polynôme de degré 5 et possède les propriétés suivantes :

- je m'annule en 0 et en 2, $X(X-2)\dots$
- je suis divisible par $x+2$, $(X+2)\dots$
- $x-3$ apparaît dans ma factorisation, $(X-3)\dots$

$\Rightarrow P(X) = X(X-2)(X+2)(X-3)(aX+b)$

- je suis le reste de la division par $x+3$ est égal à -630,
- mon évaluation en $x=1$ est égale à 6.

- $P(-3) = -630 \Leftrightarrow -3\cdot(-5)\cdot(-1)(-6)(-3a+b) = -630$

$\Leftrightarrow 90(-3a+b) = -630$

$\Leftrightarrow -3a+b = -7$

- $P(1) = 6 \Leftrightarrow 1\cdot(-1)\cdot 3\cdot(-2)(a+b) = 6$

$\Leftrightarrow 6(a+b) = 6$

$\Leftrightarrow a+b = 1$

$$\Rightarrow \begin{cases} -3a+b = -7 \\ a+b = 1 \end{cases} \Big| -1 \Rightarrow \begin{array}{r} 3a-b = 7 \\ +\ a+b = 1 \\ \hline 4a = 8 \\ a = 2 \end{array}$$

$\Rightarrow 2+b = 1 \Leftrightarrow b = -1$

$\Rightarrow P(X) = X(X-2)(X+2)(X-3)(2X-1)$

Ex 2.3.15

a) $P(X) = 2X^4 - 3X^3 - 35X^2 - 9X + 45$, $\underbrace{P(5)=0}_{\text{divisible par } X-5}$ et $\underbrace{P(-3)=0}_{\text{divisible par } X+3}$

	2	−3	−35	−9	45
5		10	35	0	−45
	2	7	0	−9	0
−3		−6	−3	9	
	2	1	−3	0	

$\Rightarrow P(X) = (X-5)(X+3)(2X^2+X-3)$ $\Delta = 25 \Rightarrow X_{1,2} = \frac{-1\pm 5}{4} = \begin{cases} 1 \\ -\frac{3}{2} \end{cases}$

$= (X-5)(X+3)\cdot 2\left(X-1\right)\left(X+\frac{3}{2}\right)$

$= (X-5)(X+3)(X-1)(2X+3)$

b) $P(x) = 2x^4 - 9x^3 + 7x^2 + 6x$

2 est une sol. de $P(x) = 0 \iff P(2) = 0 \iff P(x)$ est divisible par $x-2$

	2	−9	7	6
2		4	−10	−6
	2	−5	−3	0

$\Rightarrow P(x) = (x-2)(2x^2-5x-3)$ $\quad \Delta = 49 \quad \Rightarrow x_{1,2} = \frac{5 \pm 7}{4} = \begin{cases} 3 \\ -\frac{1}{2} \end{cases}$

$= (x-2) \cdot 2(x-3)(x+\frac{1}{2})$

$= \underline{(x-2)(x-3)(2x+1)}$

Ex 2.3.16

$p(x) = 2x^3 - 8x^2 + 8x \overset{\text{Mee}}{=} 2x(x^2-4x+4) \overset{\text{PR}}{=} \underline{2x(x-2)^2}$

Ex 2.3.17

$2x^4 + 11x^3 + 4x^2 - 29x + 12 = 0$

Il faut d'abord factoriser le polynôme.

Candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$P(1) = 2 + 11 + 4 - 29 + 12 = 0$ ✓

	2	11	4	−29	12
1		2	13	17	−12
	2	13	17	−12	0
−3		−6	−21	12	
	2	7	−4	0	

$P(x) = (x-1)(\underbrace{2x^3+13x^2+17x-12}_{Q(x)})$

candidats : idem

$Q(-2) = 2(-8) + 13 \cdot 4 + 17(-2) - 12 = -10$

$Q(-3) = 2(-27) + 13 \cdot 9 + 17(-3) - 12 = 0$ ✓

$\Rightarrow P(x) = (x-1)(x+3)(\underbrace{2x^2+7x-4}_{\Delta = 81}) = 0 \quad x_{1,2} = \frac{-7 \pm 9}{4} = \begin{cases} -4 \\ \frac{1}{2} \end{cases}$

$\Rightarrow S = \{1; -3; -4; \frac{1}{2}\}$

$\Rightarrow$ les solutions entières sont : $\underline{1, -3 \text{ et } -4}$

Ex 2.3.18

a) $p(x) = x^4 + 2x^3 - 5x^2 - 6x$

$\overset{\text{HEE}}{=} x(x^3 + 2x^2 - 5x - 6)$ candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$p(1) = 1 + 2 - 5 - 6 \neq 0$

$p(-1) = -1 + 2 + 5 - 6 = 0$ ✓

	1	2	−5	−6
−1		−1	−1	6
	1	1	−6	0

$\overset{\text{DIV}}{\Rightarrow} p(x) = x(x+1)(x^2 + x - 6) \overset{\text{SP}}{=} x(x+1)(x+3)(x-2)$

b) $x^5 + 3x^4 - 16x - 48$

$\overset{\text{GR}}{=} x^4(x+3) - 16(x+3)$

$= (x+3)(x^4 - 16)$

$\overset{\text{PR}}{=} (x+3)(x^2+4)(x^2-4)$

$\overset{\text{PR}}{=} (x+3)(x^2+4)(x+2)(x-2)$

c) $6x^4 - 5x^3 - 23x^2 + 20x - 4$ candidats : $\pm 1, \pm 2, \pm 4$

$p(1) = 6 - 5 - 23 + 20 - 4 \neq 0$

$p(-1) = 6 + 5 - 23 - 20 - 4 \neq 0$

$p(2) = 6 \cdot 16 - 5 \cdot 8 - 23 \cdot 4 + 20 \cdot 2 - 4 = 0$ ✓

	6	−5	−23	20	−4
2		12	14	−18	4
	6	7	−9	2	0
−2		−12	10	−2	
	6	−5	1	0	

$\Rightarrow p(x) = (x-2)(\underbrace{6x^3 + 7x^2 - 9x + 2}_{q(x)})$

candidats : $\underbrace{\pm 1}_{\text{Non car déjà essayé}}, \pm 2$

$q(2) = 6 \cdot 8 + 7 \cdot 4 - 9 \cdot 2 + 2 \neq 0$

$q(-2) = -6\cdot 8 + 7\cdot 4 + 9\cdot 2 + 2 = 0$

$\Rightarrow \quad p(x) = (x-2)(x+2)(6x^2-5x+1)$

$\Delta = 25 - 4\cdot 6 = 1$

$x_{1,2} = \dfrac{5 \pm 1}{12} = \begin{cases} 1/2 \\ 1/3 \end{cases}$

$\Rightarrow 6\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right) = (2x-1)(3x-1)$

$\Rightarrow \quad p(x) = (x-2)(x+2)(2x-1)(3x-1)$

Ex 2.3.19

a)

	1	1	1	1	1
1		1	2	3	4
	1	2	3	4	5

$Q(x) = x^3+2x^2+3x+4 \qquad R = 5$

b)

	1	0	0	0	0	1
-1		-1	1	-1	1	-1
	1	-1	1	-1	1	0

$Q(x) = x^4-x^3+x^2-x+1 \qquad R = 0$

c)

	3	-8	7	1	-5	6
-2		-6	28	-70	138	-266
	3	-14	35	-69	135	-260

$Q(x) = 3x^4-14x^3+35x^2-69x+135 \quad$ et $\quad R = -260$

Ex 2.3.20

$p(x) = x^6-6x^5+15x^4-20x^3+15x^2-6x+1$

$p(1) = 1-6+15-20+15-6+1 = 0$ ✓ $\Rightarrow$ divisible par $x-1$

#

Ex 2.3.21

a)
$$\begin{array}{r|l} 3x^3 - x^2 + x + 1 & 2x^2 - 1 \\ \cline{2-2} -3x^3 \quad + \frac{3}{2}x & \frac{3}{2}x - \frac{1}{2} \\ \cline{1-1} -x^2 + \frac{5}{2}x + 1 & \text{quotient} \\ +x^2 \quad - \frac{1}{2} & \\ \cline{1-1} \frac{5}{2}x + \frac{1}{2} & \\ \text{reste} & \end{array}$$

$$\Rightarrow \quad 3x^3 - x^2 + x + 1 = (2x^2 - 1)\left(\frac{3}{2}x - \frac{1}{2}\right) + \frac{5}{2}x + \frac{1}{2}$$

b)
$$\begin{array}{r|l} 2x^3 \quad - 1 & 3x^3 - x^2 + x + 1 \\ \cline{2-2} -2x^3 + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{2}{3} & \frac{2}{3} \\ \cline{1-1} \frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3} & \end{array}$$

$$\Rightarrow \quad 2x^3 - 1 = \frac{2}{3}(3x^3 - x^2 + x + 1) + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3}$$

c)
$$\begin{array}{r|l} 7x^5 - x^4 + 6x^3 \quad - 7x & 7x^3 - x \\ \cline{2-2} -7x^5 \quad + x^3 & x^2 - \frac{1}{7}x + 1 \\ \cline{1-1} -x^4 + 7x^3 \quad - 7x & \\ +x^4 \quad - \frac{1}{7}x^2 & \\ \cline{1-1} 7x^3 - \frac{1}{7}x^2 - 7x & \\ -7x^3 \quad + x & \\ \cline{1-1} -\frac{1}{7}x^2 - 6x & \end{array}$$

$$\Rightarrow \quad 7x^5 - x^4 + 6x^3 - 7x = (7x^3 - x)\left(x^2 - \frac{1}{7}x + 1\right) - \frac{1}{7}x^2 - 6x$$

d)
$$\begin{array}{r|l} 6x^4 + 4x^3 - 7x^2 & 2x^2 - 3 \\ \cline{2-2} -6x^4 \quad + 9x^2 & 3x^2 + 2x + 1 \\ \cline{1-1} 4x^3 + 2x^2 & \\ -4x^3 \quad + 6x & \\ \cline{1-1} 2x^2 + 6x & \\ -2x^2 \quad + 3 & \\ \cline{1-1} 6x + 3 & \end{array}$$

$$\Rightarrow \quad 6x^4 + 4x^3 - 7x^2 = (2x^2 - 3)(3x^2 + 2x + 1) + 6x + 3$$

Ex 2.3.22

$p(x) = 2x^3 + 3x^2 - 11x - 6$

candidats entre 0 et -5 : $-1, -2, -3$

$p(-1) = -2 + 3 + 11 - 6 \neq 0$

$p(-2) = -16 + 12 + 22 - 6 \neq 0$

$p(-3) = -54 + 27 + 33 - 6 = 0$ ✓

	2	3	-11	-6
-3		-6	9	6
	2	-3	-2	0

$\Rightarrow p(x) = (x+3)(2x^2 - 3x - 2)$

$\Delta = 25$ $\quad x_{1,2} = \frac{3 \pm 5}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$

$= (x+3) \cdot 2 (x-2)(x+\frac{1}{2}) = \underline{(x+3)(x-2)(2x+1)}$

Ex 2.3.23

a) $p(x) = x^3 + 9x^2 + 11x - 21$

candidats : $\pm 1, \pm 3, \pm 7, \pm 21$

$p(1) = 1 + 9 + 11 - 21 = 0$ ✓

	1	9	11	-21
1		1	10	21
	1	10	21	0

$p(x) \overset{Div}{=} (x-1)(x^2 + 10x + 21)$

$\overset{SP}{=} \underline{(x-1)(x+3)(x+7)}$

b) $p(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$

candidats : $\pm 1, \pm 3, \pm 5, \pm 15$

$p(1) = 1 + 2 - 16 - 2 + 15 = 0$ ✓

	1	2	-16	-2	15
1		1	3	-13	-15
	1	3	-13	-15	0
-1		-1	-2	15	
	1	2	-15	0	

$\Rightarrow p(x) \overset{Div}{=} (x-1)(\underbrace{x^3 + 3x^2 - 13x - 15}_{q(x)})$

candidats : $\pm 1, \pm 3, \pm 5, \pm 15$

$q(1) = 1 + 3 - 13 - 15 \neq 0$

$q(-1) = -1 + 3 + 13 - 15 = 0$ ✓

$\Rightarrow p(x) \overset{Div}{=} (x-1)(x+1)(x^2 + 2x - 15)$

$\overset{SP}{=} \underline{(x-1)(x+1)(x+5)(x-3)}$

c) $p(x) = x^5 + 3x^4 - 16x - 48$

$\overset{GR}{=} x^4(x+3) - 16(x+3)$

$= (x+3)(x^4 - 16)$

$\overset{PR}{=} (x+3)(x^2+4)(x^2-4)$

$\overset{PR}{=} \underline{(x+3)(x^2+4)(x+2)(x-2)}$

d) $p(x) = x^5 - 3x^4 - 21x^3 + 43x^2 + 96x - 180$

candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6 ; \pm 9 ; \pm 10 ; \pm 12, \pm 15, \pm 20, \ldots$

$p(2) = 0$

	1	−3	−21	43	96	−180
2		2	−2	−46	−6	180
	1	−1	−23	−3	90	0
2		2	2	−42	−90	
	1	1	−21	−45	0	
−3		−3	6	45		
	1	−2	−15	0		

$q(2) = 0$

$q_2(-3) = 0$

$p(x) = (x-2)^2(x+3)(x^2-2x-15) \overset{SP}{=} \underline{(x-2)^2(x+3)^2(x-5)}$

Ex 2.3.24

a) $x^2 + 19x + 18 \overset{SP}{=} \underline{(x+18)(x+1)}$

b) $x^2 - 4x + 4 \overset{PR}{=} \underline{(x-2)^2}$

c) $2x^2 + 5x - 3 \overset{\Delta}{=} 2(x - \frac{1}{2})(x+3) = \underline{(2x-1)(x+3)}$

$\Delta = 49$

$x_{1,2} = \frac{-5 \pm 7}{4} = \begin{cases} \frac{1}{2} \\ -3 \end{cases}$

d) $3x^2 - 5x + 2 \overset{\Delta}{=} 3(x - \frac{2}{3})(x-1) = \underline{(3x-2)(x-1)}$

$\Delta = 1$

$x_{1,2} = \frac{5 \pm 1}{6} = \begin{cases} \frac{2}{3} \\ 1 \end{cases}$

e) $4x^2 - 20x + 25 \overset{PR}{=} \underline{(2x-5)^2}$

f) $x^2-9 \overset{PR}{=} (x+3)(x-3)$

g) $x^2-\frac{4}{9} \overset{PR}{=} \left(x+\frac{2}{3}\right)\left(x-\frac{2}{3}\right)$

h) $9x^2-5x \overset{MEE}{=} x(9x-5)$

i) $8x^2+6x+1 \overset{\Delta}{=} 8\left(x+\frac{1}{2}\right)\left(x+\frac{1}{4}\right)$

$= (2x+1)(4x+1)$

$\Delta = 4$

$x_{1/2} = \frac{-6 \pm 2}{16} = \begin{cases} -\frac{1}{2} \\ -\frac{1}{4} \end{cases}$

j) $\frac{1}{3}x^2-x+4$ pas factorisable car $\Delta = -\frac{13}{3} < 0$

Ex 2.3.25

a) $x^3+2x^2-x-2 = 0$

GR $x^2(x+2)-(x+2) = 0$

$(x+2)(x^2-1) = 0$

PR $(x+2)(x+1)(x-1) = 0$

$-2 \quad -1 \quad 1$

$\Rightarrow \quad S = \{-2; -1; 1\}$

b) $x^3-3x^2-4x+12 = 0$

GR $x^2(x-3)-4(x-3) = 0$

$(x-3)(x^2-4) = 0$

PR $(x-3)(x+2)(x-2) = 0$

$3 \quad -2 \quad 2$

$\Rightarrow \quad S = \{-2; 2; 3\}$

c) $4x^5-12x^4+9x^3 = 0$

MEE $x^3(4x^2-12x+9) = 0$

PR $x^3(2x-3)^2 = 0$

$0 \qquad 2x-3 = 0$

$2x = 3$

$x = 3/2$

$\Rightarrow \quad S = \{0; \frac{3}{2}\}$

d) $16x^3-16x^2-4x+4=0$

$4(4x^3-4x^2-x+1)=0$

GR $4\left[4x^2(x-1)-(x-1)\right]=0$

$4(x-1)(4x^2-1)=0$

PR $4(x-1)(2x+1)(2x-1)=0$

$1 \quad -\frac{1}{2} \quad \frac{1}{2}$

$\Rightarrow \quad S=\left\{-\frac{1}{2};\frac{1}{2};1\right\}$

Ex 2.3.26

a) $x^4+2x^3-4x^2-5x-6=0$

candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$p(2)=16+16-16-10-6=0$ ✓

	1	2	−4	−5	−6
2		2	8	8	6
	1	4	4	3	0
−3		−3	−3	−3	
	1	1	1	0	

$q(x)=x^3+4x^2+4x+3$

candidats : $\pm 1, \pm 3$

$q(-3)=-27+36-12+3=0$ ✓

$\Rightarrow (x-2)(x+3)(x^2+x+1)=0 \quad \Rightarrow \quad S=\{2;-3\}$

$\Delta=-3<0$

b) $x^4-7x^3+18x^2-20x+8=0$

candidats : $\pm 1, \pm 2, \pm 4, \pm 8$

$p(1)=1-7+18-20+8=0$ ✓

	1	−7	18	−20	8
1		1	−6	12	−8
	1	−6	12	−8	0

$\Rightarrow \quad (x-1)(x^3-6x^2+12x-8)=0$

PR $(x-1)(x-2)^3=0 \quad \Rightarrow \quad S=\{1;2\}$

c) $35x^3 + 47x^2 + 13x + 1 = 0$

candidats : ± 1

$p(-1) = -35 + 47 - 13 + 1 = 0$ ✓

	35	47	13	1
-1		-35	-12	-1
	35	12	1	0

$\Rightarrow (x+1)(35x^2+12x+1) = 0$ $\quad \Delta = 4$ $\quad x_{1/2} = \frac{-12 \pm 2}{70} = \begin{cases} -\frac{14}{70} = -\frac{1}{5} \\ -\frac{1}{7} \end{cases}$

$\Rightarrow S = \{-1 ; -\frac{1}{5} ; -\frac{1}{7}\}$

d) $x^3 + 5x^2 - 8x - 48 = 0$

candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm \dots$

$p(3) = 27 + 45 - 24 - 48 = 0$ ✓

	1	5	-8	-48
3		3	24	48
	1	8	16	0

$\Rightarrow (x-3)(x^2+8x+16) = 0$

PR $(x-3)(x+4)^2 = 0$ $\quad \Rightarrow \quad S = \{-4 ; 3\}$

$\downarrow \quad \downarrow$

$3 \quad -4$

Ex 2.3.27

✓ a) $x^3 + 2x^2 + x = x(x^2+2x+1) = x(x+1)^2$

✓ b) $2a^6 - 6a^4 + 6a^2 - 2 = 2(a^6 - 3a^4 + 3a^2 - 1) = 2(a^2-1)^3 = 2(a+1)^3(a-1)^3$

✓ c) $9a^3 - ab^2 = a(9a^2 - b^2) = a(3a+b)(3a-b)$

d) $54a^6 - 2 = 2(27a^6 - 1) = 2(3a^2-1)(9a^4+3a^2+1)$

$= 2(\sqrt{3}a+1)(\sqrt{3}a-1)(9a^4+3a^2+1)$

↳ complétion : $9a^4 + 6a^2 + 1 + 3a^2 - 6a^2$

$= (3a^2+1)^2 - 3a^2$

$= (3a^2+1+\sqrt{3}a)(3a^2+1-\sqrt{3}a)$

$= (3a^2+\sqrt{3}a+1)(3a^2-\sqrt{3}a+1)$

$54a^6 - 2 = 2(\sqrt{3}a+1)(\sqrt{3}a-1)(3a^2+\sqrt{3}a+1)(3a^2-\sqrt{3}a+1)$

✓ e) $1-(x-y)^2 = (1+x-y)[1-(x-y)] = \underline{(1+x-y)(1-x+y)}$

f) $(x^2-1)^2+4x^2 = x^4-2x^2+1+4x^2 = x^4+2x^2+1 = \underline{(x^2+1)^2}$

g) $(-3x+y)^2-(4x-z)^2 = (-3x+y+4x-z)[-3x+y-(4x-z)] = \underline{(x+y-z)(-7x+y+z)}$

✓ h) $x^3+9x^2+11x-21$

candidats: $\pm 1, \pm 3, \pm 7, \pm 21$

$p(1) = 1+9+11-21 = 0$ ✓

	1	9	11	−21
1		1	10	21
	1	10	21	0

$\Rightarrow p(x) = (x-1)(x^2+10x+21) = \underline{(x-1)(x+3)(x+7)}$

i) $xy-9x^3y = xy(1-9x^2) = xy(1+3x)(1-3x)$

✓ j) $x^4+3x^3-8x-24 = x^3(x+3)-8(x+3) = (x+3)(x^3-8) = \underline{(x+3)(x-2)(x^2+2x+4)}$

✓ k) $2a^3b-a^2b^2+b^2-2ab = b(2a^3-a^2b+b-2a)$

$= b[a^2(2a-b)-(-b+2a)]$

$= b(2a-b)(a^2-1) = \underline{b(2a-b)(a+1)(a-1)}$

✓ l) $x^3+8y^3+6x^2y+12xy^2 = x^3+6x^2y+12xy^2+8y^3 = \underline{(x+2y)^3}$

m) $b^3-a^3 = \underline{(b-a)(b^2+ab+a^2)}$

n) $3x^4-18x^3+36x^2-24x = 3x(x^3-6x^2+12x-8) = \underline{3x(x-2)^3}$

✓ o) $8a^4x^3-72b^2x^3-a^4+9b^2 = 8x^3(a^4-9b^2)-(a^4-9b^2)$

$= (a^4-9b^2)(8x^3-1)$

$= \underline{(a^2+3b)(a^2-3b)(2x-1)(4x^2+2x+1)}$

✓ p) $x^3+9x-27-3x^2 = x^3-3x^2+9x-27 = x^2(x-3)+9(x-3) = \underline{(x-3)(x^2+9)}$